Definitions:

- If n is a whole number (i.e. $n = 1, 2, 3, \ldots$), then $x^n = x \cdot x \cdot x \cdot \ldots \cdot x$ (n times).
- $x^0 = 1$ for any $x \neq 0$ (0^0 is undefined).
- If n is a whole number, then $x^{1/n} = \sqrt[n]{x}$, the n^{th} root of x (to say $\sqrt[n]{x} = a$ means $a^n = x$).
- If m and n are whole numbers, then $x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$.
- If r is any positive number and $x \neq 0$, then $x^{-r} = \frac{1}{x^r}$ (0^{-r} is undefined).

Existence of roots and radicals; issues with signs:

- If $x \geq 0$, then $\sqrt[n]{x} = x^{1/n}$ exists for every whole number n, and $\sqrt[n]{x} \geq 0$. (For example, $\sqrt{25} = 5$, not ± 5.)
- If $x < 0$, then:
  1. $\sqrt[n]{x}$ exists and is negative if n is odd;
  2. if m/n is in lowest terms, $x^{m/n}$ exists only if n is odd;
  3. $\sqrt[n]{x}$ does not exist if n is even;
  4. if m/n is in lowest terms, $x^{m/n}$ does not exist if n is even.
- $\sqrt[n]{x^n} = x$ if n is odd, but $\sqrt[n]{x^n} = |x|$ if x is even.
- $\left(\sqrt[n]{x}\right)^n = x$ (provided $\sqrt[n]{x}$ exists).

Rules for manipulating exponents and radicals: Assuming everything exists, the following are always true:

- $x^a x^b = x^{a+b}$;
- $\frac{x^a}{x^b} = x^{a-b}$;
- x^{a^b} means $x^{(a^b)}$;
- $(x^a)^b = x^{ab} = x^{ba} = (x^b)^a$;
- $(xy)^a = x^a y^a$;
- $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$;
- $\sqrt[n]{xy} = \sqrt[n]{x}\ \sqrt[n]{y}$;
- $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$;
- $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x} = \sqrt[n]{\sqrt[m]{x}}$.

Common fallacies:

- In general, $(x+y)^n \neq x^n + y^n$.
- In general, $(x-y)^n \neq x^n - y^n$.
- In general, $\sqrt[n]{x+y} \neq \sqrt[n]{x} + \sqrt[n]{y}$.
- In general, $\sqrt[n]{x-y} \neq \sqrt[n]{x} - \sqrt[n]{y}$.

Exponents and radicals to "know":

- $1^n = 1$ for any n; $\sqrt[n]{1} = 1$ for any n.
- $x^1 = x$ for any x.
- Powers of 2: $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$.
  (Equivalently, $\sqrt{4} = 2$, $\sqrt[3]{8} = 2$, $\sqrt[4]{16} = 2$, $\sqrt[5]{32} = 2$, $\sqrt[6]{64} = 2$.)
- Powers of 3: $3^2 = 9$, $3^3 = 27$, $3^4 = 81$.
  (Equivalently, $\sqrt{9} = 3$, $\sqrt[3]{27} = 3$, $\sqrt[4]{81} = 3$.)
- Powers of 4: $4^2 = 16$, $4^3 = 64$.
  (Equivalently, $\sqrt{16} = 4$, $\sqrt[3]{64} = 4$.)
- Powers of 5: $5^2 = 25$, $5^3 = 125$.
  (Equivalently, $\sqrt{25} = 5$, $\sqrt[3]{125} = 5$.)
- Powers of 10: 10^n is a 1 with n zeros behind it. (Equivalently, $\sqrt[n]{10.....0} = 10$ if the number under the radical has n zeros.)
- Squares up to 15: $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $11^2 = 121$, $12^2 = 144$, $13^2 = 169$, $14^2 = 196$, $15^2 = 225$.
  (Equivalently, $\sqrt{36} = 6$, $\sqrt{49} = 7$, $\sqrt{64} = 8$, $\sqrt{81} = 9$, $\sqrt{121} = 11$, $\sqrt{144} = 12$, $\sqrt{169} = 13$, $\sqrt{196} = 14$.)