MATH 120
Exam 3 Study Guide

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Contents

3.1 Exam 3 Information

Exam 3 content

Exam 3 covers Chapters 5 and 6 in the 2023 version of my MATH 120 lecture notes.

WARNING: Before Fall 2023, trig graphs (Chapter 5) were only covered on final exams, and not on any of the midterms. For sample questions from Chapter 5, look the Fall 2023 Exam 3, or at final exams from 2022 and earlier.

Exam 3 tasks

1. **VERY IMPORTANT** $\boxed{\text{NC}}$ Compute any trig function of any special angle, given in either degrees or radians

2. Compute any expression involving trig functions of special angles

3. $\boxed{\text{NC}}$ Sketch the graph of any sinusoidal function
4. $\boxed{\text{NC}}$ Given the graph of a sinusoidal function, write an equation which has that graph
5. $\boxed{\text{NC}}$ Sketch graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$ and $y = \csc x$
6. $\boxed{\text{NC}}$ Write formulas in terms of trig functions to represent unknown quantities in pictures
7. $\boxed{\text{NC}}$ Compute the trig functions of an angle, given the coordinates of a point (x, y) on the terminal side of the angle
8. $\boxed{\text{NC}}$ Answer questions that involve the use of basic identities
9. $\boxed{\text{NC}}$ Determine the sign of a trigonometric expression based on quadrants
10. $\boxed{\text{NC}}$ Given a picture of a right triangle with the side lengths labelled, find any trig function of the angles in that triangle.
11. Evaluate expressions involving trig functions using a calculator
12. Find angle(s) whose trig function is given (i.e. solve $\sin\theta = q$, $\cos\theta = q$ and $\tan\theta = q$, $\cot\theta = q$, $\sec\theta = q$, $\csc\theta = q$)

Facts and formulas to memorize for Exam 3

Reminder: don't forget the things you learned for Exams 1 and 2.

Definitions of the trig functions:

Trig function	Definition on unit circle	Definition in (x, y) plane $(r = \sqrt{x^2 + y^2})$
$\sin\theta$	y	$\frac{y}{r}$
$\cos\theta$	x	$\frac{x}{r}$
$\tan\theta$	$\frac{y}{x}$ = slope	$\frac{y}{x}$
$\cot\theta$	$\frac{x}{y}$	$\frac{x}{y}$
$\csc\theta$	$\frac{1}{y}$	$\frac{r}{y}$
$\sec\theta$	$\frac{1}{x}$	$\frac{r}{x}$

Basic identities: Reciprocal identities

$$\cot\theta = \frac{1}{\tan\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

Periodicity $f(\theta + 360^\circ) = f(\theta)$ for any trig function f

Quotient Identities $\tan\theta = \dfrac{\sin\theta}{\cos\theta} \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1 \qquad \sec^2\theta = \tan^2\theta + 1 \qquad \csc^2\theta = \cot^2\theta + 1$$

Odd-even identities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\cot(-\theta) = -\cot\theta \qquad \sec(-\theta) = \sec\theta \qquad \csc(-\theta) = -\csc\theta$$

Trig functions of special angles in Quadrant I:

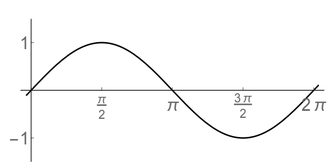
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0	1	0	DNE	1	DNE
$\frac{\pi}{6} = 30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
$\frac{\pi}{4} = 45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3} = 60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{2} = 90^\circ$	1	0	DNE	1	DNE	0

SOHCAHTOA: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

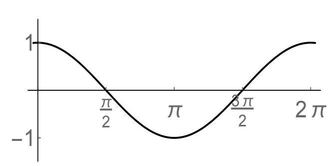
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Graphs of the six trig functions:

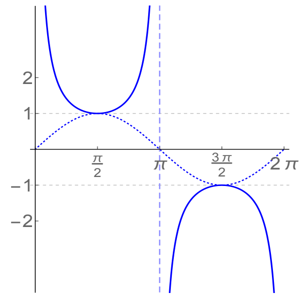
$y = \sin x$



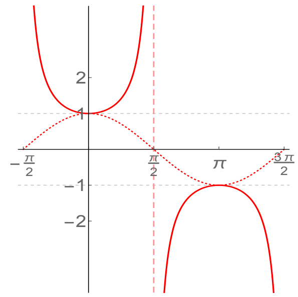
$y = \cos x$



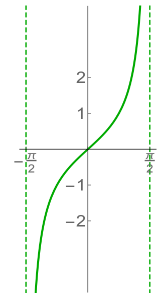
$y = \csc x$



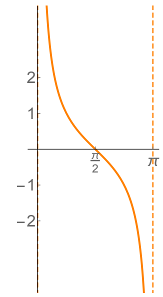
$y = \sec x$



$y = \tan x$



$y = \cot x$

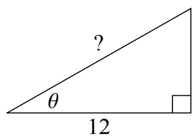


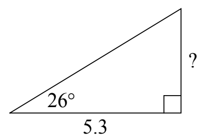
Shifts and stretches: $y = A(\sin\ \mathit{or}\ \cos)B(x \pm C) \pm D$

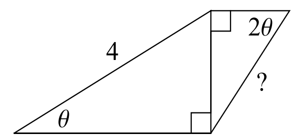
$x - C$	$\longleftrightarrow$	shift right by C units
$x + C$	$\longleftrightarrow$	shift left by C units
D	$\longleftrightarrow$	vertical shift by D units (up if $D > 0$, down if $D < 0$)
A	$\longleftrightarrow$	amplitude a.k.a. vertical stretch/shrink: graph goes up to $D + A$ and down to $D - A$ if A is negative, graph is flipped vertically
B	$\longleftrightarrow$	horizontal stretch/shrink: period becomes $\dfrac{2\pi}{B}$

