1. Clearly $n = 4$, and we are given $\Sigma x^2 = 182$. Next find the average of the list:

$$\overline{x} = \frac{3+4+6+11}{4} = \frac{24}{4} = 6.$$

Finally, the standard deviation $s$ is given by

$$s = \sqrt{\frac{\Sigma x^2 - n\overline{x}^2}{n-1}} = \sqrt{\frac{182 - 4(6^2)}{3}} = \sqrt{\frac{38}{3}}.$$

2. Let $X$ represent the lifespan of a frog. First find the $z-$score for the data point $x = 22$. This is $z = (x - \mu)/\sigma = (22 - 28)/8 = -6/8 = -0.75$. Now, we have

$$\begin{aligned} P(X \geq 22) &= 1 - P(X \leq 22) \\ &= 1 - \text{area under the standard normal to the left of } z = -0.75 \\ &= 1 - .2266 \quad \text{(from chart)} \\ &= .7734. \end{aligned}$$

3. (a) Let $E$ be the event that you draw a pink golf ball. The question is asking for $P(E')$, the probability that you do not draw a pink golf ball, which is

$$P(E') = 1 - P(E) = 1 - \frac{5}{100} = \frac{95}{100} = \frac{19}{20}.$$

(b) Let $Y$ be the event that you draw a yellow golf ball and let $T$ be the event that you draw a Titleist golf ball. If $Y$ and $T$ are independent, then $P(Y \cap T) = P(Y)P(T)$. Check to see whether this holds:

$$\begin{aligned} P(Y) &= \frac{10+10}{100} = \frac{1}{5}. \\ P(T) &= \frac{20+10+15+5}{100} = \frac{1}{2}. \\ P(Y \cap T) &= \frac{10}{100} = \frac{1}{10}. \end{aligned}$$

We see that $\frac{1}{10} = \frac{1}{5} \cdot \frac{1}{2}$, so $P(Y \cap T) = P(Y)P(T)$ and therefore $Y$ and $T$ are independent.

(c) This probability is the following fraction:

$$P = \frac{\#\text{ of groups of 1 orange, 2 white and 2 yellow golf balls}}{\text{total }\#\text{ of groups of five golf balls taken from the 100}}.$$

First notice there are $\binom{100}{5}$ groups of five golf balls (this is the denominator of the above fraction). To find the numerator, count the numbers of groups of orange, white and yellow golf balls separately and multiply. There are $\binom{15}{1}$ ways to choose one orange golf ball from the 15 orange ones, there are $\binom{60}{2}$ ways to choose two white golf balls from the 60 white ones, and there are $\binom{20}{2}$ ways to choose two yellow golf balls from the 20 yellow ones. So the total number of groups of five balls with the correct numbers of each color is

$$\binom{15}{1}\binom{60}{2}\binom{20}{2}$$

and the answer is therefore

$$\frac{\binom{15}{1}\binom{60}{2}\binom{20}{2}}{\binom{100}{5}}.$$

(d) Because you replace each ball after it is drawn, the probability of drawing an orange ball on any draw is always $p = \frac{15}{100} = .15$. So this is a binomial experiment with $n = 7$ and $k = 2$; the answer is therefore

$$P = \binom{n}{k} p^k (1-p)^{n-k} = \binom{7}{2} (.15)^2 (.85)^5.$$

(e) Divide the balls into four groups:

- $A$, the white Nike golf balls (there are 25 of these);
- $B$, the non-white Nike golf balls (there are 10 of these);
- $C$, the white non-Nike golf balls (there are $20 + 15 = 35$ of these); and
- $D$, the non-white and non-Nike golf balls (there are $10 + 15 + 5 = 30$ of these).

Now, to draw (exactly) two white and (exactly) two Nike balls in the three draws means one of two things has happened:

i. You drew two golf balls from group $A$ and one from group $D$, or
ii. You drew one golf ball from each of the groups $A$, $B$ and $C$.

