Old MATH 120 Exams

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Contents

4.1 Final Exam Information

Final exam content

The final exam covers the entire course (all 7 chapters of my MATH 120 lecture notes).

Final exam tasks

1. You should be able to handle any task listed under those listed in the study guides for Exams 1, 2 and/or 3. Of these, the most important are:

   - evaluate expressions involving trig functions of special angles;
   - solve triangles;
   - answer questions using trig identities and/or reflection properties;

- solve basic trig equations (like $\sin\theta = q$);
- convert between degrees, radians and revolutions;
- sketch sinusoidal graphs; and
- perform vector computations both numerically and using pictures.

2. New tasks coming from Chapter 7 of the lecture notes:

- $\boxed{\text{NC}}$ Simplify a trig expression using identities
- $\boxed{\text{NC}}$ Verify a trigonometric identity
- $\boxed{\text{NC}}$ Evaluating trigonometric expressions using sum/difference identities or double-angle/half-angle identities (you need not memorize these identities)

4.2 Fall 2023 Final Exam

NOTE: This exam did not include any questions from Chapter 7 of my lecture notes, as we did not have time in Fall 2023 to cover that chapter.

1. $\boxed{\text{NC}}$ Compute the exact value of each quantity:

   a) (3.6) $\sin(-3\pi)$

   b) (3.6) $\sin \dfrac{2\pi}{3}$

   c) (3.6) $\tan 135^\circ$

   d) (3.6) $\cos \dfrac{3\pi}{2}$

   e) (6.3) $\sec \dfrac{5\pi}{4}$

2. $\boxed{\text{NC}}$ Compute the exact value of each quantity:

   a) (3.6) $\cos 690^\circ$

   b) (6.3) $4 \cot \dfrac{\pi}{4}$

   c) (3.6) $\sin\left(\dfrac{5\pi}{6} + \dfrac{\pi}{6}\right)$

   d) (6.4) $\cos 325^\circ \sec(-325^\circ)$

   e) (3.6) $10 \cos 2 \cdot 30^\circ$

3. $\boxed{\text{NC}}$ Compute the exact value of each quantity:

   a) (3.6) $\tan \dfrac{4\pi}{3}$

   b) (3.6) $\cos^3 720^\circ$

   c) (6.4) $\cos^2 5^\circ + \sin^2 365^\circ$

   d) (6.3) $\csc 30^\circ \tan 90^\circ$

   e) (3.6) $\sin \dfrac{\pi}{6} + \tan \dfrac{\pi}{4}$

4. $\boxed{\text{NC}}$ In parts (a), (b) and (c), estimate the given quantity by sketching a picture on the unit circle.

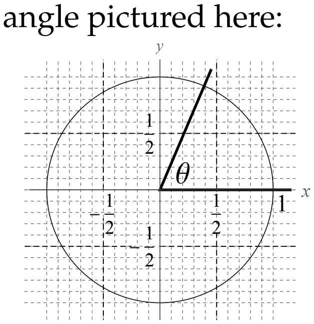
   a) (3.1) $\sin 288^\circ$

   b) (3.1) $\tan 470^\circ$

   c) (3.1) $\sin 167^\circ$

   (d) (6.1) Based on your answer to part (c), what is a reasonable estimate of $\csc 167^\circ$? (This answer does not need to be simplified.)

5. (3.1) $\boxed{\text{NC}}$ Suppose θ is the angle pictured here:

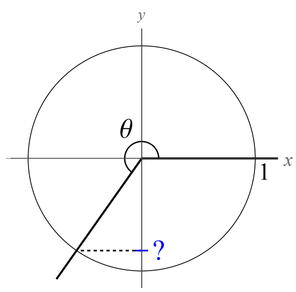


   Use this picture to estimate each quantity:

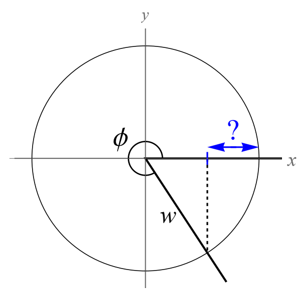
a) $\cos\theta$ b) $2\cos\theta$ c) $\cos 2\theta$

6. $\boxed{\text{NC}}$ In parts (a)-(e), give a formula in terms of the numbers and/or variables given in the picture that yields the length or coordinate indicated by the "?". Your formula should not contain any division.

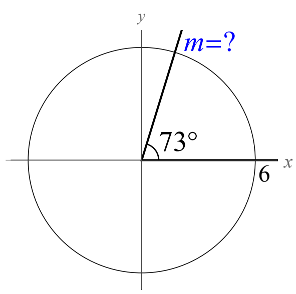
a) (3.1)



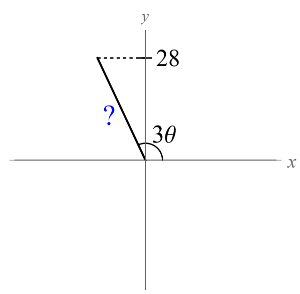
(d) (3.2)



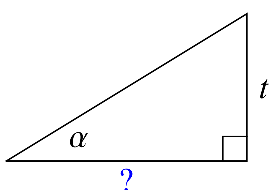
b) (3.2)



(e) (6.2)



(c) (4.3)



(f) (4.3) A 14-foot long ladder leans up against a wall. Write a formula for the distance from the bottom of the ladder to the bottom of the wall, in terms of the angle θ the ladder makes with the ground.

7. (3.4, 3.5) $\boxed{\text{NC}}$ Throughout this problem, suppose $\sin\theta = -\dfrac{3}{5}$ and $\cos\theta < 0$.

a) Compute the exact value of $\sin(360^\circ + \theta)$.

b) Compute the exact value of $\sin(\theta + 180^\circ)$.

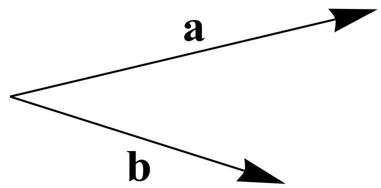
c) Compute the exact value of $\sin(180^\circ - \theta)$.

d) Compute the exact value of $\sin\theta + \sin(-\theta)$.