3.2 Fall 2023 Exam 3

1. (6.1) $\boxed{\text{NC}}$ In each picture, give a formula in terms of the numbers and/or variables given in the picture that yields the length indicated by the "?". Your formula should not contain any division.

a) 

b) 

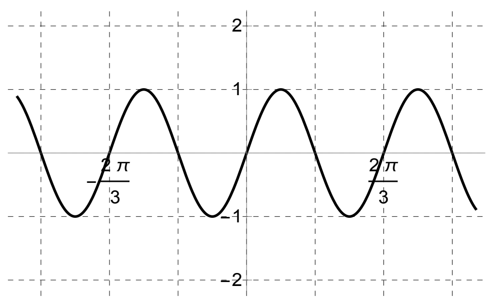
c) 

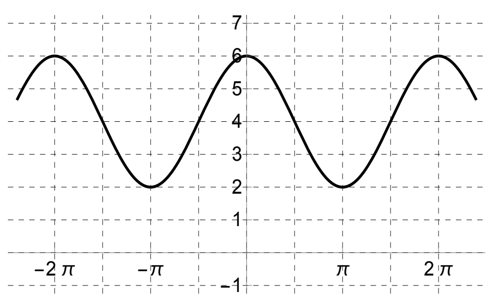
2. (6.1) $\boxed{\text{NC}}$ Suppose $\cot\theta = p$ and $\sin\theta = q$.

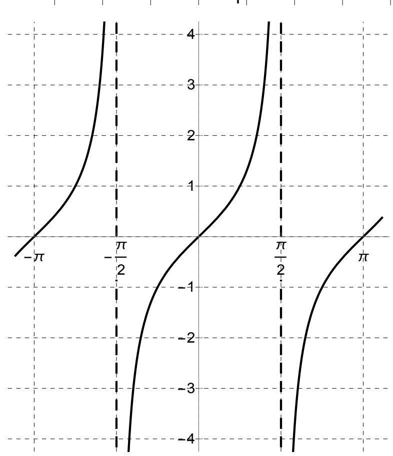
   a) Write a formula for $\tan\theta$ in terms of p and/or q.

   b) Write a formula for $\cos\theta$ in terms of p and/or q.

3. $\boxed{\text{NC}}$ Write an equation that has each given graph:

   a) (5.3) 

   b) (5.3) 

   c) (5.4) 

4. (5.3) $\boxed{\text{NC}}$ Sketch a crude graph of each function:

   a) $y = \sin(x - \pi)$

   b) $y = 2\cos x + 1$

   c) $y = -\cos x$

   d) $y = \sin \dfrac{x}{4} + 3$

5. (6.3) $\boxed{\text{NC}}$ Compute the exact value of each quantity:

   a) $\csc \dfrac{\pi}{3}$

   b) $\cot \dfrac{3\pi}{2}$

   c) $\sec 120^\circ$

   d) $\cos(-180^\circ)$

   e) $\sec \dfrac{\pi}{4}$

   f) $\sin \dfrac{5\pi}{6}$

6. (6.3) $\boxed{\text{NC}}$ Compute the exact value of each quantity. Simplify each answer:

   a) $5\cos(\pi + \pi)$

   b) $\cot \dfrac{\pi}{3} - \dfrac{\pi}{3}$

   c) $\csc^2(-45^\circ)$

   d) $\sin \dfrac{3\pi}{4} \sec \dfrac{7\pi}{6}$

   e) $\csc \dfrac{8\pi}{5} \sin \dfrac{8\pi}{5}$ f

   f) $\cot^2 471^\circ - \csc^2 471^\circ$

7. (6.1) Evaluate each expression using a calculator:

   a) $\csc 148.2^\circ$

   b) $\cot 14^\circ$

   c) $4\sec 73^\circ$

   d) $\sin 118^\circ + \cot(-132^\circ)$

   e) $\sec^2 15 \cdot 17^\circ$

8. (6.2) Find all angles between 0° and 360° that solve each given equation:

   a) $\cot\theta = 2.37$

   b) $\sec\theta = \dfrac{14}{9}$

   c) $\csc\theta = \dfrac{2}{3}$

9. (6.4) Suppose $\sin\theta = .815$ and $\cos\theta < 0$. Compute the values of all six trig functions of θ.

Solutions

1. a) We have $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{?}{12}$, so $? = \boxed{12\sec\theta}$.

   b) We have $\tan 26^\circ = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{?}{5.3}$, so $? = \boxed{5.3\tan 26^\circ}$.

   c) First, denote the vertical distance between the two right angles by w. From the left-hand triangle, $\sin\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{w}{4}$, so $w = 4\sin\theta$. From the right-hand triangle $\csc 2\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{?}{w}$, so $? = w\csc 2\theta = \boxed{4\sin\theta\csc 2\theta}$.

2. a) $\tan\theta = \dfrac{1}{\cot\theta} = \boxed{\dfrac{1}{p}}$.

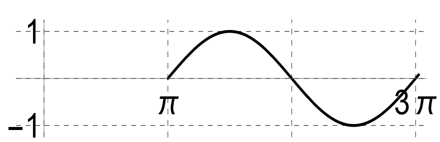
   b) The quotient identity for cotangent says $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, i.e. $p = \dfrac{\cos\theta}{q}$. Cross-multiply to get $\cot\theta = \boxed{pq}$.

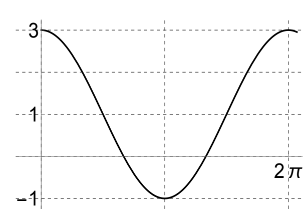
3. a) This is a sine graph (not flipped) with crosshair at $(0,0)$ (so $C = D = 0$); the amplitude is $A = 1$, and the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{3}$ (so $B = 3$). All together, this makes the equation $y = 1\sin 3(x-0)+0$, i.e. $\boxed{y = \sin 3x}$.

   b) This is a cosine graph with crosshair at $(0,4)$ (so $C = 0$, $D = 4$); the amplitude is $6-4 = 2$ so $A = 2$, and the period is 2π, so $B = 1$. All together, this makes the equation $y = 2\cos(x-0)+4$, i.e. $\boxed{y = 2\cos x + 4}$.

   c) This is the graph of $\boxed{y = \tan x}$.

