Name:

Directions: Show all work and clearly mark your final answers. All your work should be on these exam pages (do not use scrap paper). You may use a calculator, but you should indicate where you used a calculator to do complicated calculations. You should give an exact numerical answer unless otherwise specified. The time limit is 50 minutes.

1. (5 pts) Evaluate $(15 \cdot 7 + 23) \mod 29$.

2. (5 pts) What is the remainder when 500000000000000000000000008 is divided by 9?

3. (10 pts) Albert the alien comes from a planet where there are 13 days in a week, and 400 days in each year (there are no leap years). Albert notices that his 1,000th birthday falls on the second day of the week. What day will his 5,000th birthday fall on? (The days of the week are referred to as the "first day", "second day", ...)

4. (10 pts) Find the greatest common divisor of 9792 and 4464.

5. (a) (7 pts) Give *all* solutions (mod 126) of the congruence $114x \equiv 78 \mod 126$. You may use the fact that $x \equiv 4 \mod 126$ is one of the solutions (it may be the only solution). Justify your answer.

   (b) (7 pts) Give *all* solutions (mod 45) of the congruence $14x \equiv 3 \mod 45$. You may use the fact that $x \equiv 42 \mod 45$ is one of the solutions (it may be the only solution). Justify your answer.

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6. (10 pts) The probability Northwestern wins its next football game is 5%. The probability the Northwestern quarterback throws a touchdown pass in the next game is 35%. If Northwestern's chances of winning are independent of whether or not the Northwestern quarterback throws a touchdown, what is the probability that Northwestern wins its next game *or* the Northwestern quarterback throws a touchdown pass in the next game?

7. (6 pts) There are 4 red books, 4 brown books and 4 green books on a shelf which are identical except for their color. How many distinct arrangements of the books are possible?

8. There are 50 marbles in a bag. 30 are red, 15 are white, 4 are green, and 1 is blue.

   (a) (10 pts) If you randomly draw 8 marbles, one at a time with replacement, what is the probability that you draw at least 7 red marbles? (You will receive full credit if you give a formula which gives the correct answer.)

   (b) (10 pts) If you randomly draw 8 marbles from the bag simultaneously, what is the probability that you draw 5 red marbles, 2 white marbles, and 1 green marble? (You will receive full credit if you give a formula which gives the correct answer.)

   (c) (20 pts) Suppose you are playing a game where you draw two marbles at random from the bag without replacement. You win \$7 if you draw two white marbles, you win \$98 if you draw two green marbles, and you win \$18 if at least one of the marbles you draw is blue. Otherwise you win nothing. If it costs you \$1.50 to play, how much would you expect to be ahead or behind after 200 games?