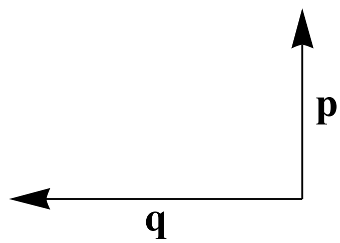
e) Compute the exact value of $\cos\theta$.

f) Compute the exact value of $\cos\theta + \cos(-\theta)$.

8. a) (2.4) $\boxed{\text{NC}}$ Vectors $\mathbf{a}$ and $\mathbf{b}$ are shown below. Sketch the vectors $\frac{1}{4}\mathbf{a}$ and $\mathbf{a} + \mathbf{b}$ on the same picture, labelling which of your answers is which.



b) (2.4) $\boxed{\text{NC}}$ Vectors $\mathbf{p}$ and $\mathbf{q}$ are shown below. Sketch the vector $2\mathbf{p} - \mathbf{q}$ on the same picture.



c) (4.4) $\boxed{\text{NC}}$ Let $\mathbf{p}$ and $\mathbf{q}$ be as in part (b). Is $\mathbf{p} \cdot \mathbf{q}$ positive, negative, or zero? Explain your answer.

9. (5.3) $\boxed{\text{NC}}$ Sketch a crude graph of each of these functions:

a) $y = 2\cos x$

b) $y = \dfrac{1}{2}\sin(x + 10)$

c) $y = -\cos x - 3$

d) $y = \sin \pi x$

10. (3.3, 6.1) Evaluate each of these expressions using a calculator. Your answers can (and should) be written as decimals.

a) $\sin(135° - 24°)$

b) $\tan^2 145°$

c) $\cot 3 \cdot 104°$

d) $-3 \sec 136°$

e) $\cos 104° \csc 358°$

11. (3.7, 6.2) In each part of this problem, find all angles between $0°$ and $360°$ that solve the equation.

a) $\cos\theta = \dfrac{5}{13}$

b) $\csc\theta = 2.13$

c) $\sin\theta = .415$

d) $\tan\theta = -\dfrac{26}{17}$

12. a) (2.9) Convert 17.4 radians to degrees, writing your answer as a decimal.

b) (2.9) Convert $613.5°$ to radians, writing your answer as a decimal.

c) (2.10) A bee flies in a circular path of radius 3 feet, with an angular velocity of $80°$ per second. What is the linear velocity of the bee?

d) (2.10) Compute the area of a circular sector taken from a circle of radius 4.5 in, if the central angle of the arc measures $53°$.

13. Throughout this problem, let $\mathbf{v} = \langle 12.3, -8.2 \rangle$ and let $\mathbf{w} = \langle 5.2, 9.3 \rangle$.

    a) (2.4) Compute $3\mathbf{w} + 2\mathbf{v}$.

    b) (2.4) Compute $\mathbf{v} \cdot \mathbf{w}$.

    c) (2.7) Compute the magnitude of $\mathbf{v}$.

    d) (3.7) Compute the direction angle of $\mathbf{w}$.

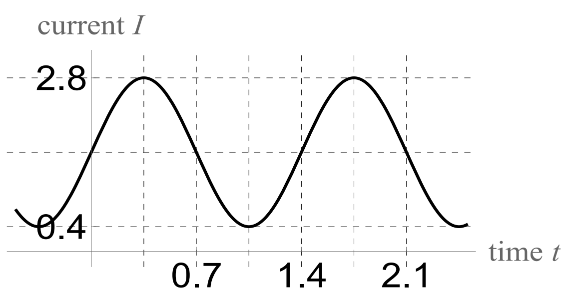
14. a) (4.1) A person looks out from an observation tower of height 120 feet and sees a piece of trash on the ground. If the angle of depression from the person to the trash is $64°$, how far from the base of the tower is the piece of trash?

    b) (3.3) A mark is made on the left edge of a wheel that has radius 19 in. If the wheel is rotated through an angle of $205°$ counterclockwise, how far to the left or right of the center of the wheel does the mark end up?

15. (4.2) Solve $\triangle KLM$, if $k = 14$, $l = 17$ and $m = 19$.

16. (4.1) Solve $\triangle DEF$, if $\angle D = 117°$, $\angle E = 26°$ and $e = 4$.

17. **(Bonus)** (5.3) The current flowing through a circuit at time t has this graph:



Write a formula for the current in terms of t, and use your formula to determine (a decimal approximation to) the current flowing through the circuit at time 100.

Solutions

1. a) $\sin(-3\pi) = \boxed{0}$ (point $(-1, 0)$ on the unit circle).

   b) $\sin\dfrac{2\pi}{3} = \boxed{\dfrac{\sqrt{3}}{2}}$ (Quadrant II; reference angle $60°$).

   c) $\tan 135° = \boxed{-1}$ (Quadrant II; reference angle $45°$).

   d) $\cos\dfrac{3\pi}{2} = \boxed{0}$ (point $(0, -1)$ on the unit circle).

   e) $\sec\dfrac{5\pi}{4} = \boxed{-\dfrac{2}{\sqrt{2}}}$ (Quadrant III; reference angle $45°$).

2. a) $\cos 690° = \boxed{\dfrac{\sqrt{3}}{2}}$ (Quadrant IV; reference angle $30°$).

   b) $4\cot\dfrac{\pi}{4} = 4(1) = \boxed{4}$.

   c) $\sin\left(\dfrac{5\pi}{6} + \dfrac{\pi}{6}\right) = \sin\dfrac{6\pi}{6} = \sin\pi = \boxed{0}$.

   d) $\cos 325° \sec(-325°) = \cos 325° \left(\dfrac{1}{\cos 325°}\right) = \boxed{1}$.

   e) $10\cos 2 \cdot 30° = 10\cos 60° = 10 \cdot \dfrac{1}{2} = \boxed{5}$.

3. a) $\tan\dfrac{4\pi}{3} = \boxed{\sqrt{3}}$ (Quadrant III; reference angle $60°$).

   b) $\cos^3 720° = (\cos 720°)^3 = 1^3 = \boxed{1}$.

   c) Let $\theta = 5°$ so that $\cos^2 5° + \sin^2 365° = \cos^2\theta + \sin^2(\theta + 360°)$. By periodicity, the $360°$ can be ignored, giving $\cos^2\theta + \sin^2\theta = \boxed{1}$.

   d) $\csc 30° \tan 90° = 2(\text{DNE}) = \boxed{\text{DNE}}$.

