MATH 120
Exam 2 Study Guide

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Contents

2.1 Exam 2 Information

Exam 2 content

Exam 2 covers Chapters 3 and 4 in the 2023 version of my MATH 120 lecture notes.

WARNING: Before 2023, tangents were on Exam 3, not Exam 2, so no questions about tangent will appear on old exams from 2022 or earlier. You can find questions about tangent on old Exam 3s.

Exam 2 tasks

1. **VERY IMPORTANT** $\boxed{\text{NC}}$ Compute the sine, cosine and/or cosine of any special angle, given in either degrees or radians
2. Compute any expression involving trig functions of special angles

3. $\boxed{\text{NC}}$ Estimate the value of sines, cosines and/or tangents, given a picture of the unit circle **(this task is new as of 2023 and is not on old exams from 2022 or earlier)**

4. $\boxed{\text{NC}}$ Estimate solutions to basic sine/cosine/tangent equations, given a picture of the unit circle **(this task is new as of 2023 and is not on old exams from 2022 or earlier)**

5. $\boxed{\text{NC}}$ Write formulas in terms of sine, cosine and/or tangent to represent unknown quantities in pictures **(this task is new as of 2023 and is not on old exams from 2022 or earlier)**

6. $\boxed{\text{NC}}$ Compute the sine and/or cosine of an angle, given the coordinates of a point (x, y) on the terminal side of the angle

7. $\boxed{\text{NC}}$ Determine the sign of an expression involving sine, cosine and/or tangent based on quadrants

8. $\boxed{\text{NC}}$ Answer questions that involve the use of basic identities and/or reflection properties

9. $\boxed{\text{NC}}$ Draw pictures to explain the meaning of basic trig problems involving sine, cosine and/or tangent

10. $\boxed{\text{NC}}$ Given a picture of a right triangle with the side lengths labelled, find the sine and/or cosine of the angles in that triangle.

11. Evaluate expressions involving sine, cosine and tangent using a calculator

12. Solve story problems involving rotation and length **(this task is new as of 2023 and is not on old exams from 2022 or earlier)**

13. Convert between the components of a vector and its magnitude and direction angle

14. Solve navigation problems involving vectors

15. Find angle(s) whose sine, cosine or tangent is given (i.e. solve $\sin\theta = q$, $\cos\theta = q$ and $\tan\theta = q$)

16. **VERY IMPORTANT** Solve triangles. This includes:

    - the Laws of Sines and/or Cosines, including the ambiguous case;
    - solving right triangles using SOHCAHTOA and the Pythagorean Theorem; and
    - story problems that apply methods of solving triangles

17. Compute the angle between two vectors using dot products
18. Determine whether or not the angle between two vectors is acute, obtuse or right from the dot product
19. Compute the area of a triangle

Facts and formulas to memorize for Exam 2

Reminder: don't forget the things you learned for Exam 1.

Definitions of sine, cosine and tangent:

Trig function	Definition on unit circle	Definition in (x, y) plane ($r = \sqrt{x^2 + y^2}$)
$\sin\theta$	y	$\frac{y}{r}$
$\cos\theta$	x	$\frac{x}{r}$
$\tan\theta$	$\frac{y}{x}$ = slope	$\frac{y}{x}$

Signs of the trig functions: ("All Scholars Take Calculus"):

$\sin\theta > 0$ in Quadrants I and II;

$\cos\theta > 0$ in Quadrants I and IV;

$\tan\theta > 0$ in Quadrants I and III

Vector conversions: $\theta_{\mathbf{v}} = \arctan\dfrac{b}{a}$ $\quad |\mathbf{v}| = \sqrt{a^2 + b^2}$ $\quad \mathbf{v} = \langle |\mathbf{v}|\cos\theta_{\mathbf{v}}, |\mathbf{v}|\sin\theta_{\mathbf{v}}\rangle$

Basic identities: Periodicity $\sin(\theta + 360°) = \sin\theta$ $\quad \cos(\theta + 360°) = \cos\theta$
$\tan(\theta + 360°) = \tan\theta$

Quotient Identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Pythagorean Identity $\cos^2\theta + \sin^2\theta = 1$

Cofunction Identities $\sin(90° - \theta) = \cos\theta$ $\quad \cos(90° - \theta) = \sin\theta$

Odd-even identities $\sin(-\theta) = -\sin\theta$ $\quad \cos(-\theta) = \cos\theta$ $\quad \tan(-\theta) = -\tan\theta$

Trig functions of special angles in Quadrant I:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6} = 30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4} = 45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3} = 60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2} = 90^\circ$	1	0	DNE

Solutions of trig equations:

$$\begin{array}{rcl} \underline{\text{equation}} & & \underline{\text{solution(s)}} \\ \sin\theta = q & \Rightarrow & \theta = \arcsin q, \theta = 180^\circ - \arcsin q \\ \cos\theta = q & \Rightarrow & \theta = \arccos q, \theta = 360^\circ - \arccos q \\ \tan\theta = q & \Rightarrow & \theta = \arctan q, \theta = 180^\circ + \arctan q \end{array}$$

Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

SOHCAHTOA: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\quad \cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\quad \tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

Magnitude and angle formulas with dot products:

$$\mathbf{v}\cdot\mathbf{v} = |\mathbf{v}|^2 \qquad \mathbf{v}\cdot\mathbf{w} = |\mathbf{v}|\,|\mathbf{w}|\,\cos\theta$$

Triangle area formulas: $A = \dfrac{1}{2}bh \quad A = \dfrac{1}{2}ab\sin C$

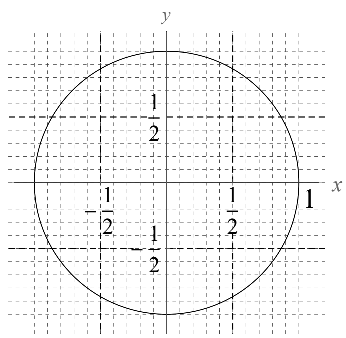
$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$

2.2 Fall 2023 Exam 2

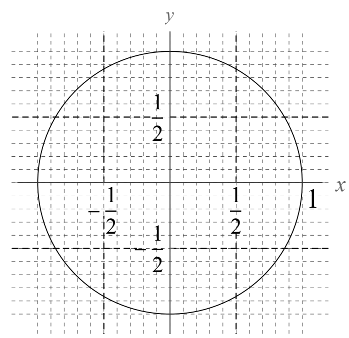
NOTE: In Fall 2023, I was running behind the course calendar and as such, I did not include Sections 4.3, 4.4 or 4.5 on this exam. For sample questions from these sections, consult the other exams in this guide.

1. (3.1) $\boxed{\text{NC}}$ Estimate each quantity by sketching a picture:

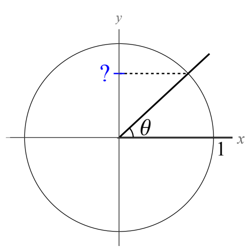
$\sin 255°$

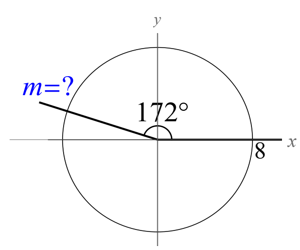


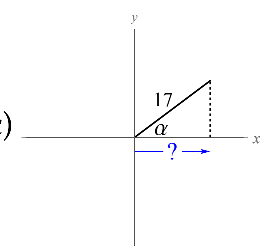
$\cos 75°$



2. (3.2) $\boxed{\text{NC}}$ In each picture, give a formula in terms of the numbers and/or variables given in the picture that yields the length or coordinate indicated by the "?":

a) 

b) 

c) 

3. (3.4) $\boxed{\text{NC}}$ Suppose $\sin\phi = \dfrac{5}{6}$ and $\cos\phi < 0$. Compute the exact value of $\cos\phi$.

4. (3.5) $\boxed{\text{NC}}$ Throughout this problem, assume θ is an angle so that $\cos\theta = \dfrac{2}{3}$. In each part of this question, you are given an expression.

- If the expression equals $\frac{2}{3}$ (i.e. it is the SAME as $\cos\theta$), write "SAME".
- If the expression equals $-\frac{2}{3}$ (i.e. it is the OPPOSITE of $\cos\theta$), write "OPPOSITE".
- If the expression is neither $\frac{2}{3}$ nor $-\dfrac{2}{3}$, write "NEITHER".

a) $\cos(\theta + 720^\circ)$
b) $\cos(-\theta)$
c) $\cos(180^\circ - \theta)$
d) $\cos(90^\circ + \theta)$
e) $\sin(90^\circ - \theta)$

5. (3.6) $\boxed{\text{NC}}$ Compute each quantity:

a) $\sin 45^\circ$
b) $\tan \pi$
c) $\cos \dfrac{5\pi}{6}$
d) $\sin \dfrac{7\pi}{6}$
e) $\cos -60^\circ$
f) $\sin 90^\circ \cos 180^\circ$
g) $4\tan^2(30^\circ + 30^\circ)$
h) $\cos 0 - \sin \dfrac{\pi}{6}$

6. (3.7) Find all angles between 0° and 360° that solve each equation:

a) $\tan\theta = \dfrac{17}{13}$
b) $\sin\theta = -.235$
c) $\cos\theta = .881$

7. (3.3) Vector $\mathbf{v}$ has magnitude 7 and direction angle 232°. Write $\mathbf{v}$ in component form (i.e. as $\mathbf{v} = \langle a, b\rangle$).

8. (3.3) Use a calculator to obtain a decimal approximation to each expression:

a) $\cos^2 26^\circ$
b) $\sin 4 \cdot 116^\circ$

9. (4.2) Solve triangle PQR if $q = 11$, $r = 18.2$ and $\angle P = 40^\circ$.

10. (4.1) Solve triangle DEF, if $\angle D = 118^\circ$, $\angle E = 33^\circ$ and $e = 5.6$.

11. a) (4.1) Your eyes are 5.5 feet above the ground. If your angle of depression when you look down at a mouse on the ground is 38°, how far away from your feet is the mouse?

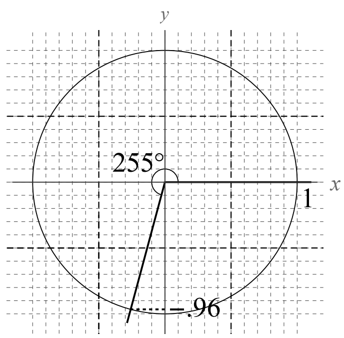
    b) (3.3) A bucket on the edge of a water wheel of radius 18 feet, starts at the bottom of the wheel, which is 2 feet below the surface of the water. if the wheel is rotated through an angle of 143° (counterclockwise), how far above the water surface is the bucket?

