1. (a) For each of the eight expressions below, circle the expression if it is defined, and draw an X through the expression if it is not defined. (Every expression should either be circled or X'd.)

$$0^0 \qquad \frac{3}{0} \qquad \frac{0}{0} \qquad \frac{0}{2} \qquad 2^0 \qquad 0^2 \qquad 0^{1/2} \qquad 0^{-2}$$

   (b) Compute $16^{-3/2}$.

   (c) Rationalize the denominator and simplify: $\frac{2\sqrt{5}}{\sqrt{6}}$.

   (d) Rewrite this expression so that it has no radical signs or negative exponents:

$$y^{-3}\sqrt[3]{x^2}$$

2. Rewrite the following compound fraction so that there are no "fractions within fractions":

$$\frac{\frac{2}{x-1}+\frac{3}{(x-1)(x+4)}}{\frac{2}{(x+4)(x-2)}-\frac{1}{(x-1)(x-2)}}$$

3. Add or subtract the following expression as indicated:

$$\frac{2}{x^2-8x+7}-\frac{1}{x^2-1}$$

4. Multiply or divide the following expressions as indicated, writing your answer in simplest form:

   (a)

$$\sqrt{96}\sqrt{15}$$

   (b)

$$(2-\sqrt{6})(2+\sqrt{6})$$

   (c)

$$(2x-3)(5y+1)$$

   (d)

$$\left(\sqrt{2}\,x+3\right)\left(2x^2-\sqrt{8}\right)$$

   (e)

$$\frac{x^2+6x+9}{x^2}\cdot\frac{x^3-x}{x^2-2x-15}$$

   (f)

$$\frac{18x^{3/2}(y^{-3})^2}{9\sqrt{x}y^2}$$

5. Factor the following expressions completely:

   (a)
   $$x^2 - 196$$

   (b)
   $$x^2 + 3x - 10$$

   (c)
   $$2x^4 + 8x^3 - 42x^2$$

6. Solve the following equations for x (if there are no solutions, say so):

   (a)
   $$\frac{4}{x+3} + 3 = \frac{5}{x+2}$$

   (b)
   $$2x^2 - 5x = -3$$

   (c)
   $$3x^2 + 7x + 5 = 0$$

1. (a) 0^0 is undefined (impossible to define this so that the rules "anything to the zero power is 1" and "zero to any power is zero" are both satisfied); $\frac{3}{0}$ is undefined (can't divide by 0); $\frac{0}{0}$ is undefined (can't divide by 0); $\frac{0}{2}$ is defined and equals 0; 2^0 is defined and equals 1; 0^2 is defined and equals 0; $0^{1/2} = \sqrt{0} = 0$ is defined; $0^{-2} = \frac{1}{0^2} = \frac{1}{0}$ is undefined (can't divide by 0).

   (b) $16^{-3/2} = \frac{1}{16^{3/2}} = \frac{1}{(\sqrt{16})^3} = \frac{1}{4^3} = \frac{1}{64}$.

   (c) $\frac{2\sqrt{5}}{\sqrt{6}} = \frac{2\sqrt{5}\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}} = \frac{2\sqrt{30}}{6} = \frac{\sqrt{30}}{3}$.

   (d) $y^{-3}\sqrt[3]{x^2} = \frac{x^{2/3}}{y^3}$.

2. Multiply through by all denominators of "interior" fractions:

$$\begin{aligned}
\frac{\frac{2}{x-1} + \frac{3}{(x-1)(x+4)}}{\frac{2}{(x+4)(x-2)} - \frac{1}{(x-1)(x-2)}} &= \frac{\left[\frac{2}{x-1} + \frac{3}{(x-1)(x+4)}\right]}{\left[\frac{2}{(x+4)(x-2)} - \frac{1}{(x-1)(x-2)}\right]} \cdot \frac{(x-1)(x+4)(x-2)}{(x-1)(x+4)(x-2)} \\
&= \frac{2(x+4)(x-2) + 3(x-2)}{2(x-1) - (x+4)} \text{ (you could stop here)} \\
&= \frac{2(x^2+2x-8) + 3x - 6}{2x - 2 - x - 4} \\
&= \frac{2x^2 + 4x - 16 + 3x - 6}{x - 6} \\
&= \frac{2x^2 + 7x - 22}{x - 6}.
\end{aligned}$$

3.