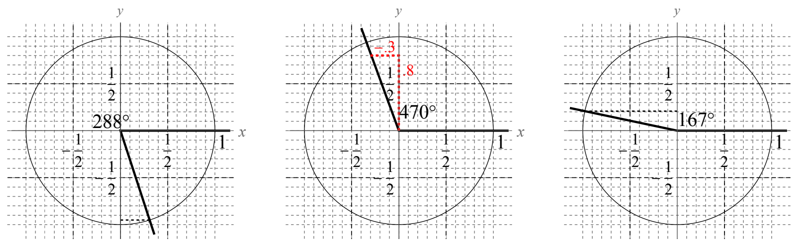
   e) $\sin\dfrac{\pi}{6} + \tan\dfrac{\pi}{4} = \dfrac{1}{2} + 1 = \boxed{\dfrac{3}{2}}$.

4. Answers may vary in these problems, since they are just estimates.

   a) $\sin 288° \approx \boxed{-.95}$ from the left-hand picture below.

   b) $\tan 470° \approx \boxed{-\dfrac{8}{3}}$ from the middle picture below.

c) $\sin 168° \approx \boxed{.2}$ from the right-hand picture below.

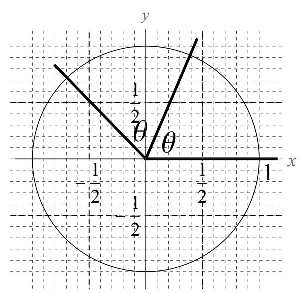


d) $\csc 168° = \dfrac{1}{\sin 167°} \approx \boxed{\dfrac{1}{.2}} = \boxed{5}$.

5. a) $\cos\theta \approx \boxed{.4}$, the x-coordinate of the point on the unit circle at θ.

b) Multiply the answer from (a) by 2 to get $2\cos\theta \approx 2(.4) = \boxed{.8}$.

c) For $\cos 2\theta$, first sketch the angle 2θ on the unit circle:



From this picture, $\cos 2\theta$ is the $x-$coordinate of the point on the unit circle at 2θ, which is about $\boxed{-.7}$.

6. a) $\boxed{? = \sin\theta}$.

b) $\boxed{? = \tan 73°}$ (the radius is irrelevant to tangent).

c) $\dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{?}{t} = \cot\alpha$, so $\boxed{? = t\cot\alpha}$.

d) The x-coordinate at the left edge of the length marked with the ? is $w\cos\phi$. The x-coordinate at the right edge of the length marked with the ? is the radius w, so we get ? by subtracting endpoints: $\boxed{? = w - w\cos\phi}$.

e) We are given $y = 28$ and $r =?$, so $\frac{r}{y} = \frac{?}{28} = \frac{r}{y} = \csc 3\theta$. Therefore $\boxed{? = 28\csc 3\theta}$.

f) Calling the distance we want "?", we have $\dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{?}{14} = \cos\theta$, so $\boxed{? = 14\cos\theta}$.

7. a) By periodicity, $\sin(360° + \theta) = \sin\theta = \boxed{-\frac{3}{5}}$.

   b) $\theta + 180°$ is opposite to θ, so its sine is the opposite of $\sin\theta$. In other words, $\sin(\theta + 180°) = -\sin\theta = \boxed{\frac{3}{5}}$.

   c) $180° - \theta$ is across the y-axis from θ. Thus it has the same sine as θ. In other words, $\sin(180° - \theta) = \sin\theta = \boxed{-\frac{3}{5}}$.

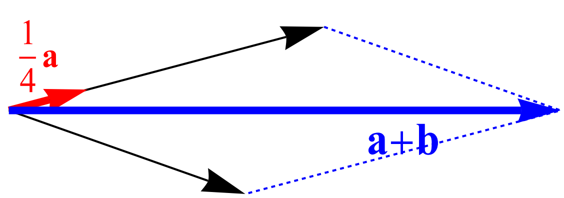
   d) By the odd-even identity for sine, $\sin\theta + \sin(-\theta) = \sin\theta - \sin\theta = \boxed{0}$.

   e) Use a Pythagorean identity:

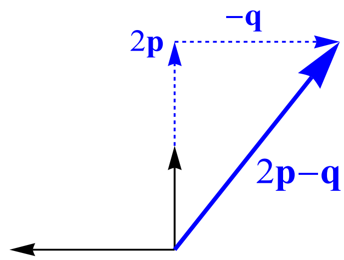
$$\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1 \\
\cos^2\theta + \left(-\frac{3}{5}\right)^2 &= 1 \\
\cos^2\theta + \frac{9}{25} &= \frac{25}{25} \\
\cos^2\theta &= \frac{16}{25} \\
\cos\theta &= \pm\sqrt{\frac{16}{25}} \\
\cos\theta &= \boxed{-\frac{4}{5}} \quad \text{(since } \cos\theta < 0\text{)}
\end{aligned}$$

   f) By the odd-even identity for cosine, $\cos\theta + \cos(-\theta) = \cos\theta + \cos\theta = -\frac{4}{5} - \frac{4}{5} = \boxed{-\frac{8}{5}}$.

8. a) For $\frac{1}{4}\mathbf{a}$, sketch a vector in the same direction as $\mathbf{a}$ but $\frac{1}{4}$ as long. For $\mathbf{a} + \mathbf{b}$, use head-to-tail addition: make a parallelogram out of $\mathbf{a}$ and $\mathbf{b}$, then $\mathbf{a} + \mathbf{b}$ is the diagonal of the parallelogram. These vectors are shown below:

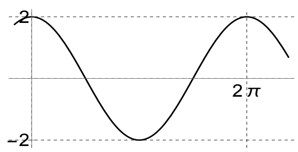


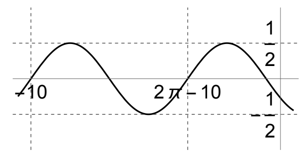
   b) Sketch $2\mathbf{p}$ by making it twice as long as $\mathbf{p}$. Then sketch $-\mathbf{q}$, which goes in the opposite direction as $\mathbf{q}$, at the end of $2\mathbf{p}$. All together, $2\mathbf{p} - \mathbf{q}$ is as shown below:



c) $\mathbf{p} \cdot \mathbf{q} = \boxed{0}$ since the vectors $\mathbf{p}$ and $\mathbf{q}$ are orthogonal.

