Name:

Directions: Show all work and clearly mark your final answers. All your work should be on these exam pages (do not use scrap paper). You may use a calculator, but you should indicate where you used a calculator to do complicated calculations. The time limit is 120 minutes.

1. (12 pts) Find the slope and the $x-$intercept of the line whose equation is $3x + 2y = 10$.

2. (8 pts) You are in charge of organizing an intramural softball tournament. 15 teams have registered for the tournament. How many games need to be played so that every team plays all the other teams exactly once?

3. (9 pts) Today, March 12, 2007, is a Monday. Assuming that there are 3 leap years between now and 2017, what day of the week will March 12, 2017 fall on?

4. (12 pts) Consider the two matrices

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 4 & -3 \end{pmatrix} \text{ and } B = \begin{pmatrix} -1 & 0 \\ -2 & -3 \end{pmatrix}.$$

Determine which of the matrix products AB or BA is defined, and find it.

5. (12 pts) Suppose a pharmaceutical company produces a drug that cures a disease, but kills .01% of those who take it. Suppose 350,000 people take the drug. Use normal approximation to estimate the probability that no more than 40 people will die from the drug.

6. (12 pts) Find all solutions (mod 75) to the linear congruence $24x \equiv 48 \mod 75$.

7. (24 pts) Find all solutions to the following systems (write your answers appropriately). If no solutions exist, say so.

(a) (12 pts) $\begin{cases} 5x - 3y = -2 \\ -x + y = 2 \\ -3x - 2y = -19 \end{cases}$

(b) (12 pts) $\begin{cases} x + 3y - 5z = 18 \\ -3x + 2y - 2z = 2 \\ 2x + y - 2z = 10 \end{cases}$

8. (16 pts) Consider the number

$$x = 123412341234123412341234123412341234123412341234123412341234$$

(the sequence of digits 1234 is written 15 times to form this number). Find the remainder when x is divided by each of the following numbers:

(a) 100

(b) 9

(c) 11

(d) 4

9. (15 pts) True or False? Circle your answer:
   (a) TRUE   FALSE 15^{-1} mod 88 exists.

   (b) TRUE   FALSE   To say two events are independent means that they cannot both happen.

   (c) TRUE   FALSE   A linear system of 12 equations in 7 variables can have exactly 3 solutions.

   (d) TRUE   FALSE   For any two events E and F, $P(E \bigcup F) = P(E) + P(F)$.

   (e) TRUE   FALSE   If you play a one-player game 100 times where the expected value is greater than the cost, you are guaranteed to win money.

10. (25 pts) Consider the following list of numbers:

$$23, 24, 26, 26, 26, 28, 28, 28, 29, 1500$$

(a) (7 pts) Find the standard deviation of the list (rounded to three decimal places).

(b) (8 pts) Find the centralized average of the list.

(c) (10 pts) Which is a better measure of central tendency to use for this list, the *mean* or the *median*? Why?

11. (20 pts) Consider the two-player game whose payoff matrix is

$$M = \begin{pmatrix} 1 & -4 \\ -2 & 2 \end{pmatrix}.$$

(a) (7 pts) Is this game strictly determined? Why or why not?

(b) (13 pts) Find the optimum strategy for the first player (i.e. the player who chooses a row).

12. (35 pts) A bag contains 25 marbles, of which 14 are red. 6 of the marbles are green and the remaining 5 marbles are blue.

   (a) (5 pts) If you draw one marble from the bag, what is the probability that you do not draw a green marble?

   (b) (10 pts) Suppose you perform an experiment in which you draw two marbles from the bag without replacement. Are the events "drawing at least one green marble" and "drawing at least one blue marble" independent? Why or why not? (You may justify your answer either with calculations or heuristic reasoning.)

   (c) (10 pts) Suppose you draw 5 marbles from the bag simultaneously. What is the probability that you draw 4 red marbles and 1 blue marble? (You will receive full credit if you give a formula which gives the correct answer.)

   (d) (10 pts) Suppose you draw 10 marbles from the bag, one at a time and putting each marble back after you draw it. What is the probability you draw 10 marbles of the same color? (You will receive full credit if you give a formula which gives the correct answer.)