4. a) $y = \sin(x-\pi)$ has crosshair at $(C,D) = (\pi,0)$, has amplitude $A = 1$ (so the top and bottom of the graph will be $y = 1$ and $y = -1$ respectively), and has period $\dfrac{2\pi}{B} = \dfrac{2\pi}{1} = 2\pi$, so one period should end at $x = C + \text{(period)} = \pi + 2\pi = 3\pi$. This makes the graph the one shown below, at left.

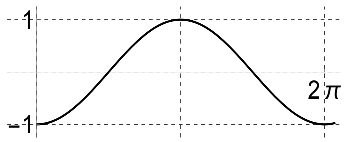


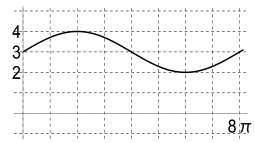


   b) $y = 2\cos x + 1$ has crosshair at $(C,D) = (0,1)$, has amplitude $A = 2$ (so the top and bottom of the graph will be $y = 1+2 = 3$ and $y = 1-2 = -1$

respectively), and has period $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$, so one period should end at $x = C + (\text{period}) = 0 + 2\pi = 2\pi$. This makes the graph the one shown above, at right.

c) $y = -\cos x$ is the graph of $y = \cos x$, flipped over (shown below at left).





d) $y = \sin\frac{x}{4} + 3$ has crosshair at $(C, D) = (0, 3)$, has amplitude $A = 1$ (so the top and bottom of the graph will be $y = 3 + 1 = 4$ and $y = 3 - 1 = 2$ respectively), and has period $\frac{2\pi}{B} = \frac{2\pi}{1/4} = 8\pi$, so one period should end at $x = C + (\text{period}) = 0 + 8\pi = 8\pi$. This makes the graph the one shown above, at right.

5. a) $\csc\frac{\pi}{3} = \boxed{\frac{2}{\sqrt{3}}}$

(reference angle $\widehat{\theta} = 60°$; Quadrant I)

b) $\cot\frac{3\pi}{2} = \frac{1}{\tan 270°} = \frac{1}{\text{DNE}} = \boxed{0}$

(line from origin to $(0, -1)$ on the unit circle is vertical so it has undefined slope)

c) $\sec 120° = \boxed{-2}$

(reference angle $\widehat{\theta} = 60°$; Quadrant II)

d) $\cos(-180°) = \boxed{-1}$

(point $(-1, 0)$ on the unit circle)

e) $\sec\frac{\pi}{4} = \boxed{\frac{2}{\sqrt{2}}}$

(reference angle $\widehat{\theta} = 45°$; Quadrant I)

f) $\sin\frac{5\pi}{6} = \boxed{\frac{1}{2}}$

(reference angle $\widehat{\theta} = 30°$; Quadrant II)

6. a) $5\cos(\pi + \pi) = 5\cos 2\pi = 5(1) = \boxed{5}$.

b) $\cot\frac{\pi}{3} - \frac{\pi}{3} = \boxed{\frac{1}{\sqrt{3}} - \frac{\pi}{3}}$.

c) $\csc^2(-45^\circ) = (\csc(-45^\circ))^2 = \left(-\frac{2}{\sqrt{2}}\right)^2 = \frac{4}{2} = \boxed{2}$.

d) $\sin\frac{3\pi}{4}\sec\frac{7\pi}{6} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{2}{\sqrt{3}}\right) = \boxed{-\frac{\sqrt{2}}{\sqrt{3}}}$.

e) $\csc\frac{8\pi}{5}\sin\frac{8\pi}{5} = \boxed{1}$.
(This applies the identity $\csc\theta = \frac{1}{\sin\theta}$.)

f) $\cot^2 471^\circ - \csc^2 471^\circ = \boxed{-1}$.
(This applies the identity $\csc^2\theta = 1 + \cot^2\theta$, so $\cot^2\theta - \csc^2\theta = -1$.)

7. a) $\csc 148.2^\circ = \frac{1}{\sin 148.2^\circ} = \frac{1}{.526956} = \boxed{1.89769}$.

b) $\cot 14^\circ = \frac{1}{\tan 14^\circ} = \frac{1}{.249328} = \boxed{4.01078}$.

c) $4\sec 73^\circ = 4\cdot\frac{1}{\cos 73^\circ} = 4\cdot\frac{1}{.292372} = 4(3.4203) = \boxed{13.6812}$.

d) $\sin 118^\circ + \cot(-132^\circ) = .882948 + \frac{1}{\tan(-132^\circ)} + .882948 + \frac{1}{1.11061} =$
$.882948 + .900404 = \boxed{1.78335}$

e) $\sec^2 15{\cdot}17^\circ = [\sec(15\cdot 17^\circ)]^2 = (\sec 255^\circ)^2 = \left(\frac{1}{\cos 255^\circ}\right)^2 = \left(\frac{1}{-.258819}\right)^2 =$
$(-3.8637)^2 = \boxed{14.9282}$.

8. a) Take reciprocals of both sides of $\cot\theta = 2.37$ to get $\tan\theta = \frac{1}{2.37} = .421941$. Now one angle that solves the equation is $\theta = \arctan .421941 = \boxed{22.88^\circ}$; a second angle is $180^\circ + 22.88^\circ = \boxed{202.88^\circ}$.

b) Take reciprocals of both sides of $\sec\theta = \frac{14}{9}$ to get $\cos\theta = \frac{9}{14}$; then $\theta = \arctan\frac{9}{14} = \boxed{50^\circ}$ is one solution. The other solution is $360^\circ - 50^\circ = \boxed{310^\circ}$.

c) Take reciprocals of both sides of $\csc\theta = \frac{2}{3}$ to get $\sin\theta = \frac{3}{2}$. Since $\frac{3}{2} > 1$, this equation has $\boxed{\text{no solution}}$.