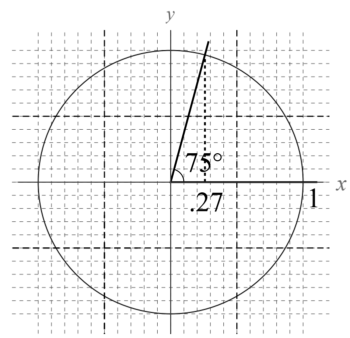
12. (3.3) A drone moves 420 feet in a direction 12° west of north, then moves 300 feet in a direction 25° north of west. How far is the drone from its original location?

Solutions

1. $\sin 255° \approx \boxed{-.96}$; this is the y-coordinate corresponding to the angle pictured below at left.

   $\cos 75° \approx \boxed{.27}$; this is the x-coordinate corresponding to the angle pictured below at right.





2. a) This is the y-coordinate on the unit circle at angle θ, which is $? = \boxed{\sin\theta}$.

   b) This is the slope at an angle of $172°$ (the radius is irrelevant), so $? = \boxed{\tan 172°}$.

   c) This is the x-coordinate on a circle of radius 17 at angle α, so $? = \boxed{17\cos\alpha}$.

3. Use the Pythagorean Identity:

$$\begin{aligned}
\cos^2\phi + \sin^2\phi &= 1 \\
\cos^2\phi + \left(\frac{5}{6}\right)^2 &= 1 \\
\cos^2\phi + \frac{25}{36} &= 1 \\
\cos^2\phi &= 1 - \frac{25}{36} = \frac{11}{36} \\
\cos\phi &= \pm\sqrt{\frac{11}{36}} = \boxed{-\sqrt{\frac{11}{36}}} = \boxed{-\frac{\sqrt{11}}{6}}.
\end{aligned}$$

   (We choose the negative square root since we are told $\cos\phi < 0$.)

4. a) $\cos(\theta + 720°)$ is the **SAME** as $\cos\theta$ by periodicity.

   b) $\cos(-\theta)$ s the **SAME** as $\cos\theta$ by the odd-even identity for cosine, or because $-\theta$ is obtained from θ by reflecting θ across the x-axis, which preserves the x-coordinate.

   c) $\cos(180° - \theta)$ is **OPPOSITE** to $\cos\theta$, because $180° - \theta$ is obtained by reflecting θ across the y-axis, which reverses the x-coordinate.

d) $\cos(90^\circ + \theta)$ is $\boxed{\textbf{NEITHER}}$; this isn't easily computed from $\cos\theta$.

e) $\sin(90^\circ - \theta)$ is the $\boxed{\textbf{SAME}}$ as $\cos\theta$ by a cofunction identity.

5. a) $\sin 45^\circ = \boxed{\dfrac{\sqrt{2}}{2}}$

b) $\tan\pi = \boxed{0}$ (slope at 180°)

c) $\cos\dfrac{5\pi}{6} = \boxed{-\dfrac{\sqrt{3}}{2}}$ (ref. angle 30°; Quadrant II)

d) $\sin\dfrac{7\pi}{6} = \boxed{-\dfrac{1}{2}}$ (ref. angle 30°; Quadrant III)

e) $\cos -60^\circ = \boxed{\dfrac{1}{2}}$ (ref. angle 60°; Quadrant IV)

f) $\sin 90^\circ \cos 180^\circ = 1(-1) = \boxed{-1}$.

g) $4\tan^2(30^\circ + 30^\circ) = 4\tan^2 60^\circ = 4(\sqrt{3})^2 = 4(3) = \boxed{12}$.

h) $\cos 0 - \sin\dfrac{\pi}{6} = 1 - \dfrac{1}{2} = \boxed{\dfrac{1}{2}}$

6. a) One angle is $\arctan\dfrac{17}{13} = \boxed{52.6^\circ}$; the other is $180^\circ + 52.6^\circ = \boxed{232.6^\circ}$.

b) One angle is $\arcsin -.235 = -13.6^\circ$; the other is $180^\circ - (-13.6^\circ) = \boxed{193.6^\circ}$. The first angle needs 360° added to it to make it between 0° and 360°, so it becomes $\boxed{346.4^\circ}$.

c) One angle is $\arccos .881 = 28.2^\circ$; the other is $360^\circ - 28.2^\circ = \boxed{331.8^\circ}$.

7. $\mathbf{v} = \langle |\mathbf{v}|\cos\theta_{\mathbf{v}}, |\mathbf{v}|\sin\theta_{\mathbf{v}}\rangle = \langle 7\cos 232^\circ, 7\sin 232^\circ\rangle = \boxed{\langle -4.31, -5.52\rangle}$.

8. a) $\cos^2 26^\circ = (\cos 26^\circ)^2 = (.8987)^2 = \boxed{.8078}$.

b) $\sin 4\cdot 116^\circ = \sin 464^\circ = \boxed{.9703}$.

9. This is an SAS triangle, so we use the Law of Cosines:

$$\begin{aligned}
p^2 &= q^2 + r^2 - 2qr\cos P \\
p^2 &= 11^2 + 18.2^2 - 2(11)(18.2)\cos 40^\circ \\
p^2 &= 145.516 \\
p &= 12.06
\end{aligned}$$

Use the Law of Cosines again to find $\angle Q$:

$$\begin{aligned}
q^2 &= p^2 + r^2 - 2pr\cos Q \\
11^2 &= 12.06^2 + 18.2^2 - 2(12.06)(18.2)\cos Q \\
121 &= 476.684 - 438.98\cos Q \\
-355.684 &= -438.98\cos Q \\
.810 &= \cos Q \\
Q &= \arccos .81 = 35.9^\circ
\end{aligned}$$

Finally, $\angle R = 180^\circ - \angle P - \angle Q = 180^\circ - 40^\circ - 35.9^\circ = 104.1^\circ$. To summarize:

$\angle P = 40^\circ$	$\angle Q = 35.9^\circ$	$\angle R = 104.1^\circ$
$p = 12.06$	$q = 11$	$r = 18.2$

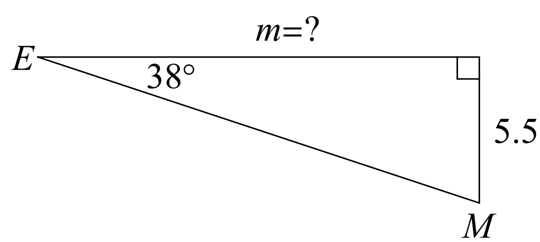
10. This is an AAS triangle, so we find the third angle: $\angle F = 180^\circ - \angle D - \angle E = 180^\circ - 118^\circ - 33^\circ = 29^\circ$. Next, use the Law of Sines twice:

$$\begin{aligned}
\frac{\sin D}{d} &= \frac{\sin E}{e} & \qquad \frac{\sin F}{f} &= \frac{\sin E}{e} \\
\frac{\sin 118^\circ}{d} &= \frac{\sin 33^\circ}{5.6} & \qquad \frac{\sin 29^\circ}{d} &= \frac{\sin 33^\circ}{5.6} \\
d &= \frac{5.6\sin 118^\circ}{\sin 33^\circ} & \qquad d &= \frac{5.6\sin 29^\circ}{\sin 33^\circ} \\
d &= 9.08 & \qquad d &= 4.98
\end{aligned}$$

To summarize:

$\angle D = 118^\circ$	$\angle E = 33^\circ$	$\angle F = 29^\circ$
$d = 9.08$	$e = 5.6$	$f = 4.98$

11. a) Here's a picture that represents this, where your eyes are at E and the mouse is at M:



Solution # 1: First, $\angle M = 180° - 90° - 38° = 52°$. Now, use the Law of Sines:

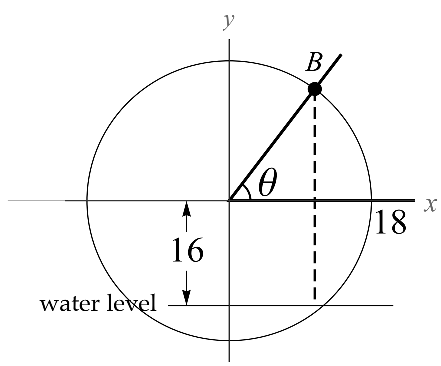
$$\frac{\sin M}{m} = \frac{\sin E}{e}$$
$$\frac{\sin 52°}{m} = \frac{\sin 38°}{5.5}$$
$$m = \frac{5.5 \sin 52°}{\sin 38°} = \boxed{7.04 \text{ ft}}.$$

Solution # 2: This is a right triangle, so we can use SOHCAHTOA:

$$\tan 38° = \frac{\text{opposite}}{\text{adjacent}} = \frac{5.5}{m}$$

By cross-multiplying, we see $m \tan 38° = 5.5$ so $m = \dfrac{5.5}{\tan 38°} = \boxed{7.04 \text{ ft}}$.

b) Here's a picture. Notice that the water level, since it is 2 feet above the bottom of the wheel, is $18 - 2 = 16$ feet below the center of the wheel. The bucket has been rotated to point B. We are interested in the length of the vertical dashed line shown in this picture.



The part of this vertical dashed line below the x-axis has length 16. For the part above the x-axis, we need the y-coordinate at angle θ, so this is $18 \sin\theta$. The last question is, what is θ? Well, the wheel rotates $143°$ but $90°$ of that is used to rotate the bucket to standard position, so $\theta = 143° - 90° = 53°$. All together, we have

$$18 \sin 53° + 16 = \boxed{30.38 \text{ ft}}.$$

12. Let $\mathbf{v}$ and $\mathbf{w}$ be the vectors corresponding to the first and second movements of the drone. We have

$$|\mathbf{v}| = 420; \theta_{\mathbf{v}} = 90° + 12° = 102°$$
$$|\mathbf{w}| = 300; \theta_{\mathbf{w}} = 180° - 25° = 155°$$

so

$$\begin{aligned}\mathbf{v} &= \langle 420\cos 102^\circ, 420\sin 102^\circ\rangle = \langle -87.32, 410.82\rangle\\ \mathbf{w} &= \langle 300\cos 155^\circ, 300\sin 155^\circ\rangle = \langle -271.89, 126.79\rangle\end{aligned}$$

Thus the drone is at position $\mathbf{v}+\mathbf{w} = \langle -359.21, 537.61\rangle$, so the distance the drone is from its original position is $||\mathbf{v}+\mathbf{w}|| = \sqrt{(-359.21)^2 + (537.61)^2} = \boxed{646.57 \text{ ft}}$.