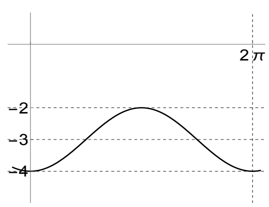
9. a) The crosshair is at $(C, D) = (0, 0)$; $A = 2$ so the graph goes up to $y = 2$ and down to $y = -2$; the period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. So the graph of $y = 2\cos x$ is below at left:

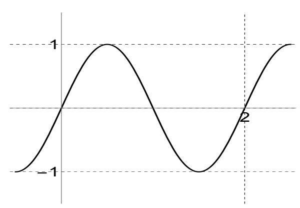




b) The crosshair is at $(C, D) = (-10, 0)$; $A = \frac{1}{2}$ so the graph goes up to $y = \frac{1}{2}$ and down to $y = -\frac{1}{2}$; the period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. So the graph of $y = \dfrac{1}{2}\sin(x + 10)$ is above at right:

c) The crosshair is at $(C, D) = (0, -3)$; $A = 1$ so the graph goes up to $y = -3 + 1 = -2$ and down to $y = -3 - 1 = -4$; the period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The graph is flipped because of the $(-)$ sign. So the graph of $y = -\cos x - 3$ is below, at left:





d) The crosshair is at $(C, D) = (0, 0)$; $A = 1$ so the graph goes up to $y = 1$ and down to $y = -1$; the period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. So the graph of $y = \sin \pi x$ is above, at right.

10. a) $\sin(135° - 24°) = \sin 111° = \boxed{.93358}$.

b) $\tan^2 145° = (\tan 145°)^2 = (-.700208)^2 = \boxed{.490291}$.

c) $\cot 3 \cdot 104° = \cot 312° = \frac{1}{\tan 312°} = \frac{1}{-1.11061} = \boxed{-.900404}$.

d) $-3 \sec 136° = -3\left(\frac{1}{\cos 136°}\right) = -3\left(\frac{1}{-1.39016}\right) = -3(-1.39016) = \boxed{4.17049}$.

e)

$$\begin{aligned}\cos 104^\circ \csc 358^\circ &= (-.241922)\left(\frac{1}{\sin 358^\circ}\right)\\ &= (-.241922)\left(\frac{1}{-.0348995}\right)\\ &= (-.241922)(-28.6537) = \boxed{6.93196}.\end{aligned}$$

11. a) One angle is $\theta = \arccos \dfrac{5}{13} = \boxed{67.4^\circ}$; the other angle is $360^\circ - 67.4^\circ = \boxed{292.6^\circ}$.

b) Take reciprocals of both sides to get $\sin\theta = .4695$; then one angle is $\theta = \arcsin .4695 = \boxed{28^\circ}$; the other angle is $180^\circ - 28^\circ = \boxed{152^\circ}$.

c) One angle is $\theta = \arcsin .415 = \boxed{24.5^\circ}$; the other angle is $180^\circ - 24.5^\circ = \boxed{155.5^\circ}$.

d) One angle is $\theta = \arctan -\frac{26}{17} = -56.8^\circ$. The other angle is $180^\circ + \theta = \boxed{123.2^\circ}$. Since -56.8° is negative, we get the second angle by $-56.8^\circ + 360^\circ = \boxed{303.2^\circ}$.

12. a) $17.4 \cdot \dfrac{180^\circ}{\pi} = \boxed{996.947^\circ}$.

b) $613.5^\circ \cdot \dfrac{\pi}{180^\circ} = \boxed{10.7}$.

c) We need to convert the angular velocity to radians per second:

$$\omega = \frac{80^\circ}{\text{sec}} \cdot \frac{\pi}{180^\circ} = 1.396 \text{ rad/sec}.$$

Now, the linear velocity is

$$v = r\omega = (3 \text{ ft})(1.396 \text{ rad/sec}) = \boxed{4.1887 \text{ ft/sec}}.$$

d) Convert the central angle θ to radians:

$$\theta = 53^\circ \cdot \frac{\pi}{180^\circ} = .925 \text{ radians}.$$

Now, from the sector area formula we have

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(4.5 \text{ in})^2(.925) = \boxed{9.3656 \text{ sq in}}.$$

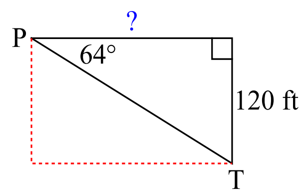
13. a) $3\mathbf{w} + 2\mathbf{v} = 3\langle 5.2, 9.3\rangle + 2\langle 12.3, -8.2\rangle = \langle 15.6, 27.9\rangle + \langle 24.6, -16.4\rangle = \boxed{\langle 40.2, 11.5\rangle}$.

b) Compute $\mathbf{v} \cdot \mathbf{w} = (12.3)5.2 + (-8.2)9.3 = \boxed{-12.3}$.

c) $|\mathbf{v}| = \sqrt{(12.3)^2 + (-8.2)^2} = \sqrt{151.29 + 67.24} = \sqrt{218.53} = \boxed{14.78}$.

d) $\theta_{\mathbf{w}} = \arctan \dfrac{9.3}{5.2} = \arctan 1.78846 = \boxed{60.79^\circ}$.

14. a) Here's a picture, where the person is at P and the trash is at T (the tower goes straight down from P, and the ground is across the bottom of the picture):



The angle at T is $180^\circ - 90^\circ - 64^\circ = 26^\circ$, so by the Law of Sines we have

$$\begin{aligned} \frac{\sin P}{p} &= \frac{\sin T}{t} \\ \frac{\sin 64^\circ}{120} &= \frac{\sin 26^\circ}{?} \\ ?\ \sin 64^\circ &= 120 \sin 26^\circ \\ ? &= \frac{120 \sin 26^\circ}{\sin 64^\circ} = \boxed{58.53 \text{ ft}}. \end{aligned}$$

(This can also be done with SOHCAHTOA, using either $\cot 64^\circ = \frac{?}{120}$ or $\tan 64^\circ = \frac{?}{120}$ and solving for the "?"... you get the same answer.)

b) If the mark is originally on the left edge of the wheel, it is initially at an angle of 180° so after the wheel is rotated, it ends up at $180^\circ + 205^\circ = 385^\circ$. The distance left or right of the center is therefore the radius times $\cos 385^\circ$, which is $19 \cos 385^\circ = \boxed{17.22 \text{ in}}$. This is $\boxed{\text{right of center}}$ since the answer is positive.