9. First, since $\sin\theta > 0$ and $\cos\theta < 0$, θ is in Quadrant II, so $\sin\theta$ and $\csc\theta$ will be positive but the other four trig functions will be negative.

At this point, you could use a triangle to find the other trig functions, but I'll use identities. Start by using the Pythagorean Identity to find $\cos\theta$:

$$\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1 \\
(\cos\theta)^2 + (.815)^2 &= 1 \\
(\cos\theta)^2 + .664225 &= 1 \\
(\cos\theta)^2 &= .335775 \\
\cos\theta &= \pm\sqrt{.335775} \\
\cos\theta &= -.579461 \text{ (negative since } \theta \text{ is in Quad. II)}
\end{aligned}$$

Now,

$$\begin{aligned}
\sin\theta &= \boxed{.815} \text{ (this was given);} \\
\cos\theta &= \boxed{-.579461} \text{ (this was figured above);} \\
\tan\theta &= \frac{\sin\theta}{\cos\theta} = \frac{.815}{-.579461} = \boxed{-1.40648}; \\
\csc\theta &= \frac{1}{\sin\theta} = \frac{1}{.815} = \boxed{1.22699}; \\
\sec\theta &= \frac{1}{\cos\theta} = \frac{1}{-.579461} = \boxed{1.72574}; \\
\cot\theta &= \frac{1}{\tan\theta} = \frac{1}{-1.40648} = \boxed{-.710995}.
\end{aligned}$$

3.3 Fall 2022 Exam 3

1. $\boxed{\text{NC}}$ Suppose $\tan\theta = \dfrac{5}{3}$ and $\cos\theta < 0$.

   a) (6.1) What trig function of θ equals $\frac{3}{5}$?

   b) (6.4) Compute $\tan(-\theta)$.

   c) (6.4) Compute $\csc\theta$.

2. (6.3) $\boxed{\text{NC}}$ Compute the exact value of each quantity:

   a) $\tan 135°$

   b) $\tan\dfrac{7\pi}{6}$

   c) $\csc\dfrac{\pi}{4}$

   d) $\cot\dfrac{\pi}{2}$

   e) $\sec\dfrac{-2\pi}{3}$

3. (6.3, 6.4) $\boxed{\text{NC}}$ Compute the exact value of each expression:

   a) $1 - 2\cot^2\dfrac{\pi}{3}$

   b) $\cot^2 215° - \csc^2 215°$

   c) $\tan\dfrac{\pi}{2} + \cot\dfrac{7\pi}{4}$

   d) $\tan\left(\dfrac{\pi}{6} + \dfrac{5\pi}{6}\right)$

   e) $\cos 5\pi \sin\dfrac{5\pi}{2}$

   f) $\sin(-70°)\csc(-70°)$

4. (6.1) Suppose $(3.42, -1.75)$ is on the terminal side of angle θ, when θ is drawn in standard position. Compute $\sec\theta$.

5. (6.2) Find <u>all</u> angles θ between $0°$ and $360°$ that solve each equation:

   a) $\sec\theta = 3.15$

   b) $\cot\theta = -2.25$

Solutions

1. a) $\boxed{\cot\theta} = \frac{3}{5}$.

   b) $\tan(-\theta) = -\tan\theta = \boxed{-\dfrac{5}{3}}$.

c) Angles θ and $\theta + 180°$ are opposite to one another, so they have the same slope. Therefore $\tan(\theta + 180°) = \tan\theta = \boxed{\dfrac{5}{3}}$.

d) θ is in Quadrant III since $\tan\theta > 0$ and $\cos\theta < 0$. Therefore $\csc\theta < 0$.
Now, sketch a right triangle with opposite side 5 and adjacent side 3. The hypotenuse of this triangle is $\sqrt{5^2 + 3^2} = \sqrt{34}$. Finally,

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \boxed{-\frac{\sqrt{34}}{5}}.$$

2. a) $\tan 135° = \boxed{-1}$ (ref. angle 45°; Quadrant II)

   b) $\tan\dfrac{7\pi}{6} = \boxed{\dfrac{1}{\sqrt{3}}}$ (ref. angle 30°; Quadrant III)

   c) $\csc\dfrac{\pi}{4} = \boxed{\dfrac{2}{\sqrt{2}}}$ (45°; Quadrant I)

   d) $\cot\dfrac{\pi}{2} = \dfrac{1}{\tan 90°} = \dfrac{1}{\text{DNE}} = \boxed{0}$.

   e) $\sec\dfrac{-2\pi}{3} = \boxed{-2}$ (ref. angle 60°; Quadrant III)

3. a) $1 - 2\cot^2\dfrac{\pi}{3} = 1 - 2\left(\dfrac{1}{\sqrt{3}}\right)^2 = 1 - 2\left(\dfrac{1}{3}\right) = 1 - \dfrac{2}{3} = \boxed{\dfrac{1}{3}}$.

   b) $\cot^2 215° - \csc^2 215° = \boxed{-1}$ (from identity $\csc^2\theta = 1 + \cot^2\theta$).

   c) $\tan\dfrac{\pi}{2} + \cot\dfrac{7\pi}{4} = \text{ DNE } + (-1) = \boxed{\text{DNE}}$.

   d) $\tan\left(\dfrac{\pi}{6} + \dfrac{5\pi}{6}\right) = \tan\dfrac{6\pi}{6} = \tan\pi = \boxed{0}$.

   e) $\cos 5\pi \sin\dfrac{5\pi}{2} = (-1)(1) = \boxed{-1}$.

   f) $\sin(-70°)\csc(-70°) = \sin(-70°)\frac{1}{\sin(-70°)} = \boxed{1}$.