$$\begin{aligned}
\frac{2}{x^2 - 8x + 7} - \frac{1}{x^2 - 1} &= \frac{2}{(x-7)(x-1)} - \frac{1}{(x+1)(x-1)} \\
&= \frac{2(x+1)}{(x-7)(x-1)(x+1)} - \frac{(x-7)}{(x+1)(x-1)(x-7)} \\
&= \frac{2(x+1) - (x-7)}{(x-7)(x-1)(x+1)} \\
&= \frac{x+9}{(x-7)(x-1)(x+1)}.
\end{aligned}$$

4. (a) Start by factoring the numbers under the radical signs:

$$\begin{aligned}
\sqrt{96}\sqrt{15} = \sqrt{4}\sqrt{24}\sqrt{5}\sqrt{3} &= \sqrt{4}\sqrt{4}\sqrt{6}\sqrt{5}\sqrt{3} \\
&= \sqrt{4}\sqrt{4}\sqrt{2}\sqrt{3}\sqrt{5}\sqrt{3} = 2 \cdot 2 \cdot 3 \cdot \sqrt{2}\sqrt{5} = 12\sqrt{10}.
\end{aligned}$$

   (b) This follows the "$(a+b)(a-b) = a^2 - b^2$" pattern: $(2-\sqrt{6})(2+\sqrt{6}) = 2^2 - (\sqrt{6})^2 = 4 - 6 = -2$.

(c) FOIL: $(2x-3)(5y+1) = 2x(5y) + 2x(1) - 3(5y) - 3 = 10xy + 2x - 15y - 3$.

(d) FOIL and reduce the square roots:

$$\begin{aligned}\left(\sqrt{2}\,x+3\right)\left(2x^2-\sqrt{8}\right) &= 2\sqrt{2}x^3 - \sqrt{16}x + 6x^2 - 3\sqrt{8} \\ &= 2\sqrt{2}x^3 - 4x + 6x^2 - 3\sqrt{4}\sqrt{2} \\ &= 2\sqrt{2}x^3 - 4x + 6x^2 - 6\sqrt{2}.\end{aligned}$$

(e)

$$\begin{aligned}\frac{x^2+6x+9}{x^2}\cdot\frac{x^3-x}{x^2-2x-15} &= \frac{(x+3)(x+3)}{x^2}\cdot\frac{x(x^2-1)}{(x-5)(x+3)} \\ &= \frac{(x+3)(x+3)}{x^2}\cdot\frac{x(x+1)(x-1)}{(x-5)(x+3)} \\ &= \frac{(x+3)(x+1)(x-1)}{x(x-5)}.\end{aligned}$$

(f) $\frac{18x^{3/2}(y^{-3})^2}{9\sqrt{x}y^2} = \frac{2x^{3/2}y^{-6}}{x^{1/2}y^2} = 2x^{3/2-1/2}y^{-6-2} = 2xy^{-8}$.

5. (a) This is a difference of squares: $x^2 - 196 = x^2 - 14^2 = (x+14)(x-14)$.

(b) $x^2 + 3x - 10 = (x+5)(x-2)$.

(c) $2x^4 + 8x^3 - 42x^2 = 2x^2(x^2+4x-21) = 2x^2(x+7)(x-3)$.

6. (a) Find common denominators, clear denominators, solve the quadratic (I did it here by factoring) and check solution:

$$\begin{aligned}\frac{4}{x+3} + 3 &= \frac{5}{x+2} \\ \frac{4(x+2)}{(x+3)(x+2)} + \frac{3(x+3)(x+2)}{(x+3)(x+2)} &= \frac{5(x+3)}{x+2} \\ 4(x+2) + 3(x+3)(x+2) &= 5(x+3) \\ 4x+8+3(x^2+5x+6) &= 5x+15 \\ 4x+8+3x^2+15x+18 &= 5x+15 \\ 3x^2+19x+26 &= 5x+15 \\ 3x^2+14x+11 &= 0 \\ (3x+11)(x+1) &= 0 \\ 3x+11 = 0, x+1 &= 0 \\ x = \frac{-11}{3}, x &= -1\end{aligned}$$

(Both these solutions check in the original equation.)

(b) Here is the solution by factoring (the quadratic formula or completing the square also works):

$$\begin{aligned} 2x^2 - 5x &= -3 \\ 2x^2 - 5x + 3 &= 0 \\ (x-1)(2x-3) &= 0 \\ x - 1 = 0, 2x - 3 &= 0 \\ x = 1, x &= \frac{3}{2}. \end{aligned}$$

(c) Use the quadratic formula:

$$\Delta = b^2 - 4ac = 7^2 - 4 \cdot 3 \cdot 5 = 49 - 60 = -11 < 0$$

Since the discriminant is negative, this equation has no solution.