2.3 Fall 2022 Exam 2

1. $\boxed{\text{NC}}$ For each given problem, draw a picture illustrating the problem on the provided picture of the unit circle. (You do not have to actually solve these problems.) Each picture should appropriately indicate:
   - any given information in the problem;
   - what the problem is asking for (labelled clearly with a "?"); and
   - in parts (b), (c) and (d), an indication of where θ is in your picture.

   a) (3.1) Compute $\sin 342°$.

   b) (3.4) Compute $\sin\theta$, if $\cos\theta = .875$ and $\sin\theta < 0$.

   c) (3.5) Suppose $\sin\theta = \frac{2}{5}$ and $\cos\theta < 0$. Find $\sin(-\theta)$.

   d) (3.5) Suppose $\cos\theta = \frac{3}{4}$ and $\sin\theta > 0$. Find $\cos(180° - \theta)$.

   e) (3.7) Find all values of θ between $0°$ and $360°$ such that $\sin\theta = -.4$.

2. $\boxed{\text{NC}}$ Throughout this problem, assume $\cos\theta = -\frac{1}{6}$ and $\sin\theta > 0$.

   a) (3.2) What quadrant does θ lie in?

   b) (3.5) Compute $\cos(-\theta)$.

   c) (3.5) Compute $\cos(180° + \theta)$.

   d) (3.1) Compute $\cos(\theta - 720°)$.

   e) (3.4) Compute the exact value of $\sin\theta$.

3. (3.6) $\boxed{\text{NC}}$ Compute the exact value of each quantity:

   a) $\sin 540°$

   b) $\cos 45°$

   c) $\sin \frac{5\pi}{6}$

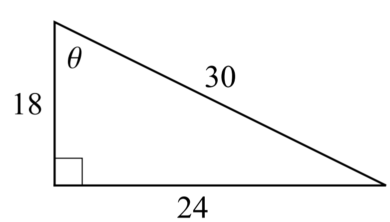
   d) $\cos(-150°)$

   e) $\cos \frac{4\pi}{3}$

   f) $\cos 90°$

   g) $\sin \frac{11\pi}{4}$

4. (4.3) $\boxed{\text{NC}}$ Compute $\sin\theta$, where θ is as in the triangle shown below:



5. (3.6) $\boxed{\text{NC}}$ Compute the exact value of each expression:

a) $\sin \frac{3\pi}{4} + \cos \frac{7\pi}{4}$

b) $8\cos^2 120°$

c) $\sin \frac{\pi}{2} + \frac{\pi}{4}$

d) $1 + \cos(5\pi - \pi)$

6. a) (3.7) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sin\theta = \frac{13}{29}$.
   b) (3.7) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = 1.123$.
   c) (3.7) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = -.425$.

7. (4.5) Find (a decimal approximation to) the area of a triangle whose side lengths are 10 cm, 14 cm and 19 cm.

8. (4.1) Solve triangle ABC, if $\angle A = 27°$, $a = 11$ and $b = 50$.

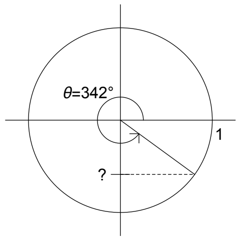
9. (4.2) Solve triangle PQR if $p = 19$, $r = 27$ and $\angle Q = 21.5°$.

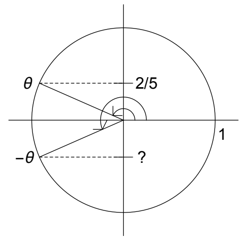
10. a) (4.1) At your current position, you notice your angle of elevation to the top of a tree in the distance is $51°$. You walk 40 feet away from the tree, and then turn around and look at the top of the tree. Now, your angle of elevation to the top of the tree is $43°$. At this point, how far are you from the top of the tree?
    b) (4.1 or 4.3) Adam visits Everglades National Park and stands on an observation deck so that his eyes are 90 feet above the ground. He looks down and sees an alligator in the swamp below. If the angle of depression from Adam to the alligator is $35°$, how far away is the alligator from the bottom of the observation deck?

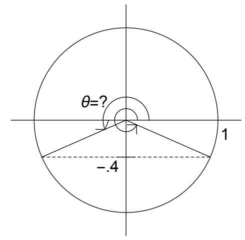
11. Throughout this problem, let $\mathbf{v} = \langle 13, 17\rangle$.
    a) (3.7) Compute the direction angle of $\mathbf{v}$.
    b) (4.4) If the vector $\langle a, 5\rangle$ is orthogonal to $\mathbf{v}$, what is the value of a?

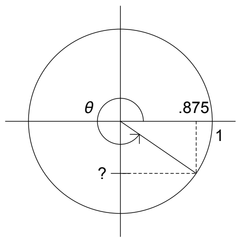
12. (3.3) A boat leaves a port and sails 5 miles on a heading $25°$ north of west. The boat then turns and sails 7 miles on a heading $10°$ east of north. Finally, the boat sails 9 miles on a heading $28°$ south of east. At this point, how far from the port is the boat?

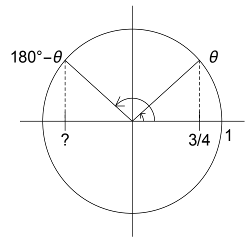
Solutions

1. a) 

c) 

e) 

b) 

d) 

2. a) Since $\cos\theta < 0$ and $\sin\theta > 0$, θ is in $\boxed{\textbf{Quadrant II}}$.

   b) $\cos(-\theta) = \cos\theta = \boxed{-\frac{1}{6}}$.

   c) $180° + \theta$ is opposite to θ, so $\cos(180° + \theta) = -\cos\theta = \boxed{\frac{1}{6}}$.

   d) $\theta - 720°$ is in the same direction as θ, so $\cos(\theta - 720°) = \cos\theta = \boxed{-\frac{1}{6}}$.

   e) Use the Pythagorean identity:

$$\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1 \\
\left(-\frac{1}{6}\right)^2 + \sin^2\theta &= 1 \\
\frac{1}{36} + \sin^2\theta &= 1 \\
\sin^2\theta &= \frac{35}{36} \\
\sin\theta &= \pm\sqrt{\frac{35}{36}} = \boxed{\frac{\sqrt{35}}{6}} \quad \text{since } \sin\theta > 0.
\end{aligned}$$

3. a) $\sin 540° = \boxed{0}$ (quadrantal; at the point $(-1, 0)$ on the unit circle)

   b) $\cos 45° = \boxed{\frac{\sqrt{2}}{2}}$

   c) $\sin\frac{5\pi}{6} = \boxed{\frac{1}{2}}$ (Quadrant II; ref. angle $30°$)

d) $\cos(-150°) = \boxed{-\dfrac{\sqrt{3}}{2}}$ (Quadrant II; ref. angle $30°$)

e) $\cos\frac{4\pi}{3} = \boxed{-\dfrac{1}{2}}$ (Quadrant III; ref. angle $60°$)

f) $\cos 90° = \boxed{0}$ (quadrantal; at the point $(0, 1)$ on the unit circle)

g) $\sin\frac{11\pi}{4} = \boxed{\dfrac{\sqrt{2}}{2}}$ (Quadrant II; ref. angle $45°$)

4. $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{24}{30} = \boxed{\dfrac{4}{5}}$.

5. a) $\sin\frac{3\pi}{4} + \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \boxed{\sqrt{2}}$.

   b) $8\cos^2 120° = 8\left(-\frac{1}{2}\right)^2 = 8\left(\frac{1}{4}\right) = \boxed{2}$.

   c) $\sin\frac{\pi}{2} + \frac{\pi}{4} = \boxed{1 + \dfrac{\pi}{4}}$.

   d) $1 + \cos(5\pi - \pi) = 1 + \cos 4\pi = 1 + 1 = \boxed{2}$.

6. a) One answer is $\theta = \arcsin\frac{13}{29} = \boxed{26.63°}$; the other answer is $180° - 26.63° = \boxed{153.37°}$.

   b) There is no solution, since $1.123 > 1$.

   c) One answer is $\theta = \arccos -.425 = \boxed{115.15°}$; the other answer is $360° - \theta = \boxed{244.85°}$.

7. Use Heron's formula. First, the semiperimeter is $s = \frac{10+14+19}{2} = 21.5$. Next, the area is

$$\begin{aligned} A = \sqrt{s(s-a)(s-b)(s-c)} &= \sqrt{21.5(21.5-10)(21.5-14)(21.5-19)} \\ &= \sqrt{21.5(11.5)(7.5)(2.5)} \\ &= \sqrt{4635.94} = \boxed{68.09 \text{ sq cm}}. \end{aligned}$$

8. This is an SSA/ASS triangle, so start with the Law of Sines (this is the ambiguous case):

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 27°}{11} &= \frac{\sin B}{50} \\ \frac{.45399}{11} &= \frac{\sin B}{50} \\ 11\sin B &= 22.6995 \\ \sin B &= 2.063 \end{aligned}$$

Since $2.063 > 1$, there is no solution for B to this equation, meaning that $\boxed{\textbf{no such triangle exists}}$.

9. This is an SAS triangle, so use the Law of Cosines:

$$\begin{aligned} q^2 &= p^2 + r^2 - 2pr\cos Q \\ q^2 &= 19^2 + 27^2 - 2(19)(27)\cos 21.5^\circ \\ q^2 &= 135.392 \\ q &= \boxed{11.64} \end{aligned}$$

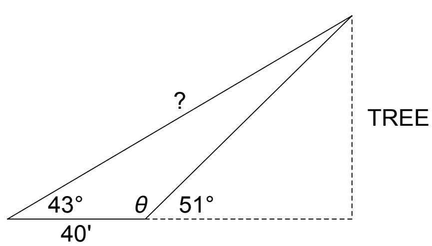
Now use the Law of Cosines again to find $\angle P$ (you could find R instead):

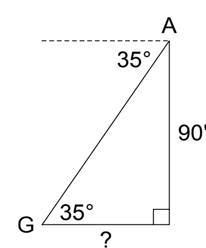
$$\begin{aligned} p^2 &= q^2 + r^2 - 2qr\cos P \\ 19^2 &= 11.64^2 + 27^2 - 2(11.64)(27)\cos P \\ 361 &= 864.49 - 628.56\cos P \\ -503.49 &= -628.56\cos P \\ .81012 &= \cos P \\ P &= \arccos .81012 = \boxed{36.77^\circ} \end{aligned}$$

Finally, $\angle R = 180^\circ - \angle P - \angle Q = 180^\circ - 21.5^\circ - 36.77^\circ = \boxed{121.73^\circ}$.