4. We have $x = 3.42$ and $y = -1.75$ and need to find r: $r = \sqrt{x^2 + y^2} = \sqrt{3.42^2 + (-1.75)^2} = 3.8417$. Thus $\sec\theta = \dfrac{r}{x} = \dfrac{3.8417}{3.42} = \boxed{1.123}$.

5. a) Take reciprocals of both sides to get $\cos\theta = \frac{1}{3.15} = .3175$. Thus $\theta = \arccos .3175 = \boxed{71.5°}$ is one answer, and the other is $\theta = 360° - 71.5° = \boxed{288.5°}$.

   b) Take reciprocals of both sides to get $\tan\theta = \frac{1}{-2.25} = -.444$. Thus $\theta = \arctan -.444 = -24°$ would be an answer, but it isn't between 0° and 360°. So add 360° to get $-24° + 360° = \boxed{336°}$ as one solution. The other solution is $180° + \theta = 180° + (-24°) = \boxed{156°}$.

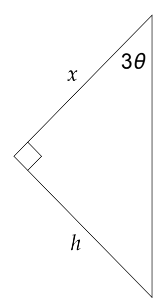
3.4 Fall 2019 Exam 3

1. $\boxed{\text{NC}}$ Throughout this problem, assume $\cot\theta = \frac{-1}{3}$ and $\sin\theta < 0$.
   a) (6.4) What is $\cot(-\theta)$?
   b) (6.2) What quadrant does θ lie in?
   c) (6.4) Find the exact value of $\cos\theta$.

2. (6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:
   a) $\csc\dfrac{\pi}{2}$
   b) $\cos 240°$
   c) $\tan 7\pi$
   d) $\sec\dfrac{3\pi}{2}$
   e) $\tan\dfrac{3\pi}{4}$
   f) $\cot\dfrac{13\pi}{6}$

3. (6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:
   a) $\tan\dfrac{-\pi}{2}$
   b) $\cos^3\dfrac{-\pi}{3}$
   c) $\cos\dfrac{7\pi}{6}\sec\dfrac{\pi}{4}$
   d) $\cot(30° - 90°)$
   e) $4\sin\dfrac{5\pi}{6} + 1$

4. (6.1) $\boxed{\text{NC}}$ In the diagram below, write an equation for x in terms of the other quantities in the picture. Your formula for x should not contain division.



5. a) (6.2) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\tan\theta = .615$.
   b) (6.2) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sec\theta = 4.38$.

Solutions

1. a) $\cot(-\theta) = -\cot\theta = \boxed{\frac{1}{3}}$.

   b) Since $\cot\theta < 0$ and $\sin\theta < 0$, θ lies in $\boxed{\text{Quadrant IV}}$.

   c) First, since θ is in Quadrant IV, $\cos\theta$ is positive. Draw a triangle, labelling the adjacent side to θ as 1 and the opposite side to θ as 3. Solve for the hypotenuse using the Pythagorean Theorem to get $\sqrt{10}$. Thus $\cos\theta = -\frac{\text{adj}}{\text{hyp}} = \boxed{\frac{1}{\sqrt{10}}}$.

2. a) $\csc\frac{\pi}{2} = \frac{1}{\sin\frac{\pi}{2}} = \frac{1}{1} = \boxed{1}$.

   b) $\cos 240° = \boxed{-\frac{1}{2}}$ (Quadrant III; reference angle $60°$).

   c) $\tan 7\pi = \boxed{0}$ (the point $(-1, 0)$ on the unit circle).

   d) $\sec\frac{3\pi}{2} = \frac{1}{\cos\frac{3\pi}{2}} = \frac{1}{0}$ $\boxed{\text{DNE}}$ (the point $(0, -1)$ on the unit circle).

   e) $\tan\frac{3\pi}{4} = \boxed{-1}$ (Quadrant II; reference angle $45°$).

   f) $\cot\frac{13\pi}{6} = \boxed{\sqrt{3}}$ (Quadrant I; reference angle $30°$).

3. a) $\tan\frac{-\pi}{2}$ $\boxed{\text{DNE}}$ (the point $(0, -1)$ on the unit circle).

   b) $\cos^3\frac{-\pi}{3} = (\cos\frac{-\pi}{3})^3 = \left(\frac{1}{2}\right)^3 = \boxed{\frac{1}{8}}$.

   c) $\cos\frac{7\pi}{6}\sec\frac{\pi}{4} = \left(\frac{-\sqrt{3}}{2}\right)\left(\sqrt{2}\right) = \boxed{-\frac{\sqrt{6}}{2}}$.

   d) $\cot(30° - 90°) = \cot(-60°) = \boxed{-\frac{1}{\sqrt{3}}}$ (Quadrant IV; reference angle $60°$).

   e) $4\sin\frac{5\pi}{6} + 1 = 4\left(\frac{1}{2}\right) + 1 = 2 + 1 = \boxed{3}$.

4. $\cot 3\theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{h}$, so $\boxed{x = h\cot 3\theta}$.

5. a) $\theta = \arctan .615 = \boxed{31.6°}$; the second angle is $\theta + 180° = \boxed{211.6°}$.

   b) First, take reciprocals to get $\cos\theta = \frac{1}{4.38} = .228$. Then $\theta = \arccos .228 = \boxed{76.8°}$; the other angle is $360° - 76.8° = \boxed{283.2°}$

3.5 Fall 2018 Exam 3

No calculator allowed on these problems

1. $\boxed{\text{NC}}$ Classify the following statements as true or false:

   a) (6.4) If θ is in Quadrant II, then $\sec\theta > 0$.

   b) (6.4) If $\tan\theta = -2$, then $\tan(-\theta) = 2$.

   c) (6.4) For any angle θ, $\sec^2\theta + 1 = \tan^2\theta$.

   d) (6.2) It is possible to solve the equation $\csc\theta = \dfrac{1}{4}$ for θ.

   e) (6.2) In a right triangle, $\csc\theta$ is the hypotenuse divided by the opposite side.