15. The given information is SSS, so we use the Law of Cosines twice to find angles K and L:

$$\begin{aligned} k^2 &= l^2 + m^2 - 2lm \cos K \\ 14^2 &= 17^2 + 19^2 - 2(17)(19) \cos K \\ 196 &= 650 - 646 \cos K \\ -454 &= -646 \cos K \\ .7027 &= \cos K \\ K &= \arccos .7027 \\ K &= 45.3^\circ \end{aligned}$$

$$\begin{aligned} l^2 &= k^2 + m^2 - 2km \cos L \\ 17^2 &= 14^2 + 19^2 - 2(14)(19) \cos L \\ 289 &= 557 - 532 \cos L \\ -268 &= -532 \cos L \\ .5037 &= \cos L \\ L &= \arccos .5037 \\ L &= 59.8^\circ \end{aligned}$$

Finally, $\angle M = 180^\circ - \angle K - \angle L = 180^\circ - 45.3^\circ - 59.8^\circ = 74.9^\circ$. So the answer is

$\angle K = 45.3°$	$\angle L = 59.8°$	$\angle M = 74.9°$
$k = 14$	$l = 17$	$m = 19$

16. The given information is AAS, so find the third angle and then use the Law of Sines twice. First, $\angle F = 180° - \angle D - \angle E = 180° - 117° - 26° = 37°$. Now,

$$\begin{aligned} \tfrac{\sin D}{d} &= \tfrac{\sin E}{e} \\ \tfrac{\sin 117°}{d} &= \tfrac{\sin 26°}{4} \\ d \sin 26° &= 4 \sin 117° \\ d &= \tfrac{4 \sin 117°}{\sin 26°} \\ d &= 8.13 \end{aligned} \qquad \begin{aligned} \tfrac{\sin F}{f} &= \tfrac{\sin E}{e} \\ \tfrac{\sin 37°}{f} &= \tfrac{\sin 26°}{4} \\ f \sin 26° &= 4 \sin 37° \\ f &= \tfrac{4 \sin 37°}{\sin 26°} \\ f &= 5.49 \end{aligned}$$

This gives

$\angle D = 117°$	$\angle E = 26°$	$\angle F = 37°$
$d = 8.13$	$e = 4$	$f = 5.49$

17. **(Bonus)** Suppose this graph has equation $y = A \sin B(x - C) + D$.
   - First, notice that D is halfway in between the top and bottom of the graph, so $D = \dfrac{2.8 + 0.4}{2} = 1.6$.
   - Next, the amplitude A is the top of the graph minus D, which is $2.8 - 1.6 = 1.2$.
   - $C = 0$ since the sine graph starts at $x = 0$ (no horizontal shift).
   - Last, the period is $1.4 = \dfrac{2\pi}{B}$ so $1.4B = 2\pi$ so $B = \dfrac{2\pi}{1.4} = 4.488$.

   Therefore the graph has formula $\boxed{y = 1.2 \sin 4.488t + 1.6}$.

   To find the current at time $t = 100$, plug 100 in for t in this formula. This gives

$$\begin{aligned} 1.2 \sin 4.488(100) + 1.6 &= 1.2 \sin 448.8 + 1.6 \\ &= 1.2(.432938) + 1.6 \\ &= \boxed{2.223}. \end{aligned}$$

   Keep in mind that the 448.8 is radians, so before taking the sine one should either change your calculator to radians or convert 448.8 radians to degrees: $448.8 \cdot \dfrac{180°}{\pi} = 25714.3°$.

4.3 Fall 2022 Final Exam

1. $\boxed{\text{NC}}$ Compute the exact value of each quantity:

   a) $\cos 7\pi$
   b) $\sin \frac{3\pi}{4}$
   c) $\tan 90^\circ$
   d) $\sin \frac{-5\pi}{3}$
   e) $\sec 240^\circ$
   f) $\cos \frac{5\pi}{6}$

2. $\boxed{\text{NC}}$ Compute the exact value of each quantity:

   a) $2 \csc -2 \cdot \frac{\pi}{3}$
   b) $\sin \frac{5\pi}{6} + \frac{\pi}{6}$
   c) $\cos^2 135^\circ$
   d) $9 - 2\tan^2 \frac{\pi}{3}$
   e) $\sin 22.5^\circ$
   f) $\sec^2 320^\circ - \tan^2 320^\circ$

3. $\boxed{\text{NC}}$ In each part of this problem, you are to do two things:

   - sketch a picture that explains the problem (this picture should include any given information, indicate where θ is, and mark what is being asked for with a "?"), and
   - solve the problem.

   a) Suppose $\cos\theta = \frac{1}{3}$ and $\sin\theta < 0$. Compute $\cos(180^\circ - \theta)$.
   b) Suppose $\sin\theta = \frac{2}{3}$ and $\cos\theta < 0$. Compute $\csc\theta$.
   c) Suppose $\tan\theta = 3$ and $\sin\theta > 0$. Compute $\tan(\theta + 180^\circ)$.
   d) Suppose $\sin\theta = \frac{3}{7}$ and $\cos\theta > 0$. Compute $\sin(\theta - 60^\circ)$.

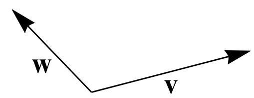
4. $\boxed{\text{NC}}$ Throughout this problem, suppose $\tan\theta = \frac{2}{5}$ and $\cos\theta < 0$.

   a) Compute the exact value of $\tan(360^\circ - \theta)$.
   b) Compute the exact value of $\tan(\theta + 720^\circ)$.
   c) Compute the exact value of $2\tan\theta$.
   d) Compute the exact value of $\tan 2\theta$.
   e) Compute the exact value of $\sin\theta$.