10. a) First, the picture in the problem is the one below at left. You need first to find θ, which is $180^\circ - 51^\circ = 129^\circ$. Now, the angle at the top of the triangle is $180^\circ - 43^\circ - 129^\circ = 8^\circ$. Finally, we can use the Law of Sines to determine the side with the "?":

$$\begin{aligned} \frac{\sin 129^\circ}{?} &= \frac{\sin 8^\circ}{40} \\ \frac{.7771}{?} &= \frac{.1392}{40} \\ .1392(?) &= 31.0858 \\ ? &= \boxed{223.36 \text{ ft}}. \end{aligned}$$





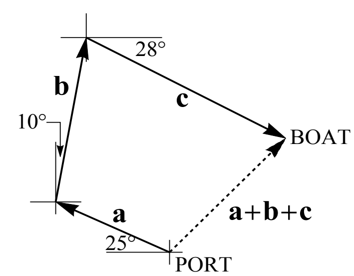
b) This is modeled by the picture above at right, where Adam is at point A and the alligator is at point G. Since the bottom of the triangle is parallel to the dashed line, the angle G is also $35°$. So we use SOHCAHTOA:

$$\begin{aligned} \tan 35° &= \frac{\text{opp}}{\text{adj}} = \frac{90}{?} \\ .7 &= \frac{90}{?} \\ .7(?) &= 90 \\ ? &= \boxed{128.57 \text{ ft}}. \end{aligned}$$

11. a) $\theta_{\mathbf{v}} = \arctan \frac{b}{a} = \arctan \frac{17}{13} = \boxed{52.6°}$. (Since θ is in Quadrant I, we do not need to add $180°$ to this answer.)

 b) Since $\langle a, 5\rangle \perp \langle 13, 17\rangle$, we know $\langle a, 5\rangle \cdot \langle 13, 17\rangle = 0$, i.e. $13a + 5(17) = 0$. Solve for a to get $a = \boxed{-\frac{85}{13}} = \boxed{-6.538}$.

12. If we think of the three parts of the boat's trip as vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, we obtain the following picture:



We need to find the magnitude of the boat's final position, which is $|\mathbf{a} + \mathbf{b} + \mathbf{c}|$. To do this, first convert each vector to component form:

$$\left.\begin{aligned} |\mathbf{a}| &= 5 \\ \theta_{\mathbf{a}} &= 180° - 25° = 155° \end{aligned}\right\} \Rightarrow \mathbf{a} = \langle 5\cos 155°, 5 \sin 155°\rangle = \langle -4.53, 2.113\rangle$$

$$\left.\begin{aligned} |\mathbf{b}| &= 7 \\ \theta_{\mathbf{b}} &= 90° + 10° = 100° \end{aligned}\right\} \Rightarrow \mathbf{b} = \langle 7\cos 100°, 7 \sin 100°\rangle = \langle 1.215, 6.894\rangle$$

$$\left.\begin{aligned} |\mathbf{c}| &= 9 \\ \theta_{\mathbf{c}} &= 360° - 28° = 332° \end{aligned}\right\} \Rightarrow \mathbf{c} = \langle 9\cos 332°, 9 \sin 332°\rangle = \langle 7.947, -4.225\rangle$$

Next, add the three vectors to get the position of the boat:

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \langle -4.53 + 1.215 + 7.947, 2.113 + 6.894 - 4.225\rangle = \langle 4.63, 4.782\rangle\,.$$

Finally, to get the distance from the boat to the port, compute the magnitude:

$$|\mathbf{a} + \mathbf{b} + \mathbf{c}| = \sqrt{(4.63)^2 + (4.782)^2} = \boxed{6.656 \text{ mi}}.$$

2.4 Fall 2019 Exam 2

1. $\boxed{\text{NC}}$ For each given problem, draw a picture illustrating the problem on the provided picture of the unit circle. Each picture should appropriately indicate:
   - any given information in the problem;
   - what the problem is asking for (labelled clearly with a "?"); and
   - in parts (b), (c) and (d), an indication of where θ is in your picture.

   You do not have to actually solve these problems.

   a) (3.1) Find $\sin 222°$.
   b) (3.4) Find $\sin\theta$, if $\cos\theta = -\frac{2}{3}$.
   c) (3.5) Suppose $\cos\theta = \frac{3}{5}$ and $\sin\theta > 0$. Find $\cos(-\theta)$.
   d) (3.5) Suppose $\sin\theta = .675$ and $\cos\theta < 0$. Find $\sin(\theta - 360°)$.
   e) (3.7) Find all values of θ between $0°$ and $360°$ such that $\sin\theta = -.25$.

2. $\boxed{\text{NC}}$ Throughout this problem, assume $\sin\theta = \frac{-2}{3}$ and $\cos\theta > 0$.

   a) (3.2) What quadrant does θ lie in?
   b) (3.5) Find $\sin(-\theta)$.
   c) (3.5) Find $\sin(180° - \theta)$.
   d) (3.1) Find $\sin(\theta + 360°)$.
   e) (3.4) Find the exact value of $\cos\theta$.

3. $\boxed{\text{NC}}$ (3.6) Find the exact value of each quantity:

   a) $\sin 0°$ c) $\sin\frac{7\pi}{4}$ e) $\sin\frac{-\pi}{2}$ g) $\sin 30°$
   b) $\cos 150°$ d) $\cos\frac{-2\pi}{3}$ f) $\cos 180°$ h) $\cos\frac{\pi}{4}$

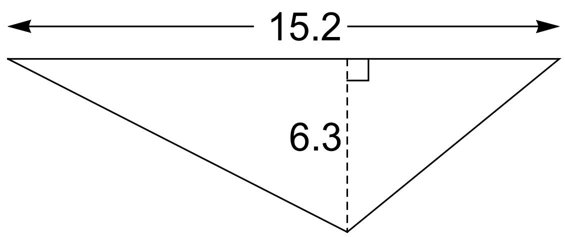
4. (3.3) Use a calculator to compute decimal approximations to each of the following expressions:

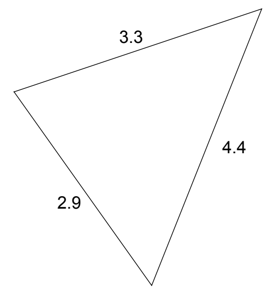
   a) $\sin(28° - 83°)$ b) $\cos 37°$ c) $3\sin^2 145°$

5. a) (3.7) Find all angles θ between $0°$ and $360°$ such that $\cos\theta = .338$.
   b) (3.7) Find all angles θ between $0°$ and $360°$ such that $\sin\theta = 1.485$.
   c) (3.7) Find all angles θ between $0°$ and $360°$ such that $\sin\theta = -.325$.

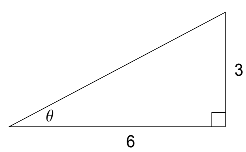
6. (3.2) Suppose the point $(13, -6)$ is on the terminal side of an angle when drawn in standard position. Find $\sin\theta$.

7. (4.5) Find the area of each triangle pictured below:





8. (4.3) Find $\cos\theta$, where θ is as indicated in the triangle below:



9. No information from part (a) of this problem is needed in part (b).

   a) (4.2) Three hunters: James, Kate and Louie, go into the woods and climb into their tree stands. Kate notices that the distance from her stand to James' is 330 feet, and the distance from her stand to Louie's stand is 275 feet. If the angle between Kate's line of sight toward James and her line of sight to Louie is $85°$, how far apart are James' and Louie's stands?

   b) (4.1 or 4.3) Kate sees a deer on the ground, 72 feet away from the base of her tree stand. If her eyes are 15 feet above the ground, what is the angle of depression from her to the deer?

10. (4.1) Solve triangle ABC if $a = 25.9$, $\angle B = 115°$ and $\angle C = 43°$.

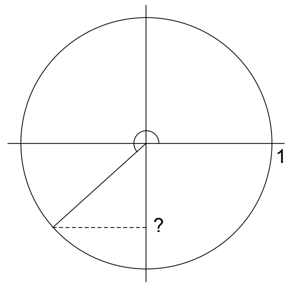
11. (3.7) Compute the direction angle of $\mathbf{w} = \langle 13, 8\rangle$.

12. (3.3) A snowmobile is driven through a large, empty pasture, starting at a gate on the south edge of the pasture. The snowmobile travels 3.75 miles at an angle $20°$ north of west, then travels 2.85 miles at an angle $36°$ east of north. Then the snowmobile travels 1.2 mile due south. At this point, how far is the snowmobile from the gate where it began its trip?

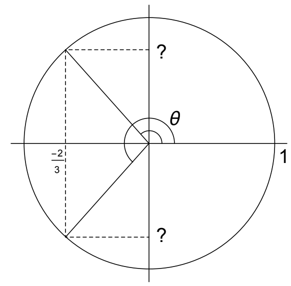
Solutions

1. Here are the pictures:

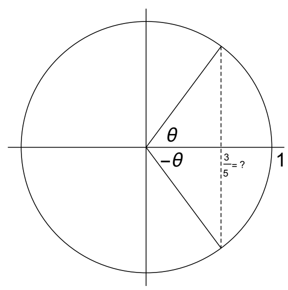
(a)



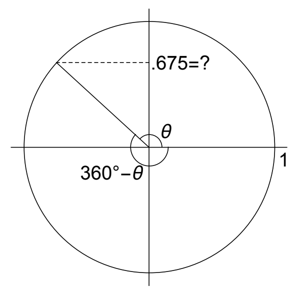
(b)



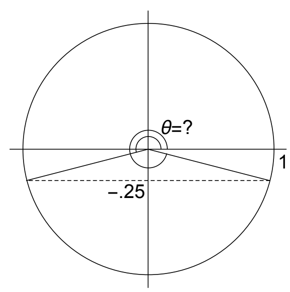
(c)



(d)



(e)



2. a) Since $\sin\theta < 0$ and $\cos\theta > 0$, θ must be in $\boxed{\textbf{Quadrant IV}}$.

   b) Since you get from θ to $-\theta$ by reflecting across the x-axis, $\sin(-\theta) = -\sin\theta = -(\frac{-2}{3}) = \boxed{\dfrac{2}{3}}$.

   c) Since you get from θ to $180° - \theta$ by reflecting across the y-axis, $\sin(180° - \theta) = \sin\theta = \boxed{-\dfrac{2}{3}}$.

   d) Since θ and $\theta + 360°$ are in the same place (i.e. coterminal), $\sin(\theta + 360°) = \sin\theta = \boxed{-\dfrac{2}{3}}$.

e) Solve for $\cos\theta$ using the Pythagorean Identity:

$$\begin{aligned}\cos^2\theta + \sin^2\theta &= 1\\ \cos^2\theta + \left(\frac{-2}{3}\right)^2 &= 1\\ \cos^2\theta + \frac{4}{9} &= 1\\ \cos^2\theta &= \frac{5}{9}\\ \cos\theta &= \pm\sqrt{\frac{5}{9}}.\end{aligned}$$

Note we are given that $\cos\theta > 0$, so $\cos\theta = \boxed{\sqrt{\frac{5}{9}}} = \boxed{\frac{\sqrt{5}}{3}}$.