2. $\boxed{\text{NC}}$ Throughout this problem, assume $\tan\theta = -3$ and $\cos\theta < 0$.

   a) (6.4) What is $\tan(-\theta)$?

   b) (6.4) What is $\tan(\theta + 2\pi)$?

   c) (6.2) What quadrant does θ lie in?

   d) (6.4) Find the exact value of $\sin\theta$.

3. (6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\csc\dfrac{4\pi}{3}$

   b) $\tan 120°$

   c) $\tan\dfrac{-\pi}{2}$

   d) $\sec 0$

   e) $\sin\dfrac{3\pi}{4}$

   f) $\cot 270°$

4. (6.3, 6.4) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\sin^2\dfrac{7\pi}{12} + \cos^2\dfrac{7\pi}{12}$

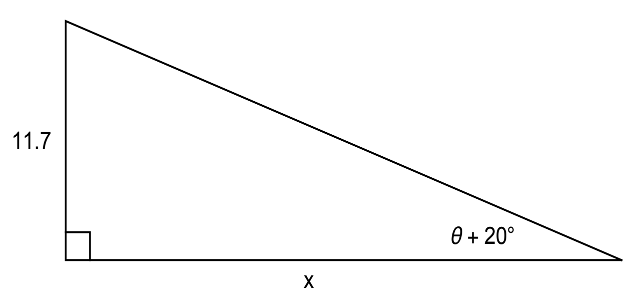
   b) $8\csc(180° - 30°)$

   c) $\sec 120° + \sec 45°$

   d) $\cot^2\dfrac{\pi}{6}$

   e) $\tan\dfrac{3\pi}{4} - 1$

5. In the diagram below, write an equation for x in terms of the other quantities in the picture. Your formula for x should not contain division.



6. a) Find all angles θ between $0°$ and $360°$ such that $\tan\theta = .773$.
   b) Find all angles θ between $0°$ and $360°$ such that $\csc\theta = 4.38$.

7. Suppose $\cos\theta = .285$ and $\sin\theta > 0$. Find $\tan\theta$ and $\sec\theta$.

Solutions

1. a) FALSE ($\sec\theta$ and $\cos\theta$ are positive in Quadrants I and IV).
   b) TRUE ($\tan(-\theta) = -\tan\theta$).
   c) FALSE ($\tan^2\theta + 1 = \sec^2\theta$, not the other way around).
   d) FALSE ($\frac{1}{4}$ is between -1 and 1)
   e) TRUE ($\csc\theta = \frac{1}{\sin\theta}$ is the hypotenuse divided by the opposite side).

2. a) $\tan(-\theta) = -\tan\theta = -(-3) = \boxed{3}$.
   b) $\tan(\theta + 2\pi) = \tan\theta = \boxed{-3}$.
   c) Since $\tan\theta < 0$ and $\cos\theta < 0$, θ is in $\boxed{\text{Quadrant II}}$.
   d) Draw a right triangle and label the opposite side 3 and the adjacent side 1. Solve for the hypotenuse using the Pythagorean Theorem to get $\sqrt{3^2 + 1^2} = \sqrt{10}$. Then $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \boxed{\dfrac{3}{\sqrt{10}}}$ (this answer is positive since θ is in Quadrant II).

3. a) $\csc\frac{4\pi}{3} = \boxed{-\dfrac{2}{\sqrt{3}}}$ (Quadrant III; reference angle $\frac{\pi}{3} = 60°$).

   b) $\tan 120° = \boxed{-\sqrt{3}}$ (Quadrant II; reference angle $60°$).

   c) $\tan\frac{-\pi}{2}$ $\boxed{\text{DNE}}$ (slope of a vertical line does not exist).

   d) $\sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = \boxed{1}$

   e) $\sin\frac{3\pi}{4} = \boxed{\dfrac{\sqrt{2}}{2}}$ (Quadrant II; reference angle $\frac{\pi}{4} = 45°$).

   f) $\cot 270° = \frac{1}{\text{DNE}} = \boxed{0}$ (the slope of a vertical line DNE; cotangent is 1 over the slope).

4. a) $\sin^2\frac{7\pi}{12} + \cos^2\frac{7\pi}{12} = \boxed{1}$ (Pythagorean identity).

   b) $8\csc(180° - 30°) = 8\csc 150° = 8 \cdot 2 = \boxed{16}$.

   c) Do each part separately to get $\sec 120° + \sec 45° = \boxed{-2 + \sqrt{2}}$.

   d) $\cot^2\frac{\pi}{6} = \left(\sqrt{3}\right)^2 = \boxed{3}$.

   e) $\tan\frac{3\pi}{4} - 1 = -1 - 1 = \boxed{-2}$.

5. $\cot(\theta + 20°) = \frac{\text{adj}}{\text{opp}} = \frac{x}{11.7}$, so $\boxed{x = 11.7\cot(\theta + 20°)}$.

   NOTE: the parenthesis are important in this answer.

6. a) One such angle is $\theta = \arctan .773 = \boxed{37.7°}$; the other angle is $\theta + 180° = \boxed{217.7°}$.

   b) Take reciprocals of both sides of $\csc\theta = 4.38$ to get $\sin\theta = \frac{1}{4.38} = .2283$. Then, one angle is $\theta = \arcsin .2283 = \boxed{13.2°}$; the other angle is $180° - \theta = \boxed{166.8°}$.

7. First, $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{.285} = \boxed{3.509}$.

   Second, draw a triangle with adjacent side $.285$ and hypotenuse 1; use the Pythagorean Theorem to find the opposite side, which is $\sqrt{1^2 - .285^2}$ which is $\sqrt{1 - .081225} = \sqrt{.918775} = .958$. Thus $\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{.958}{.285} = \boxed{3.36}$.