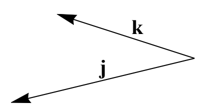
5. (7.1) $\boxed{\text{NC}}$ Simplify the following expression using trigonometric identities:

$$\frac{1 - \sin^2\theta}{\cos(-\theta)}$$

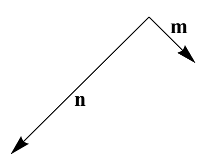
6. a) (2.4) $\boxed{\text{NC}}$ Vectors $\mathbf{v}$ and $\mathbf{w}$ are pictured below. On the same picture, sketch $2\mathbf{v} + \mathbf{w}$.



b) (2.4) $\boxed{\text{NC}}$ Vectors $\mathbf{j}$ and $\mathbf{k}$ are pictured below. On the same picture, sketch $\mathbf{j} - \frac{1}{2}\mathbf{k}$.



c) (2.4) $\boxed{\text{NC}}$ Vectors $\mathbf{m}$ and $\mathbf{n}$ are pictured below. On the same picture, sketch $-3\mathbf{m}$.



7. $\boxed{\text{NC}}$ Sketch a crude graph of each of these functions:

a) (5.3) $y = \cos x + 4$

b) (5.3) $y = 3\sin(x - \pi)$

c) (5.4) $y = \tan x$

d) (5.3) $y = -4\sin 8\pi x$

8. (3.3, 6.1) Evaluate each of these expressions using a calculator. Your answers can (and should) be written as decimals.

a) $\sin(47° + 85°)$

b) $\cos 119° \tan 198°$

c) $\cot -67°$

d) $\sec 282.5°$

e) $5\cos^2 842°$

f) $13 - 4\tan 85°$

9. a) (2.9) Convert 4.25 radians to degrees, writing your answer as a decimal.
   b) (2.9) Convert $83.25°$ to radians, writing your answer as a decimal.
   c) (3.2) If $(12.35, -8.9)$ is on the terminal side of angle θ when θ is drawn in standard position, what is $\cos\theta$?
   d) (2.10) Compute the length of a circular arc taken from a circle of radius 19 in, if the central angle of the arc measures $66°$.

10. (3.7, 6.2) In each part of this problem, you are to find all angles between $0°$ and $360°$ that solve the equation. If the equation has no solution, say so.

    a) $\sin\theta = \dfrac{19}{34}$
    b) $\tan\theta = 2.13$
    c) $\cos\theta = 1.15$
    d) $\sec\theta = -\dfrac{27}{12}$
    e) $\sin\theta = 1$

11. a) (2.4) Suppose $\mathbf{v} = \langle -19, 15\rangle$ and $\mathbf{w} = \langle 7, -10\rangle$. Compute $\mathbf{v} - 2\mathbf{w}$.
    b) (2.4) Compute $\mathbf{v}\cdot\mathbf{w}$, where $\mathbf{v}$ and $\mathbf{w}$ are as in part (a) of this problem.
    c) (4.4) Suppose $\mathbf{a}$ and $\mathbf{b}$ are vectors, both having magnitude 12. If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $106°$, what is $\mathbf{a}\cdot\mathbf{b}$?

12. a) (4.1 or 4.3) To measure the height of a cell phone tower, a surveyor walks 35 feet away from the base of the tower and looks up at the top of the tower, determining that his angle of elevation to the top of the tower is $56°$. If the surveyor's eyes are 5.75 feet off the ground, what is the height of the tower?
    b) (4.1) Two people, Angela and Brian, are standing on a shoreline, 0.35 miles apart. They both look out at a boat on the water. Angela estimates that the angle between her line of sight to Brian and her line of sight to the boat is $43°$. Brian estimates that the angle between his line of sight to Angela and his line of sight to the boat is $38°$. How far is the boat from Brian?

13. (4.2) Solve $\triangle KLM$, if $k = 77.35$, $l = 103.91$ and $m = 140.06$. (If there is no such triangle with these measurements, you should say so, and if there are two triangles, you should solve both triangles.)

14. (4.1) Solve $\triangle DEF$, if $\angle D = 52°$, $d = 21$ and $f = 24$. (If there is no such triangle with these measurements, you should say so, and if there are two triangles, you should solve both triangles.)

Solutions

1. a) $\cos 7\pi = \boxed{-1}$ ($180°$ is at $(-1,0)$ on the unit circle)

   b) $\sin \frac{3\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$ (Quadrant II, ref. angle $45°$)

   c) $\tan 90°$ $\boxed{\text{DNE}}$ (slope of vertical line)

   d) $\sin \frac{-5\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$ (Quadrant I, ref. angle $60°$)

   e) $\sec 240° = \boxed{-2}$ (Quadrant III, ref. angle $60°$)

   f) $\cos \frac{5\pi}{6} = \boxed{-\frac{\sqrt{3}}{2}}$ (Quadrant II, ref. angle $30°$)

2. a) $2\csc -2\cdot\frac{\pi}{3} = 2\csc\frac{-2\pi}{3} = 2\left(-\frac{2}{\sqrt{3}}\right) = \boxed{-\frac{4}{\sqrt{3}}}$

   b) $\sin\frac{5\pi}{6} + \frac{\pi}{6} = \boxed{\frac{1}{2} + \frac{\pi}{6}}$

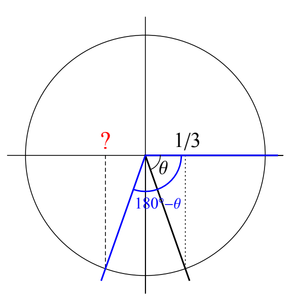
   c) $\cos^2 135° = \left(-\frac{\sqrt{2}}{2}\right)^2 = \frac{\sqrt{4}}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

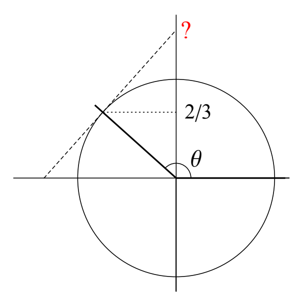
   d) $9 - 2\tan^2\frac{\pi}{3} = 9 - 2\left(\sqrt{3}\right)^2 = 9 - 2(3) = \boxed{3}$

   e) Use a half-angle identity: $\sin 22.5° = \sin\frac{45°}{2} = \pm\sqrt{\frac{1-\cos 45°}{2}} = \boxed{\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}}$.