3. a) $\sin 0^\circ = \boxed{0}$ (point on unit circle is $(1,0)$)
   b) $\cos 150^\circ = \boxed{-\frac{\sqrt{3}}{2}}$ (ref. angle 30°; Quadrant II)
   c) $\sin\frac{7\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$ (ref. angle 45°; Quadrant IV)
   d) $\cos\frac{-2\pi}{3} = \boxed{-\frac{1}{2}}$ (ref. angle 60°; Quadrant III)
   e) $\sin\frac{-\pi}{2} = \boxed{-1}$ (point on unit circle is $(0,-1)$)
   f) $\cos 180^\circ = \boxed{-1}$ (point on unit circle is $(-1,0)$)
   g) $\sin 30^\circ = \boxed{\frac{1}{2}}$ (ref. angle 30°; Quadrant I)
   h) $\cos\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$ (ref. angle 45°; Quadrant I)

4. a) $\sin(28^\circ - 83^\circ) = \sin(-55^\circ) = \boxed{-.8191}$.
   b) $\cos 37^\circ = \boxed{.7896}$.
   c) $3\sin^2 145^\circ = 3(\sin 145^\circ)^2 \approx 3(.5736)^2 = 3(.3289) = \boxed{.9869}$.

5. a) One angle is $\arccos .338 = \boxed{70.24^\circ}$. The other angle is $360^\circ - 70.24^\circ = \boxed{289.76^\circ}$.
   b) Since $1.485 > 1$, there are $\boxed{\text{no such angles}}$.
   c) $\arcsin -.325 = -18.97^\circ$. This is not between 0° and 360°, but we can fix this by adding 360° to get $-18.97^\circ + 360^\circ = \boxed{341.03^\circ}$. The other angle is $180^\circ - (-18.97^\circ) = \boxed{198.97^\circ}$.

6. First, $r = \sqrt{x^2+y^2} = \sqrt{13^2+(-6)^2} = \sqrt{169+36} = \sqrt{205}$. Then, $\sin\theta = \frac{y}{r} = \frac{-6}{\sqrt{205}} = \frac{-6}{14.317} = \boxed{-.4191}$.

7. a) The area is $\frac{1}{2}bh = \frac{1}{2}(15.2)(6.3) = \boxed{47.88 \text{ sq units}}$.

   b) First, the semiperimeter is $s = \frac{a+b+c}{2} = \frac{2.9+3.3+4.4}{2} = 5.3$. Then, by Heron's formula, the area is

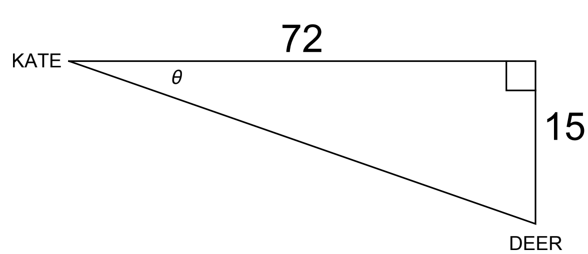
$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{5.3(5.3-2.9)(5.3-3.3)(5.3-4.4)} \\ &= \sqrt{22.896} \\ &= \boxed{4.784 \text{ sq units}}. \end{aligned}$$

8. First, the hypotenuse is $\sqrt{3^2+6^2} = \sqrt{9+36} = \sqrt{45}$. Then, $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{6}{\sqrt{45}} = \frac{6}{6.708} = \boxed{.8944}$.

9. a) Draw a triangle where the hunters' positions are J, K and L. We are given $l = 330$, $j = 275$ and $\angle K = 85°$ and asked to find k, so this is an SAS situation, which requires the Law of Cosines:

$$\begin{aligned} k^2 &= j^2 + l^2 - 2jl\cos K \\ k^2 &= 275^2 + 330^2 - 2(275)(330)\cos 85° \\ k^2 &= 75625 + 108900 - 181500(.0871) \\ k^2 &= 168706.23 \\ k &= \sqrt{168706.23} \\ k &= \boxed{410.74 \text{ ft}}. \end{aligned}$$

   b) Keep in mind that angles of depression are measured <u>from the horizontal</u>. So the appropriate picture is this (we want to find θ):



   From this picture, we see $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{15}{72} = .2083$. Therefore $\theta = \arctan .2083 = \boxed{11.76°}$.

10. First, the third angle is $A = 180° - 115° - 43° = \boxed{22°}$.

Since we know all three angles, this is a situation for the Law of Sines (and is <u>not</u> the ambiguous case since we will only be solving for side lengths). Here are the computations:

$$\begin{aligned} \frac{\sin B}{b} &= \frac{\sin A}{a} \\ \frac{\sin 115^\circ}{b} &= \frac{\sin 22^\circ}{25.9} \\ \frac{.9063}{b} &= \frac{.3746}{25.9} \\ .3746b &= 23.473 \\ b &= \boxed{62.66} \end{aligned} \qquad \begin{aligned} \frac{\sin C}{c} &= \frac{\sin A}{a} \\ \frac{\sin 43^\circ}{c} &= \frac{\sin 22^\circ}{25.9} \\ \frac{.6819}{c} &= \frac{.3746}{25.9} \\ .3746c &= 17.66 \\ c &= \boxed{47.14} \end{aligned}$$

To summarize:

$\angle A = 22^\circ$	$\angle B = 115^\circ$	$\angle C = 43^\circ$
$a = 25.9$	$b = 62.66$	$c = 471.4$

11. The direction angle of $\mathbf{w}$ is $\theta = \arctan \frac{b}{a} = \arctan \frac{8}{13} = \boxed{31.6^\circ}$.

12. Denote the three legs of the snowmobile trip by $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{x}$. Convert each of these to component form:

$$|\mathbf{v}| = 3.75; \theta = 160^\circ \Rightarrow \mathbf{v} = \langle 3.75\cos 160^\circ, 3.75\sin 160^\circ \rangle = \langle -3.52, 1.28 \rangle$$
$$|\mathbf{w}| = 2.85; \theta = 54^\circ \Rightarrow \mathbf{w} = \langle 2.85\cos 54^\circ, 2.85\sin 54^\circ \rangle = \langle 1.68, 2.31 \rangle$$
$$|\mathbf{x}| = 1.2; \theta = 270^\circ \Rightarrow \mathbf{x} = \langle 1.2\cos 270^\circ, 1.2\sin 270^\circ \rangle = \langle 0, -1.2 \rangle .$$

Now, add these three vectors to get the final position of the snowmobile: $\mathbf{v}+\mathbf{w}+\mathbf{x} = \langle -1.84, 2.38 \rangle$. The distance from the gate is the magnitude of this vector, which is $\sqrt{(-1.84)^2 + 2.38^2} = \sqrt{9.05} = \boxed{3.01 \text{ miles}}$.

2.5 Fall 2018 Exam 2

1. $\boxed{\text{NC}}$ Throughout this problem, assume $\sin\theta = \dfrac{7}{8}$ and $\cos\theta < 0$.

   a) (3.5) What is $\sin(-\theta)$?
   b) (3.1) What is $\sin(\theta + 720°)$?
   c) (3.2) What quadrant does θ lie in?
   d) (3.5) What is $\cos(90° - \theta)$?
   e) (3.4) Find the exact value of $\cos\theta$.

2. $\boxed{\text{NC}}$ (3.6) Find the exact value of each quantity:

   a) $\sin 225°$
   b) $\cos 150°$
   c) $\sin -\frac{5\pi}{6}$
   d) $\cos 90°$
   e) $\sin \frac{5\pi}{2}$
   f) $\cos \pi$
   g) $\sin 540°$
   h) $\cos \frac{\pi}{4}$

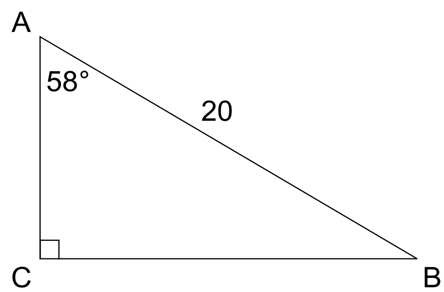
3. (3.3) Use a calculator to compute decimal approximations to each of the following expressions:

   a) $\sin(-453°)$
   b) $\cos 141°$
   c) $\cos^2 55°$
   d) $2 \sin 4 \cdot 50°$

4. a) (3.7) Find all angles θ between $0°$ and $360°$ such that $\cos\theta = .238$.
   b) (3.7) Find all angles θ between $0°$ and $360°$ such that $\sin\theta = -1.35$.
   c) (3.7) Find all angles θ between $0°$ and $360°$ such that $\sin\theta = .685$.
   d) (3.7) Find all angles θ between $0°$ and $360°$ such that $\sin\theta = 1$.

5. a) (3.2) Suppose the point $(-12, 19)$ is on the terminal side of an angle when drawn in standard position. Find $\cos\theta$.
   b) (4.3) Suppose that θ is an angle in a right triangle. If the opposite side to angle θ has length 12 and the adjacent side to θ has length 5, find $\sin\theta$.
   c) (4.5) Find the area of a triangle whose three sides have lengths 13, 18 and 26 units.
   d) (4.1 or 4.3) To measure the height of a palm tree, you walk 72 feet away from the base of the tree, turn around and look at the top of the tree. If your angle of elevation to the top of the tree is $35°$ and your eyes are 4.85 feet off the ground, how tall is the palm tree?

6. (4.1 or 4.3) Solve triangle ABC, given the information in the picture below:



7. (4.2) Solve triangle KLM if $k = 11$, $l = 9$ and $\angle M = 26°$.

8. (4.1) Solve triangle JKL if $\angle J = 43°$, $l = 15$ and $j = 12$.

9. a) (4.2) Suppose $\mathbf{v} = \langle 3, -2 \rangle$ and $\mathbf{w} = \langle 5, 2 \rangle$. Find the angle between $\mathbf{v}$ and $\mathbf{w}$.

   b) (3.3) A pelican leaves its nest in search of food. First, it flies 3 miles at an angle $40°$ south of west. Then, it flies for 3 miles due south. Then, it flies 3.7 miles at an angle $25°$ north of east to get to its current location. Currently, how far is the pelican from its nest?

Solutions

1. a) $\sin(-\theta) = -\sin\theta = \boxed{-\frac{7}{8}}$.

   b) $\sin(\theta + 720°) = \sin\theta = \boxed{\frac{7}{8}}$.

   c) Since $\sin\theta > 0$ and $\cos\theta < 0$, θ is in $\boxed{\text{Quadrant II}}$.

   d) $\cos(90° - \theta) = \sin\theta = \boxed{\frac{7}{8}}$.

   e) Draw a right triangle and label the opposite side 7 and the hypotenuse 8. Solve for the adjacent side using the Pythagorean Theorem to get $\sqrt{8^2 - 7^2} = \sqrt{64 - 49} = \sqrt{15}$. Thus $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \boxed{-\frac{\sqrt{15}}{8}}$ (it is negative because we are given $\cos\theta < 0$).