3.6 Fall 2017 Exam 3

1. $\boxed{\text{NC}}$ Throughout this problem, assume $\cot\theta = \dfrac{5}{3}$.

   a) (6.4) What is $\cot(-\theta)$?

   b) (6.1) What is $\tan\theta$?

   c) (6.2) Which two quadrants might θ be in?

   d) (6.4) If $\sin\theta < 0$, find $\cos\theta$.

2. (6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\csc 135°$

   b) $\tan 60°$

   c) $\cot 180°$

   d) $\sec(-225°)$

   e) $\cot \dfrac{3\pi}{2}$

   f) $\sec \dfrac{\pi}{6}$

   g) $\tan \dfrac{3\pi}{4}$

3. (6.3, 6.4) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\sec^2 \dfrac{4\pi}{3}$

   b) $8 \sin \dfrac{\pi}{6}$

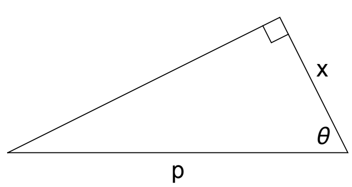
   c) $\sin^2 215° + \cos^2 215°$

   d) $\tan(2 \cdot 90°)$

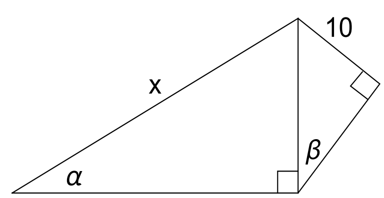
   e) $\cos 2\pi + 1$

4. (6.2) $\boxed{\text{NC}}$ In each diagram below, write an equation for x in terms of the other given quantities in the picture. Your formula for x should not contain division in it.

a)



b)



5. (6.4) Suppose that θ is some angle such that when drawn in standard position, $(-7, 12)$ is on the terminal side of θ. Find $\sec\theta$ and $\tan\theta$.

6. (6.1) Use a calculator to compute decimal approximations of these quantities:

   a) $\csc 23°$

   b) $\tan 140° + \cot 140°$

c) $\cos^2 213°$

7. a) (6.2) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\tan\theta = 4.125$.

   b) (6.2) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sec\theta = 3.38$.

Solutions

1. a) $\cot(-\theta) = -\cot\theta = \boxed{-\frac{5}{3}}$.

   b) $\tan\theta = \frac{1}{\cot\theta} = \boxed{\frac{3}{5}}$.

   c) $\cot\theta$ is positive in $\boxed{\text{Quadrants I and III}}$.

   d) Since $\cot\theta > 0$ and $\sin\theta < 0$, θ is in Quadrant III, so $\cos\theta$ will be $\boxed{\text{negative}}$.

   Now draw a triangle and label the adjacent side as 5 and the opposite side as 3. Solve for the hypotenuse using the Pythagorean Theorem to get $\sqrt{5^2+3^2} = \sqrt{34}$. Finally, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \boxed{-\frac{5}{\sqrt{34}}}$.

2. a) $\csc 135° = \boxed{\sqrt{2}}$.

   b) $\tan 60° = \boxed{\sqrt{3}}$.

   c) $\cot 180°$ $\boxed{\text{DNE}}$.

   d) $\sec(-225°) = \sec 225° = \boxed{-\sqrt{2}}$.

   e) $\cot\frac{3\pi}{2} = \boxed{0}$.

   f) $\sec\frac{\pi}{6} = \boxed{\frac{2}{\sqrt{3}}}$.

g) $\tan \frac{3\pi}{4} = \boxed{-1}$.

3. a) $\sec^2 \frac{4\pi}{3} = (-2)^2 = \boxed{4}$.
   b) $8 \sin \frac{\pi}{6} = 8 \cdot \frac{1}{2} = \boxed{4}$.
   c) $\sin^2 215° + \cos^2 215° = \boxed{1}$.
   d) $\tan(2 \cdot 90°) = \tan 180° = \boxed{0}$.
   e) $\cos 2\pi + 1 = 1 + 1 = \boxed{2}$.

4. a) $\dfrac{x}{p} = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta$, so $\boxed{x = p\cos\theta}$.
   b) Label the vertical segment common to both triangles as w. Then $w = 10\csc\beta$ and $x = w\csc\alpha$, so by substitution $\boxed{x = 10\csc\beta\csc\alpha}$.

5. We have $x = -7$ and $y = 12$ so $r = \sqrt{x^2+y^2} = \sqrt{(-7)^2+12^2} = \sqrt{49+144} = \sqrt{193}$. Therefore

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{193}}{-7} \approx -1.984 \qquad \text{and} \qquad \tan\theta = \frac{y}{x} = \boxed{\frac{12}{-7}} \approx \boxed{-1.714}.$$

6. a) $\csc 23° = \dfrac{1}{\sin 23°} = \boxed{2.5593}$.
   b) $\tan 140° + \cot 140° = -.8391 + (-1.1917) = \boxed{-2.0308}$.
   c) $\cos^2 213° = (-.8386)^2 = \boxed{-.7033}$.

7. a) $\theta = \arctan 4.125 = \boxed{76.37°}$. The other angle is $180° + \theta = \boxed{256.37°}$.
   b) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{3.38} = .2958$. Therefore $\theta = \arccos .2958 = \boxed{72.79°}$. The other angle is $360° - \theta = \boxed{287.21°}$.

3.7 Fall 2016 Exam 3

1. $\boxed{\text{NC}}$ Throughout this problem, assume $\sec\theta = 4$.

   a) (6.4) What is $\sec(-\theta)$?

   b) (6.1) What is $\cos\theta$?

   c) (6.4) What is $\sec(\theta - 360°)$?

   d) (6.2) Which two quadrants might θ be in?

   e) (6.4) If $\tan\theta < 0$, find $\cot\theta$.