   f) $\sec^2 320° - \tan^2 320° = \boxed{1}$ (from the identity $\sec^2\theta = 1 + \tan^2\theta$)

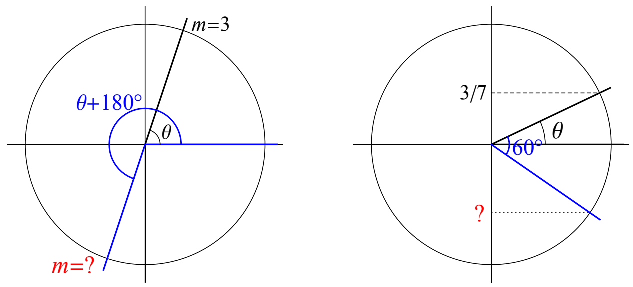
3. a) $\cos(180° - \theta) = -\cos\theta = \boxed{-\frac{1}{3}}$. The picture is below, at left.





   b) $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{2/3} = \boxed{\frac{3}{2}}$. The picture is above, at right.

c) $\tan(\theta + 180^\circ) = \tan\theta = \boxed{3}$. The picture is below, at left.



d) Suppose $\sin\theta = \frac{3}{7}$ and $\cos\theta > 0$. First, find $\cos\theta$ via a triangle or the Pythagorean identity:

$$\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1 \\
\cos^2\theta + \left(\frac{3}{7}\right)^2 &= 1 \\
\cos^2\theta + \frac{9}{49} &= 1 \\
\cos^2\theta &= 1 - \frac{9}{49} = \frac{40}{49} \\
\cos\theta &= \pm\sqrt{\frac{40}{49}} \\
&= \frac{\sqrt{40}}{7}.
\end{aligned}$$

Then, use the difference identity for sine:

$$\begin{aligned}
\sin(\theta - 60^\circ) &= \sin\theta\cos 60^\circ - \cos\theta\sin 60^\circ \\
&= \frac{3}{7}\cdot\frac{1}{2} - \frac{\sqrt{40}}{7}\cdot\frac{\sqrt{3}}{2} \\
&= \boxed{\frac{3-\sqrt{120}}{14}}.
\end{aligned}$$

The picture for this is above, at right.

4. a) $\tan(360^\circ - \theta) = \tan(-\theta) = -\tan\theta = \boxed{-\frac{2}{5}}$.

b) $\tan(\theta + 720^\circ) = \tan\theta = \boxed{\frac{2}{5}}$.

c) $2\tan\theta = 2\left(\frac{2}{5}\right) = \boxed{\frac{4}{5}}$.

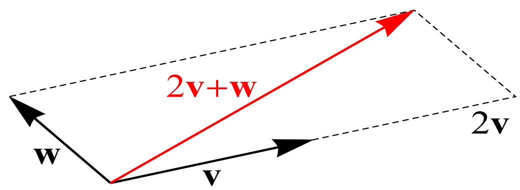
d) Use a double-angle identity:

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{2}{5}\right)}{1-\left(\frac{2}{5}\right)^2} = \frac{\frac{4}{5}}{1-\frac{4}{25}} = \frac{\frac{4}{5}}{\frac{21}{25}} = \frac{4}{5}\cdot\frac{25}{21} = \boxed{\frac{20}{21}}.$$

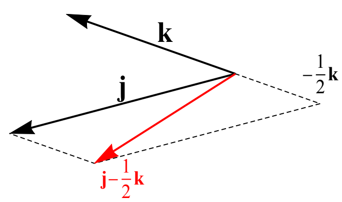
e) First, this angle is in Quadrant III (since $\tan\theta > 0$ and $\cos\theta < 0$) so $\sin\theta$ is negative. Now, draw a triangle and label the opposite side 2 and the adjacent side 5. Solving for the hypotenuse using the Pythagorean Theorem, we get that the hypotenuse is $\sqrt{29}$, so $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \boxed{-\frac{2}{\sqrt{29}}}$.

5. $\dfrac{1-\sin^2\theta}{\cos(-\theta)} = \dfrac{\cos^2\theta}{\cos(-\theta)} = \dfrac{\cos^2\theta}{\cos\theta} = \boxed{\cos\theta}$.

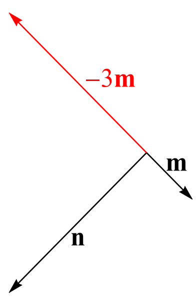
6. a)



b)



c)

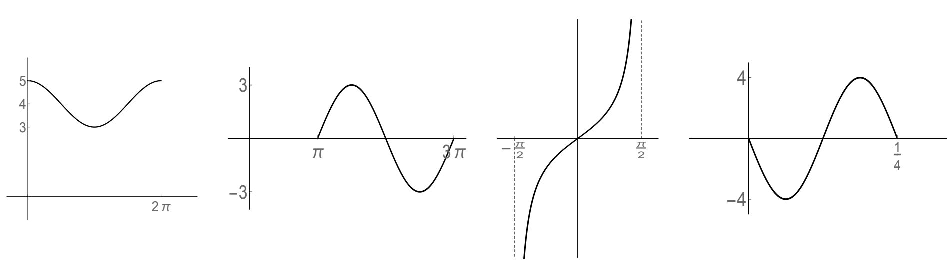


7. a) This is the graph of $y = \cos x$ shifted up 4 units, as shown below at left.

b) This is the graph of $y = \sin x$ shifted right π units, then stretched vertically by a factor of 3, as shown below second from left.

c) $y = \tan x$ is the third graph from the left shown below.

d) $y = -4\sin 8\pi x$ is the graph of $y = \sin x$, flipped over, vertically stretched by a factor of 4, and with a period of $\frac{2\pi}{B} = \frac{2\pi}{8\pi} = \frac{1}{4}$. This graph is shown below at right:



8. a) $\sin(47^\circ + 85^\circ) = \sin 132^\circ = \boxed{.743145}$

   b) $\cos 119^\circ \tan 198^\circ = -.48481(.32492) = \boxed{-.157524}$

   c) $\cot -67^\circ = \dfrac{1}{\tan -67^\circ} = \dfrac{1}{-2.35585} = \boxed{-.424475}$

   d) $\sec 282.5^\circ = \frac{1}{\cos 282.5^\circ} = \frac{1}{.21644} = \boxed{4.62023}$

   e) $5\cos^2 842^\circ = 5(-.529919)^2 = 5(.280814) = \boxed{1.40407}$

   f) $13 - 4\tan 85^\circ = 13 - 4\,() = 13 - 4(11.4301) = 13 - 45.7202 = \boxed{-32.7202}$

