

Name:

Directions: This exam has nine questions, numbered 1 to 9, spread across pages 2-9 of this document. 200 points are possible. Show all work and clearly mark your final answers. Calculators may be used (indicate where you used a calculator to perform complicated calculations), but other study aids are not permitted.

Grading:

Page	Maximum Points	Score
2	30	
3	24	
4	16	
5	40	
6	25	
7	30	
8	20	
9	15	
Total	200	

1. (15 pts) Suppose that the augmented matrix corresponding to some linear system of equations has 15 columns and 19 rows.
 - (a) (5 pts) How many equations are in this system?

 - (b) (5 pts) How many variables are in this system?

 - (c) (5 pts) What are the possible numbers of solutions to this system of equations?

2. (15 pts) Suppose the number of cars on a stretch of the Edens Expressway is normally distributed with mean 400 and standard deviation 40. Use the attached table of values for the normal distribution (or a calculator) to approximate the probability that there are more than 428 cars on that stretch of the Edens Expressway.

3. (24 pts) Complete each of these sentences in such a way that gives a precise mathematical definition of the italicized terms:

(a) (6 pts) Two lines are called *perpendicular* if their slopes...

(b) (6 pts) Two events E and F are called *independent* if ...

(c) (6 pts) A matrix is said to be in *row-echelon form* if ...

(d) (6 pts) A *binomial experiment* is ...

4. (16 pts) Consider a game where you flip three coins (a nickel, a dime and a quarter) and you get to keep any of the coins that land heads (assume each coin flips heads with probability $1/2$). If you have to pay 20 cents per game to play, how much will you expect to win or lose if you play this game 50 times?

5. (30 pts (10 each)) Each of the matrices below is in row-echelon form. For each of the following matrices, find all solutions to the corresponding system of equations, writing the answers appropriately. If there is no solution, say so:

$$(a) \begin{pmatrix} 1 & -3 & 4 \\ 0 & 0 & 1 \end{pmatrix} \quad (b) \begin{pmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (c) \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

6. (10 pts) Find the mean and median of the list 3, 4, 1, 2, 0, 0, 1, 7, 9 (there are 9 numbers in the list).

7. (55 pts) Adam and Eve each have an identical standard deck of 52 playing cards. (Answers to all parts of this question may be written in terms of combinatorics notation.)
- (a) (5 pts) Suppose Eve chooses one card from Adam's deck at random. What is the probability she does not choose the 4 of spades?
- (b) (10 pts) Suppose Eve chooses a card at random from her deck, puts it back in the deck, then chooses another card at random, puts that card back, etc. until she has chosen 10 cards. What is the probability that of the 10 cards, she has chosen 3 hearts?
- (c) (10 pts) Suppose Adam chooses six cards from his deck, all at once. What is the probability that of his six cards, two are hearts and two are spades?

- (d) (10 pts) Suppose Adam deals a hand of four cards from his deck at random. What is the probability that his hand is a four-of-a-kind (that is, all his cards have the same value)?
- (e) (10 pts) Suppose Adam deals himself a ten-card hand from his deck, and Eve deals herself a ten-card hand from her deck. What is the probability that their hands have exactly two cards in common?
- (f) (10 pts) Suppose Adam and Eve take their two decks and shuffle them together, making one big deck of 104 cards. Then they deal those 104 cards across the floor (face up) from left to right. In how many distinguishable ways can they do this?

8. (20 pts) On a given day, an ATM is either “out of order” or “working properly”. If it is working properly on a day, then it is 90% likely to be working properly on the next day. If it is out of order on a day, then it is 30% likely to be out of order on the next day.

(a) (10 pts) What is the probability that the ATM is working two days from now, given that it is out of order today? (Exact answer required.)

(b) (10 pts) What is the probability that the ATM is working properly on some distant day in the future? (Exact answer required.)

9. (15 pts)

(a) (5 pts) Give an example of a system of two equations in two variables which has no solution (you do not need to verify that your system has no solution).

(b) (5 pts) Give an example of a system of three equations in two variables which has exactly one solution (and specify what that solution is).

(c) (5 pts) Give an example of a system of three equations in two variables which has more than one solution (you do not need to verify that your system has more than one solution).