2. a) $\sin 225° = \boxed{-\frac{\sqrt{2}}{2}}$ (Quadrant III; reference angle $45°$)

   b) $\cos 150° = \boxed{-\frac{\sqrt{3}}{2}}$ (Quadrant II; reference angle $30°$)

   c) $\sin -\frac{5\pi}{6} = \boxed{-\frac{1}{2}}$ (Quadrant III; reference angle $\frac{\pi}{6} = 30°$)

   d) $\cos 90° = \boxed{0}$ (top of unit circle is $(0, 1)$)

   e) $\sin\frac{5\pi}{2} = \sin 5 \cdot 90° = \boxed{1}$ (top of unit circle is $(0, 1)$)

   f) $\cos\pi = \boxed{-1}$ (left-hand edge of unit circle is $(-1, 0)$)

   g) $\sin 540° = \boxed{0}$ (left-hand edge of unit circle is $(-1, 0)$)

   h) $\cos\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$ (Quadrant I; reference angle $45°$)

3. a) $\sin(-453°) = \boxed{-.99863}$.

   b) $\cos 141° = \boxed{-.777146}$.

   c) $\cos^2 55° = (.573576)^2 = \boxed{.32899}$.

   d) $2\sin 4 \cdot 50° = 2\sin 200° = 2(-.34202) = \boxed{-.68404}$.

4. a) One such angle is $\theta = \arccos .238 = \boxed{76.23°}$; the other angle is $360° - \theta = \boxed{283.77°}$.

   b) There are $\boxed{\text{no such angles}}$ since $-1.35 < -1$.

   c) One such angle is $\theta = \arcsin .685 = \boxed{43.24°}$; the other angle is $180° - \theta = \boxed{136.76°}$.

d) One such angle is $\theta = \arcsin 1 = \boxed{90^\circ}$; the other angle is $180^\circ - \theta = 90^\circ$ (this is redundant).

5. a) First, $r = \sqrt{x^2+y^2} = \sqrt{(-12)^2+19^2} = \sqrt{144+361} = \sqrt{505} \approx 22.47$. Then $\cos\theta = \frac{x}{r} = \frac{-12}{22.47} = \boxed{-.533993}$.

   b) First, find the hypotenuse r using the Pythagorean Theorem: $r^2 = x^2+y^2 = 5^2+12^2 = 25+144 = 169$, so $r = \sqrt{169} = 13$. Then, $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \boxed{\frac{12}{13}}$.

   c) Use Heron's formula: first, $s = \frac{1}{2}(13+18+26) = \frac{1}{2}(57) = 28.5$. Then,

$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{28.5(16.5)10.5(2.5)} = \boxed{111.104 \text{ sq units}}.$$

   d) We have a right triangle with adjacent side 72. Therefore we have $\tan 35^\circ = \frac{\text{opp}}{72}$, i.e. $.7 = \frac{\text{opp}}{72}$, i.e. the opposite side is $72(.7) = 50.41$ feet. The height of the tree is therefore $50.41 + 4.85 = \boxed{55.2649 \text{ ft}}$.

6. First, $\angle B = 180^\circ - 90^\circ - 58^\circ = \boxed{32^\circ}$.

   Next, $\sin 58^\circ = \frac{a}{20}$, i.e. $a = 20\sin 58^\circ = 20(.848) = \boxed{16.96}$.

   Last, $\cos 58^\circ = \frac{b}{20}$, i.e. $b = 20\cos 58^\circ = 20(.53) = \boxed{10.6}$.

7. The given information is SAS, so use the Law of Cosines to find m:

$$\begin{aligned}
m^2 &= k^2 + l^2 - 2kl\cos M\\
m^2 &= 11^2 + 9^2 - 2(11)9\cos 26^\circ\\
m^2 &= 121 + 81 - 198(.89879)\\
m^2 &= 24.0396\\
\Rightarrow m &= \sqrt{24.0396} = \boxed{4.9}.
\end{aligned}$$

   Next, use the Law of Cosines to find $\angle K$:

$$\begin{aligned}
k^2 &= l^2 + m^2 - 2lm\cos K\\
11^2 &= 9^2 + 4.9^2 - 2(9)4.9\cos K\\
121 &= 81 + 24.01 - 88.2\cos K\\
121 &= 105.01 - 88.2\cos K\\
15.99 &= -88.2\cos K\\
-.181293 &= \cos K\\
K &= \arccos -.181293 = \boxed{100.4^\circ}
\end{aligned}$$

   Finally, $\angle L = 180^\circ - 26^\circ - 100.4^\circ = \boxed{53.6^\circ}$.

8. The given information is SSA, so start with the Law of Sines to find $\angle L$:

$$\begin{aligned}
\frac{\sin L}{l} &= \frac{\sin J}{j} \\
\frac{\sin L}{15} &= \frac{\sin 43^\circ}{12} \\
12 \sin L &= 15 \sin 43^\circ \\
\sin L &= \frac{15 \sin 43^\circ}{12} = .8525 \\
\Rightarrow L &= \arcsin .8525 = 58.48 \text{ or } L = 180^\circ - 58.48^\circ = \boxed{121.52^\circ}.
\end{aligned}$$

This means we have two triangles (this is the ambiguous case of the Law of Sines). In the first triangle, $\angle L = 58.48^\circ$ so $\angle K = 180^\circ - 43^\circ - 58.48^\circ = \boxed{78.52^\circ}$; we use the Law of Sines to find k:

$$\begin{aligned}
\frac{\sin L}{l} &= \frac{\sin K}{k} \\
\frac{\sin 58.48^\circ}{15} &= \frac{\sin 78.52^\circ}{k} \\
k \sin 58.48^\circ &= 15 \sin 78.52^\circ \\
k(.842458) &= 14.7 \\
k &= \boxed{17.24}.
\end{aligned}$$

In the second triangle, $\angle L' = 121.52^\circ$ so $\angle K' = 180^\circ - 43^\circ - 121.52^\circ = \boxed{15.48^\circ}$; we use the Law of Sines to find k':

$$\begin{aligned}
\frac{\sin L'}{l'} &= \frac{\sin K'}{'k} \\
\frac{\sin 121.52^\circ}{15} &= \frac{\sin 15.48^\circ}{k'} \\
k' \sin 121.52^\circ &= 15 \sin 15.48^\circ \\
k'(.852458) &= 4 \\
k' &= \boxed{4.69}.
\end{aligned}$$

9. a) Use the formula $\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}|\,|\mathbf{w}| \cos\theta$. First, $|\mathbf{w}| = \sqrt{5^2 + 2^2} = \sqrt{29} \approx 5.385$. Then, using previous parts of this problem,

$$\begin{aligned}
\mathbf{v} \cdot \mathbf{w} &= |\mathbf{v}|\,|\mathbf{w}| \cos\theta \\
11 &= 3.606(5.385) \cos\theta \\
.56653 &= \cos\theta \\
\theta &= \arccos .56653 = \boxed{55.49^\circ}.
\end{aligned}$$

b) The first leg of the pelican's trip has magnitude 3 and direction angle $180^\circ + 40^\circ = 220^\circ$. Thus this leg is

$$\langle 3 \cos 220^\circ, 3 \sin 220^\circ \rangle = \langle -2.298, -1.928 \rangle .$$

The second leg has magnitude 2 and direction angle $270°$, so it is

$$\langle 2\cos 270°, 2\sin 270° \rangle = \langle 0, -3 \rangle .$$

The last leg has magnitude 3.7 and direction angle $25°$, so it is

$$\langle 3.7\cos 25°, 3.7\sin 25° \rangle = \langle 3.353, 1.564 \rangle .$$

Adding these three vectors together, we get the pelican's current position, which is

$$\langle -2.298 + 0 + 3.353, -1.928 - 3 + 1.565 \rangle = \langle 1.055, -3.363 \rangle .$$

Last, the distance the pelican is from its nest is the magnitude of its current position, which is

$$\sqrt{1.055^2 + (-3.363)^2} = \sqrt{12.4228} = \boxed{3.52 \text{ miles}}.$$

2.6 Fall 2017 Exam 2

1. $\boxed{\text{NC}}$ Throughout this problem, assume $\cos\theta = \dfrac{3}{4}$.

   a) (3.5) What is $\cos(-\theta)$?

   b) (3.1) What is $\cos(\theta - 360°)$?

   c) (3.2) What are the two quadrants θ could lie in?

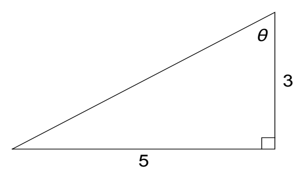
   d) (3.4) If $\sin\theta < 0$, find $\sin\theta$.

2. $\boxed{\text{NC}}$ (3.6) Find the exact value of each quantity:

   a) $\sin 180°$ c) $\sin -450°$ e) $\sin \frac{\pi}{4}$ g) $\sin \frac{-5\pi}{6}$

   b) $\cos 60°$ d) $\cos 135°$ f) $\cos 3\pi$ h) $\cos \frac{\pi}{2}$

3. (4.3) Find $\cos\theta$ and $\sin\theta$, if θ is as in the following picture. (Either an exact answer or a decimal is OK, but make sure to tell me which answer is $\sin\theta$ and which is $\cos\theta$.)

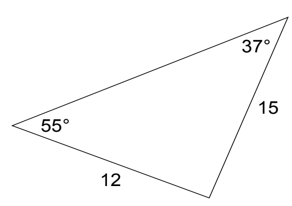


4. a) (3.7) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sin\theta = -.265$.

   b) (3.7) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = 2$.

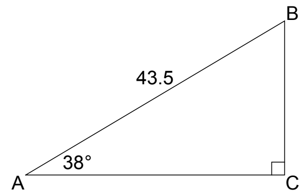
   c) (3.7) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = .43$.

5. (4.5) Find the area of this triangle:



6. (4.1 or 4.3) A 20-foot long ladder leans up against the side of a building. If the bottom of the ladder makes an angle of $70°$ with the ground, how high up the side of the building does the ladder go?

7. (4.1 or 4.3) Solve triangle ABC, given the information in the picture below:



8. (4.2) Solve triangle KLM if $k = 18$, $l = 13$ and $m = 10$.

9. (4.1) Solve triangle ABC if $\angle A = 70°$, $\angle B = 58°$ and $a = 22$.