2. (6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\sec 45°$

   b) $\tan 150°$

   c) $\cot 180°$

   d) $\csc(-120°)$

   e) $\sec 60°$

3. (6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\tan 5\pi$

   b) $\csc \frac{\pi}{6}$

   c) $\tan \frac{3\pi}{4}$

   d) $\cos \frac{\pi}{2}$

   e) $\sin \frac{2\pi}{3}$

4. (6.3, 6.4) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\sin^2 125° + \cos^2 125°$

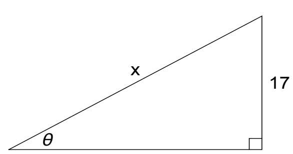
   b) $\cot(45° + 45°)$

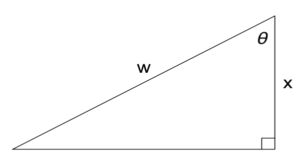
   c) $\tan^2 \frac{5\pi}{6}$

   d) $3 \sin \frac{\pi}{2}$

   e) $\sin 3 \cdot \frac{\pi}{2}$

5. (6.2) In each diagram below, write an equation for x in terms of the other given quantities in the picture.

   a) 

   b) 

6. (6.4) Suppose that $\sin\theta = .614$ and that $\tan\theta > 0$. Find the values of all six trig functions of θ.

7. (6.1) Use a calculator to compute these quantities:

a) $\csc 75°$

b) $\cot 40° + \cot 70°$

c) $\tan^2 125°$

d) $3 \sec 40°$

8. a) (6.2) Find all angles θ between $0°$ and $360°$ such that $\tan\theta = .72$.

   b) (6.2) Find all angles θ between $0°$ and $360°$ such that $\sec\theta = 2.8$.

   c) (6.2) Find all angles θ between $0°$ and $360°$ such that $\csc\theta = .35$.

Solutions

1. a) $\sec(-\theta) = \sec\theta = \boxed{4}$.

   b) $\cos\theta = \frac{1}{\sec\theta} = \boxed{\dfrac{1}{4}}$.

   c) $\sec(\theta - 360°) = \sec\theta = \boxed{4}$.

   d) θ must be in $\boxed{\text{Quadrant I or Quadrant IV}}$.

   e) Since $\tan\theta < 0$, we now know θ is in Quadrant IV, so $\cot\theta < 0$. Sketch a triangle with hypotenuse 4 and adjacent side of length 1. From the Pythagorean Theorem, the opposite side is $\sqrt{4^2 - 1^2} = \sqrt{15}$, so $\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \boxed{\dfrac{1}{\sqrt{15}}}$.

2. a) $\sec 45° = \boxed{\sqrt{2}}$.

   b) $\tan 150° = \boxed{-\dfrac{1}{\sqrt{3}}}$ (reference angle $30°$, Quadrant II).

   c) $\cot 180°$ $\boxed{\text{DNE}}$.

d) $\csc(-120°) = \boxed{-\dfrac{2}{\sqrt{3}}}$ (reference angle $60°$, Quadrant III).

e) $\sec 60° = \boxed{2}$.

3. a) $\tan 5\pi = \boxed{0}$

   b) $\csc \frac{\pi}{6} = \boxed{2}$

   c) $\tan \frac{3\pi}{4} = \boxed{-1}$ (reference angle $45°$, Quadrant II)

   d) $\cos \frac{\pi}{2} = \boxed{0}$

   e) $\sin \frac{2\pi}{3} = \boxed{\dfrac{\sqrt{3}}{2}}$ (reference angle $60°$, Quadrant II)

4. a) $\sin^2 125° + \cos^2 125° = \boxed{1}$ (by a Pythagorean identity)

   b) $\cot(45° + 45°) = \cot 90° = \boxed{0}$

   c) $\tan^2 \frac{5\pi}{6} = \left(\frac{-1}{\sqrt{3}}\right)^2 = \boxed{\dfrac{1}{3}}$

   d) $3 \sin \frac{\pi}{2} = 3(1) = \boxed{3}$

   e) $\sin 3 \cdot \frac{\pi}{2} = \boxed{-1}$

5. a) $\dfrac{x}{17} = \dfrac{\text{hyp}}{\text{opp}} = \csc\theta$ so $\boxed{x = 17\csc\theta}$.

   b) $\dfrac{x}{w} = \dfrac{\text{adj}}{\text{hyp}} = \cos\theta$ so $\boxed{x = w\cos\theta}$.

6. Since $\sin\theta > 0$ and $\tan\theta > 0$, θ is in Quadrant I, so all the trig functions are positive. We are given $\sin\theta = \boxed{.614}$, so $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{.614} = \boxed{1.629}$. From the identity $\sin^2\theta + \cos^2\theta = 1$, we can solve for $\cos\theta$ to get $\cos\theta = \boxed{.789}$. Then $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{.789} = \boxed{1.267}$. Last, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{.614}{.789} = \boxed{.777}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{.789}{.614} = \boxed{1.285}$.

7. a) $\csc 75° = \boxed{1.03528}$.

   b) $\cot 40° + \cot 70° = 1.19175 + .36397 = \boxed{1.55572}$.

   c) $\tan^2 125° = (-1.42815)^2 = \boxed{2.03961}$.

   d) $3\sec 40° = 3(1.30541) = \boxed{3.91622}$.

8. a) $\theta = \arctan .72 = \boxed{35.7539°}$; a second angle is $\theta + 180° = \boxed{215.7539°}$.

   b) If $\sec\theta = 2.8$, then $\cos\theta = \frac{1}{2.8}$ so $\theta = \arccos\frac{1}{2.8} = \boxed{69.07°}$. A second angle is $360° - \theta = \boxed{290.93°}$.

   c) There are $\boxed{\text{no such angles}}$, because $\csc\theta$ cannot be between -1 and 1.