These cases are disjoint, so you can find the probabilities of them separately (using similar reasoning as in part (c) of this problem) and add them to get the answer:

$$P(\text{Case i}) = \frac{\binom{25}{2}\binom{30}{1}}{\binom{100}{3}}.$$

$$P(\text{Case ii}) = \frac{\binom{25}{1}\binom{10}{1}\binom{35}{1}}{\binom{100}{3}}.$$

$$\begin{aligned} \text{Answer} &= P(\text{Case i}) + P(\text{Case ii}) \\ &= \frac{\binom{25}{2}\binom{30}{1} + \binom{25}{1}\binom{10}{1}\binom{35}{1}}{\binom{100}{3}}. \end{aligned}$$

4. (a) The total area under the density function and above the $x-$axis must be 1; this region is a trapezoid so we have

$$A = \frac{1}{2}h(a+b) = \frac{1}{2}c(5+1) = 3c = 1;$$

solve this to get $c = \frac{1}{3}$.

(b) This probability is the area under the density function from $x = 4$ to $x = 6$; this region is a triangle so the area is

$$A = \frac{1}{2}bh = \frac{1}{2}(2)\frac{1}{3} = \frac{1}{3}.$$

(c) The events "$X \leq 2$" and "$X > 4$" are disjoint, so their probabilities add and we have

$$P(X \leq 2 \text{ or } X > 4) = P(X \leq 2) + P(X > 4).$$

From part (b) of this problem, we know $P(X > 4) = \frac{1}{3}$; to find $P(X \leq 2)$ we need to find the area under the density function from $x = 0$ to $x = 2$; this region is a triangle with base length 1. The height can be found using similar triangles:

$$\frac{H}{B} = \frac{h}{b} \Rightarrow \frac{1/3}{2} = \frac{h}{1} \Rightarrow 2h = \frac{1}{3} \Rightarrow h = \frac{1}{6}.$$

Next, $P(X \leq 2) = \frac{1}{2}bh = \frac{1}{2}(1)\frac{1}{6} = \frac{1}{12}$. Finally,

$$P(X \leq 2 \text{ or } X > 4) = P(X \leq 2) + P(X > 4) = \frac{1}{3} + \frac{1}{12} = \frac{5}{12}.$$

5. Throughout this problem, let $S$ be the event that it snows tomorrow and let $F$ be the event that the temperature is below freezing tomorrow. The given information in the problem is $P(S) = .2$, $P(F|S) = .9$ and $P(F|S') = .6$.

(a) This is asking for $P(S \cap F)$ which by the multiplication principle is

$$P(S \cap F) = P(F|S)P(S) = (.9)(.2) = .18.$$

(b) This is asking for $P(S|F)$ which can be found using Bayes' Law. In particular, since we are given $P(S) = .2$, we also know $P(S') = 1 - .2 = .8$. So by Bayes' Law,

$$\begin{aligned} P(S|F) &= \frac{P(F|S)P(S)}{P(F|S)P(S) + P(F|S')P(S')} \\ &= \frac{(.9)(.2)}{(.9)(.2) + (.6)(.8)} \\ &= \frac{.18}{.18 + .48} \\ &= \frac{.18}{.66} = \frac{18}{66} = \frac{3}{11}. \end{aligned}$$

(c) The question is asking for $P(S \cup F)$. First, from the hint, find $P(F)$, the probability that it is below freezing tomorrow. This is done with the Law of Total Probability:

$$\begin{aligned} P(F) &= P(F|S)P(S) + P(F|S')P(S') \\ &= (.9)(.2) + (.6)(.8) = .66. \end{aligned}$$

Now by the Inclusion-Exclusion Rule,

$$\begin{aligned} P(S \cup F) &= P(S) + P(F) - P(S \cap F) \\ &= .2 + .66 - .18 = .68. \end{aligned}$$

6. This is a conditional probability:

$$
\begin{aligned}
P(\text{bottom side blue} \mid \text{top side blue}) &= \frac{P(\text{bottom side blue and top side blue})}{P(\text{top side blue})} \\
&= \frac{1/3}{1/2} = \frac{2}{3}.
\end{aligned}
$$

(The numerator of the above fraction comes from the fact that there are three cards, of which one has both sides blue. The denominator comes from the fact that there are six total sides (on the three cards) which could be face up, of which three are blue. So the denominator of the above fraction is 3/6 which is 1/2.)