10. a) (3.3) Suppose $\mathbf{w}$ is a vector with magnitude 7 and direction angle $285°$. Find the components of $\mathbf{w}$.

    b) (3.3) Nemo the fish swims away from his home on a trajectory $50°$ north of east for 2 miles, then changes course and swims due east for 1 mile. After that, Nemo changes course again and swims for 3 miles on a trajectory $35°$ south of east. At this point, what distance would Nemo have to swim if he wanted to swim directly home?

Solutions

1. a) $\cos(-\theta) = \cos\theta = \boxed{\frac{3}{4}}$.

   b) $\cos(\theta - 360°) = \cos\theta = \boxed{\frac{3}{4}}$.

   c) Since $\cos\theta > 0$, θ must be in Quadrants $\boxed{\text{I or IV}}$.

   d) Start with the Pythagorean identity:

$$\cos^2\theta + \sin^2\theta = 1$$
$$\left(\frac{3}{4}\right)^2 + \sin^2\theta = 1$$
$$\frac{9}{16} + \sin^2\theta = 1$$
$$\sin^2\theta = \frac{7}{16} \Rightarrow \cos\theta = \pm\sqrt{\frac{7}{16}}.$$

   Since we are told $\sin\theta < 0$, $\sin\theta = -\sqrt{\frac{7}{16}} = \boxed{-\frac{\sqrt{7}}{4}}$.

2. a) $\sin 180° = \boxed{0}$ (point on unit circle is $(-1, 0)$).

   b) $\cos 60° = \boxed{\frac{1}{2}}$

   c) $\sin -450° = \sin -90° = -\sin 90° = \boxed{-1}$ (point on unit circle is $(0, 1)$).

   d) $\cos 135° = \boxed{-\frac{\sqrt{2}}{2}}$ (Quadrant II; reference angle $45°$)

   e) $\sin \frac{\pi}{4} = \sin 45° = \boxed{\frac{\sqrt{2}}{2}}$ (Quadrant I)

   f) $\cos 3\pi = \cos 540° = \cos 180° = \boxed{-1}$ (point on unit circle is $(-1, 0)$)

   g) $\sin \frac{-5\pi}{6} = \sin(-150°) = \boxed{-\frac{1}{2}}$ (Quadrant III; reference angle $\frac{\pi}{6} = 30°$)

   h) $\cos \frac{\pi}{2} = \cos 90° = \boxed{0}$ (point on unit circle is $(0, 1)$)

3. First, use the Pythagorean Theorem to find the hypotenuse, which I'll call c:

$$3^2 + 5^2 = c^2 \Rightarrow c = \sqrt{3^2 + 5^2} = \sqrt{34} \approx 5.83.$$

   Then $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \approx \frac{3}{5.83} = \boxed{.5145}$ and $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \approx \frac{5}{5.83} = \boxed{.8575}$.

4. a) From a calculator, $\theta = \arcsin -.265 \approx \boxed{-15.37°}$. Add $360°$ to this to get $344.63°$; take $180° -$ the calculator answer to get the second answer which is $\boxed{196.37°}$.

b) There are $\boxed{\text{no such angles}}$ (because $2 > 1$).

c) From a calculator, $\theta = \arccos .43 \approx \boxed{64.5^\circ}$. A second angle which solves the equation is $360^\circ - \theta = \boxed{295.5^\circ}$.

5. First, the third angle is $180^\circ - 55^\circ - 37^\circ = 88^\circ$. Then, by the SAS Area Formula, $A = \frac{1}{2}ab\sin C = \frac{1}{2}(12)(15)\sin 88^\circ \approx \boxed{89.94 \text{ sq units}}$.

6. In this problem, we have a right triangle where the hypotenuse is 20 and the angle from the hypotenuse to the ground is 70°. We are asked to find the opposite side, which I'll call b. By SOHCAHTOA, we get

$$\sin 70^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{20}.$$

Solve for b by cross-multiplying to get $b = 20\sin 70^\circ = \boxed{18.79 \text{ ft}}$.

7. First, the third angle is $B = 90^\circ - 38^\circ = \boxed{52^\circ}$.

Second, find side a using a trig function (either sine or cosine):

$$\begin{aligned} \sin 38^\circ &= \frac{a}{43.5} \\ .6157 &= \frac{a}{4}3.5 \\ \boxed{26.78} &= a. \end{aligned}$$

Last, find the remaining side using the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (26.78)^2 + b^2 &= (43.5)^2 \\ b &= \sqrt{(43.5)^2 - (26.78)^2} = \sqrt{1175} \approx \boxed{34.27}. \end{aligned}$$

8. The given information is SSS, so start with the Law of Cosines to find any angle (say K):

$$\begin{aligned} k^2 &= l^2 + m^2 - 2lm\cos K \\ 18^2 &= 13^2 + 10^2 - 2(13)(10)\cos K \\ 324 &= 169 + 100 - 260\cos K \\ 324 &= 269 - 260\cos K \\ 55 &= -260\cos K \\ -.2115 &= \cos K \\ \boxed{102.2^\circ} &= K \end{aligned}$$

Now use the Law of Cosines again to find a second angle (say L):

$$\begin{aligned} l^2 &= k^2 + m^2 - 2km\cos L \\ 13^2 &= 18^2 + 10^2 - 2(18)(10)\cos L \\ 169 &= 324 + 100 - 360\cos L \\ 169 &= 424 - 360\cos L \\ -255 &= -360\cos L \\ .708 &= \cos L \\ \boxed{44.9^\circ} &= L \end{aligned}$$

Last, find the third angle (for me,, this is M): $M = 180^\circ - 102.2^\circ - 44.9^\circ = \boxed{32.9^\circ}$.

9. The given information is SAA/AAS, which requires the Law of Sines.

   First, the third angle is $C = 180^\circ - 70^\circ - 58^\circ = \boxed{52^\circ}$.

   Second, find side length b using the Law of Sines:

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 70^\circ}{22} &= \frac{\sin 58^\circ}{b} \\ b\sin 70^\circ &= 22\sin 58^\circ \\ .939b &= 18.65 \\ b &= \boxed{19.85} \end{aligned}$$

   Last, find side length c using the Law of Sines (you could use the Law of Cosines):

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin C}{c} \\ \frac{\sin 70^\circ}{22} &= \frac{\sin 52^\circ}{c} \\ c\sin 70^\circ &= 22\sin 52^\circ \\ .939c &= 17.33 \\ c &= \boxed{18.45} \end{aligned}$$

10. a) $\mathbf{w} = \langle 7\cos 285^\circ, 7\sin 285^\circ\rangle = \boxed{\langle 1.811, -6.761\rangle}$.

    b) Nemo's journey can be described by three vectors: the first part of his trip is

$$\mathbf{a} = \langle 2\cos 50^\circ, 2\sin 50^\circ\rangle = \langle 1.285, 1.532\rangle\,;$$

    the second part of his trip is

$$\mathbf{b} = \langle 1\cos 0^\circ, 1\sin 0^\circ\rangle = \langle 1, 0\rangle\,;$$

the last part of his trip is

$$\mathbf{c} = \langle 3\cos 325°, 3\sin 325° \rangle = \langle 2.457, -1.72 \rangle .$$

(The angle is $360° - 35° = 325°$ because the trajectory is <u>south</u> of east.) That means Nemo's eventual position is

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \langle 1.285 + 1 + 2.457, 1.532 + 0 - 1.72 \rangle = \langle 4.742, -.188 \rangle .$$

The magnitude of this vector, which is the distance from Nemo to his home, is

$$\sqrt{4.742^2 + (-.188)^2} = \sqrt{22.5219} \approx \boxed{4.745 \text{ miles}}.$$

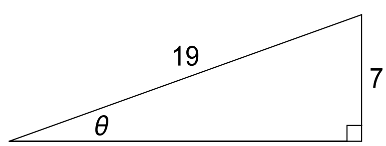
2.7 Fall 2016 Exam 2

1. $\boxed{\text{NC}}$ Throughout this problem, assume $\sin\theta = \frac{-2}{3}$.
   a) (3.5) What is $\sin(-\theta)$?
   b) (3.4) If $\cos\theta > 0$, find $\cos\theta$.
   c) (3.2) If $\cos\theta > 0$, what quadrant is θ in?
   d) (3.1) What is $\sin(\theta + 360°)$?
   e) (3.5) What is $\sin(\theta + 180°)$?

2. (3.6) $\boxed{\text{NC}}$ Find the exact value of each quantity:
   a) $\sin 45°$
   b) $\cos 120°$
   c) $\sin 180°$
   d) $\cos 300°$
   e) $\sin 570°$
   f) $\cos(-30°)$

3. (3.6) $\boxed{\text{NC}}$ Find the exact value of each quantity:
   a) $\sin\frac{5\pi}{6}$
   b) $\sin 8\pi$
   c) $\sin\frac{-3\pi}{4}$
   d) $\cos\frac{10\pi}{3}$
   e) $\sin\frac{\pi}{2}$
   f) $\cos\frac{-7\pi}{2}$

4. (4.3) Find $\cos\theta$, if θ is as in the following picture:

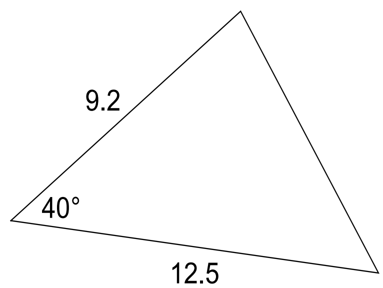


5. (3.4) Suppose $\sin\theta = .55$. Find all possible values of $\cos\theta$.

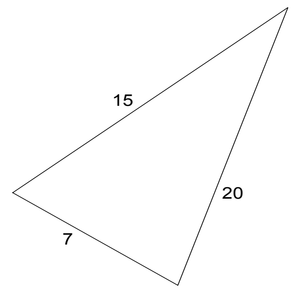
6. (3.7)
   a) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sin\theta = .3$.
   b) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sin\theta = 1.3$.
   c) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = .75$.
   d) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = 1$.

7. (4.5) Find the area of each indicated triangle:

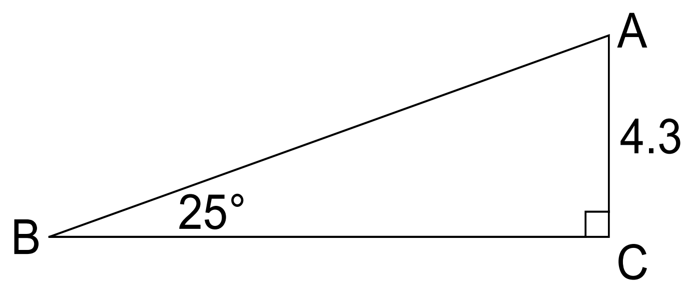
a)



b)



8. (4.1 or 4.3) Solve triangle ABC, given the information in the picture below:



9. (4.2) Solve triangle GHI if $g = 17$, $h = 25$ and $\angle I = 62°$.

