

## Outline of how to put a matrix in row-echelon form:

1. First column
  - a. Get a 1 somewhere in the first column by adding a multiple of one row to another (if necessary).
  - b. Swap rows to put the 1 in the upper left-hand corner (if necessary).
  - c. Add multiples of Row 1 to the other rows to make the other numbers in the first column become 0.
2. Second column
  - a. Don't do any operations with Row 1. In fact, completely ignore Row 1 from now on.
  - b. Get a 1 or a 0 in the second column by adding a multiple of one row to another (but don't use Row 1).
  - c. If you have put a 1 in the second column:
    - i. Swap rows (if necessary) to put that 1 in Row 2.
    - ii. Add a multiple of Row 2 to Row 3 to make the entry in column 2, row 3 become a 0.
  - d. If you put a 0 in the second column:
    - i. Swap rows (if necessary) to put that 0 in Row 3.
    - ii. Multiply Row 2 by a constant to make the entry in column 2, row 2 become a 1.
3. Third column
  - a. Multiply Row 3 by a constant to make the entry at row 3, column 3 become a 1.

**Example:** Consider  $\begin{array}{cccc} 0 & 6 & 4 & -12 \\ 3 & 3 & 0 & 9 \\ 2 & 0 & -3 & 10 \end{array}$

**Step 1a)** I need a 1 somewhere in the first column. Looking at the numbers in the first column, I notice that  $3 - 2$  is 1. So I can add  $-1$  times Row 3 to Row 2 and this will put a 1 in Row 2.

$$\begin{array}{cccc} 0 & 6 & 4 & -12 \\ 1 & 3 & 3 & -1 \\ 2 & 0 & -3 & 10 \end{array}$$

**Step 1b)** To put the 1 in the upper left corner, swap Rows 1 and 2.

$$\begin{array}{cccc} 1 & 3 & 3 & -1 \\ 0 & 6 & 4 & -12 \\ 2 & 0 & -3 & 10 \end{array}$$

**Step 1c)** I only need to get rid of the 2 in the lower left corner. To do this I will add  $-2$  times Row 1 to Row 3.

$$\begin{array}{cccc} 1 & 3 & 3 & -1 \\ 0 & 6 & 4 & -12 \\ 0 & -6 & -9 & 12 \end{array}$$

**Step 2b)** Looking at the numbers in the second column, I see that  $6 + (-6)$  is 0. So I can make a 0 in Row 3 by adding 1 times Row 2 to Row 3.

Repeated from  
the other side:

$$\begin{array}{cccc} 1 & 3 & 3 & -1 \\ 0 & 6 & 4 & -12 \\ 0 & -6 & -9 & 12 \end{array}$$

**Step 2b)** Looking at the numbers in the second column, I see that  $6 + (-6)$  is 0. So I can make a 0 in Row 3 by adding 1 times Row 2 to Row 3.

$$\begin{array}{cccc} 1 & 3 & 3 & -1 \\ 0 & 6 & 4 & -12 \\ 0 & 0 & -5 & 0 \end{array}$$

**Step 2c)** doesn't apply because I put a 0 in column 2, not a 1.

**Step 2d)** I need to multiply row 2 by  $1/6$  because that will make the 6 become a 1.

$$\begin{array}{cccc} 1 & 3 & 3 & -1 \\ 0 & 1 & 2/3 & -2 \\ 0 & 0 & -5 & 0 \end{array}$$

**Step 3a)** Multiply Row 3 by  $-1/5$  to make the  $-5$  a 1.

$$\begin{array}{cccc} 1 & 3 & 3 & -1 \\ 0 & 1 & 2/3 & -2 \\ 0 & 0 & 1 & 0 \end{array}$$

Done. This matrix is in row-echelon form.