10. (4.1) Solve triangle ABC if $a = 17$, $b = 10$ and $\angle B = 40°$.

11. (4.1 or 4.3) A person wants to measure the height of a flagpole. She walks 50 feet away from the base of the flagpole, turns around and notices that her angle of elevation to the top of the pole is $48°$. If her eyes are 5 feet above the ground, how tall is the flagpole?

12. Throughout this question, assume that $\mathbf{u} = \langle -5, 3 \rangle$, $\mathbf{v} = \langle -1, 8 \rangle$ and that $\mathbf{w}$ is a vector whose magnitude is 7 and whose direction angle is $58°$.

    a) (4.4) Compute the angle between $\mathbf{u}$ and $\mathbf{v}$.
    b) (3.7) Compute the direction angle of $\mathbf{v}$.
    c) (3.3) Write $\mathbf{w}$ in component form.

13. A bird leaves its nest and flies 20 miles on a heading $37°$, measured east of north. The bird then flies 9 miles on a bearing $72°$, measured west of north.

    a) (3.3) How far is the bird from its nest?
    b) (3.7) If the bird wanted to return to its nest, would it be more accurate to say it should fly southeast, or southwest? Explain your answer.

Solutions

1. a) $\sin(-\theta) = -\sin\theta = \boxed{\dfrac{2}{3}}$.

   b) Start with the Pythagorean identity:

$$\begin{aligned}\cos^2\theta + \sin^2\theta &= 1\\ \cos^2\theta + \left(\frac{-2}{3}\right)^2 &= 1\\ \cos^2\theta + \frac{4}{9} &= 1\\ \cos^2\theta &= \frac{5}{9} \Rightarrow \cos\theta = \pm\sqrt{\frac{5}{9}}.\end{aligned}$$

   Since we are told $\cos\theta > 0$, $\cos\theta = \sqrt{\frac{5}{9}} = \boxed{\dfrac{\sqrt{5}}{3}}$.

   c) Since $\sin\theta < 0$ and $\cos\theta > 0$, θ must be in $\boxed{\text{Quadrant IV}}$.

   d) $\sin(\theta + 360°) = \sin\theta = \boxed{-\dfrac{2}{3}}$.

   e) $\theta + 180°$ is in Quadrant II, and has the opposite y-coordinate as θ, so $\sin(\theta + 180°) = \boxed{\dfrac{2}{3}}$.

2. a) $\sin 45° = \boxed{\dfrac{\sqrt{2}}{2}}$.

   b) $\cos 120° = \boxed{-\dfrac{1}{2}}$ (Quadrant II; reference angle $60°$)

   c) $\sin 180° = \boxed{0}$ (point on unit circle is $(-1, 0)$)

   d) $\cos 300° = \boxed{\dfrac{1}{2}}$ (Quadrant IV; reference angle $60°$)

   e) $\sin 570° = \sin 210° = \boxed{-\dfrac{1}{2}}$ (Quadrant II; reference angle $30°$)

   f) $\cos(-30°) = \cos 30° = \boxed{\dfrac{\sqrt{3}}{2}}$

3. a) $\sin\frac{5\pi}{6} = \sin 150° = \boxed{\dfrac{1}{2}}$ (Quadrant II; reference angle $30°$)

   b) $\sin 8\pi = \sin 0 = \boxed{0}$

   c) $\sin\frac{-3\pi}{4} = \sin(-135°) = \boxed{\dfrac{-\sqrt{2}}{2}}$ (Quadrant III; reference angle $45°$)

d) $\cos\frac{10\pi}{3} = \cos\frac{4\pi}{3} = \cos 240^\circ = \boxed{-\frac{1}{2}}$ (Quadrant III; reference angle 60°)

e) $\sin\frac{\pi}{2} = \sin 90^\circ = \boxed{1}$ (point on unit circle is $(0,1)$)

f) $\cos\frac{-7\pi}{2} = \cos\frac{7\pi}{2} = \cos 270^\circ = \boxed{0}$ (point on unit circle is $(0,-1)$)

4. First, use the Pythagorean Theorem to find the adjacent side, which I'll call a:

$$a^2 + 7^2 = 19^2 \Rightarrow a = \sqrt{19^2 - 7^2} = \sqrt{312} \approx 17.66.$$

Then $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \approx \frac{17.66}{19} = \boxed{.9297}$.

5. Start with the Pythagorean identity:

$$\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1\\
\cos^2\theta + (.55)^2 &= 1\\
\cos^2\theta + .3025 &= 1\\
\cos^2\theta &= .6975\\
\cos\theta &= \pm\sqrt{.6975} \approx \boxed{\pm .8352}.
\end{aligned}$$

6. a) From a calculator, $\theta = \arcsin .3 \approx \boxed{17.46^\circ}$. A second angle which solves the equation is $180^\circ - \theta \approx \boxed{162.54^\circ}$.

b) There are $\boxed{\text{no such angles}}$ (because $1.3 > 1$).

c) From a calculator, $\theta = \arccos .75 \approx \boxed{41.4^\circ}$. A second angle which solves the equation is $360^\circ - \theta = \boxed{318.6^\circ}$.

d) The only such angle is $\theta = \boxed{0}$ (if you also included $\theta = 360^\circ$, that's okay, but 360° is really the same angle as 0°).

7. a) By the SAS Area Formula, $A = \frac{1}{2}ab\sin C = \frac{1}{2}(9.2)(12.5)\sin 40^\circ \approx \boxed{36.96 \text{ sq units}}$.

b) Use Heron's Formula (so first, find the semiperimeter):

$$s = \frac{a+b+c}{2} = \frac{7+15+20}{2} = 21.$$

Then the area is

$$\begin{aligned}
A = \sqrt{s(s-a)(s-b)(s-c)} &= \sqrt{21(21-7)(21-15)(21-20)}\\
&= \sqrt{21(14)(6)(1)}\\
&= \sqrt{1764}\\
&= \boxed{42 \text{ sq units}}.
\end{aligned}$$

8. First, the third angle is $A = 90° - 25° = \boxed{65°}$.
   Second, find the hypotenuse c using a trig function (either sine or cosine):

$$\begin{aligned} \sin 25° &= \frac{4.3}{c} \\ .4226 &= \frac{4.3}{c} \\ .4226c &= 4.3 \\ c &= \frac{4.3}{.4226} = \boxed{10.175}. \end{aligned}$$

   Last, find the remaining side using the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + (4.3)^2 &= (10.175)^2 \\ a &= \sqrt{(10.175)^2 - (4.3)^2} = \sqrt{85.03} \approx \boxed{9.22}. \end{aligned}$$

9. The given information is SAS, so start with the Law of Cosines to find side i:

$$\begin{aligned} i^2 &= g^2 + h^2 - 2gh\cos I \\ i^2 &= 17^2 + 25^2 - 2(17)(25)\cos 62° \\ i^2 &= 289 + 625 - 850(.4695) \\ i^2 &= 514.949 \\ i &= \sqrt{514.949} \approx \boxed{22.69}. \end{aligned}$$

   Now use the Law of Cosines again to find $\angle G$ (you could find $\angle H$ first instead):

$$\begin{aligned} g^2 &= h^2 + i^2 - 2hi\cos G \\ 17^2 &= 25^2 + 22.69^2 - 2(25)(22.69)\cos G \\ &\vdots \\ .75 &= \cos G \\ G &= \arccos .75 = \boxed{41.4°}. \end{aligned}$$

   Last, find $\angle H$: $H = 180° - 62° - 41.4° = \boxed{76.6°}$.

10. The given information is SSA, which is the ambiguous case of the Law of Sines:

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin A}{17} &= \frac{\sin 40°}{10} \\ 10\sin A &= 17(.6427) \\ \sin A &= \frac{17(.6427)}{10} = 1.092. \end{aligned}$$

   This equation has no solution (because $1.092 > 1$), so there is $\boxed{\textbf{no such triangle}}$.

11. Draw a right triangle where the opposite side is the flagpole. We are given that the adjacent side is 50 feet and the angle to the horizontal is $48°$. That means, labelling the adjacent side as a, that

$$\begin{aligned}\tan 48° &= \frac{\text{opposite}}{\text{adjacent}}\\ 1.11 &= \frac{a}{50}\\ 1.11(50) &= a\\ 55.5 &= a.\end{aligned}$$

Adding the 5 feet to account for the height of the viewer's eyes, we see that the height of the flagpole is $55.5 + 5 = \boxed{60.5 \text{ ft}}$.

12. a) First, $|\mathbf{u}| = \sqrt{(-5)^2 + 3^2} = \sqrt{34} \approx 5.83$. Second, $|\mathbf{v}| = \sqrt{(-1)^2 + 8^2} = \sqrt{65} = 8.06$. Therefore,

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= |\mathbf{u}|\,|\mathbf{v}|\,\cos\theta\\ 29 &= (5.83)(8.06)\cos\theta\\ .6171 &= \cos\theta\\ \theta &= \arccos .6171 = \boxed{51.89°}.\end{aligned}$$

b) $\theta = \arctan \frac{b}{a} = \arctan \frac{8}{-1} = \arctan(-8) = -82.875°$. But this is in the wrong quadrant ($\mathbf{v}$ is in Quadrant II), so we need to add $180°$ to get $\boxed{97.125°}$.

c) $\mathbf{w} = \langle |\mathbf{w}|\cos\theta, |\mathbf{w}|\sin\theta\rangle = \langle 7\cos 58°, 7\sin 58°\rangle = \boxed{\langle 3.709, 5.936\rangle}$.

13. a) Let $\mathbf{v}$ be the first part of the bird's journey; we have

$$\mathbf{v} = \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta\rangle = \langle 20\cos 53°, 20\sin 53°\rangle = \langle 12.03, 15.97\rangle.$$

Let $\mathbf{w}$ be the second part of the bird's journey; we have

$$\mathbf{w} = \langle |\mathbf{w}|\cos\theta, |\mathbf{w}|\sin\theta\rangle = \langle 9\cos 162°, 9\sin 162°\rangle = \langle -8.559, 2.781\rangle.$$

Therefore the bird's position is $\mathbf{v} + \mathbf{w} = \langle 3.47, 18.75\rangle$. The distance from the bird to the nest is

$$|\mathbf{v} + \mathbf{w}| = |\langle 3.47, 18.75\rangle| = \sqrt{(3.47)^2 + (18.75)^2} = \boxed{19.06 \text{ mi}}.$$

b) The first component of the bird's position is positive, so it has to fly $\boxed{\text{southwest}}$ (although just barely west of south) to get back to its nest.