9. a) 4.25 radians $= 4.25 \cdot \dfrac{180^\circ}{\pi} = \boxed{243.5^\circ}$.

   b) $83.25^\circ \cdot \dfrac{\pi}{180^\circ} = \boxed{1.45255}$ radians.

   c) First, $r = \sqrt{x^2+y^2} = \sqrt{12.35^2 + (-8.9)^2} = \sqrt{152.52 + 79.21} = \sqrt{231.73} = 15.22$. Then, $\cos\theta = \dfrac{x}{r} = \dfrac{12.35}{15.22} = \boxed{.8113}$.

   d) In radians, the angle is $\theta = 66^\circ \cdot \frac{\pi}{180^\circ} = 1.15192$. Thus the arc length is $s = r\theta = (19 \text{ in})(1.15192) = \boxed{21.8864 \text{ in}}$.

10. a) One solution is $\theta = \arcsin \frac{19}{34} = \boxed{33.97^\circ}$. The other solution is $180^\circ - 33.97^\circ = \boxed{146.03^\circ}$.

   b) One solution is $\theta = \arctan 2.13 = \boxed{64.85^\circ}$. The other solution is $180^\circ + 64.85^\circ = \boxed{244.85^\circ}$.

   c) $\cos\theta = 1.15$ has $\boxed{\textbf{no solution}}$ since $1.15 > 1$.

   d) Take reciprocals of both sides to get $\cos\theta = -\dfrac{12}{27} = -.4444$. Now, one solution is $\theta = \arccos(-.4444) = \boxed{116.388^\circ}$; the other solution is $360^\circ - 116.388^\circ = \boxed{243.612^\circ}$.

   e) The only solution is $\theta = \arcsin 1 = \boxed{90^\circ}$ (the other solution is $180^\circ - 90^\circ = 90^\circ$, which is a repeat of the first solution).

11. a) $\mathbf{v} - 2\mathbf{w} = \langle -19, 15\rangle - 2\langle 7, -10\rangle = \langle -19, 15\rangle - \langle 14, -20\rangle = \boxed{\langle -33, 35\rangle}$.

   b) $\mathbf{v}\cdot\mathbf{w} = -19(7) + 15(-10) = -133 - 150 = \boxed{-283}$.

   c) $\mathbf{a}\cdot\mathbf{b} = ||\mathbf{a}||\,||\mathbf{b}||\cos\theta = 12 \cdot 12\cos 106^\circ = 144(-.275637) = \boxed{-39.6918}$.

12. a) Sketch a right triangle with one leg horizontal and one leg vertical. The horizontal leg gets labelled 35 and the vertical leg is the height of the tower, and the angle at the end of the horizontal leg is the angle of elevation, so we have

$$\tan 56^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{35}.$$

   Therefore $x = 35\tan 56^\circ = 51.89$ ft. Finally, add in the height of the surveyor's eyes to get the total height of the tower, which is $51.89 + 5.75 = \boxed{57.64 \text{ ft}}$.

b) If you draw a triangle with Angela at A, Brian at B and the boat at C, we have $\angle A = 43^\circ$, $\angle B = 38^\circ$ and $c = .35$. This is an ASA triangle, so we find a, the distance from Brian (point B) to the boat (point C) by first finding angle C, then using the Law of Sines. First,

$$\angle C = 180^\circ - \angle A - \angle B = 180^\circ - 43^\circ - 38^\circ = 99^\circ.$$

Then

$$\begin{aligned}
\frac{\sin C}{c} &= \frac{\sin A}{a} \\
\frac{\sin 99^\circ}{.35} &= \frac{\sin 43^\circ}{a} \\
a \sin 99^\circ &= .35 \sin 43^\circ \\
a &= \frac{.35 \sin 43^\circ}{\sin 99^\circ} = \boxed{.2417 \text{ mi}}.
\end{aligned}$$

13. With this SSS triangle, use the Law of Cosines to find an angle:

$$\begin{aligned}
k^2 &= l^2 + m^2 - 2lm \cos K \\
77.35^2 &= 103.91^2 + 140.06^2 - 2(103.91)(140.06) \cos K \\
5983.02 &= 30414.1 - 29107.3 \cos K \\
-24431.1 &= -29107.3 \cos K \\
.839345 &= \cos K \\
K &= \arccos .839345 = \boxed{32.9^\circ}
\end{aligned}$$

Then, use the Law of Cosines again, similar to what was done above to find $\angle K$, to find $\angle L$:

$$\begin{aligned}
l^2 &= k^2 + m^2 - 2km \cos L \\
103.91^2 &= 77.35^2 + 140.06^2 - 2(77.35)(140.06) \cos L \\
&\vdots \\
L &= \boxed{46.9^\circ}
\end{aligned}$$

Finally, $\angle M = 180^\circ - K - L = 180^\circ - 32.9^\circ - 46.9^\circ = \boxed{100.2^\circ}$.

$\angle K = 32.9^\circ$	$\angle L = 46.9^\circ$	$\angle M = 100.2^\circ$
$k = 77.35$	$l = 103.91$	$m = 140.06$

14. This SSA triangle is the ambiguous case of the Law of Sines. To get started, we find

the two possible values of $\angle F$:

$$\begin{aligned}
\frac{\sin F}{f} &= \frac{\sin D}{d} \\
\frac{\sin F}{24} &= \frac{\sin 52^\circ}{21} \\
21 \sin F &= 24 \sin 52^\circ \\
\sin F &= \frac{24 \sin 52^\circ}{21} = .9 \\
F &= \arcsin .9 = 64.2^\circ
\end{aligned}$$

The other possible angle is $F' = 180^\circ - 64.2^\circ = 115.8^\circ$.

In either situation, we find the remaining angle:

$$\begin{aligned}
E &= 180^\circ - D - F = 180^\circ - 52^\circ - 64.2^\circ = 63.8^\circ \\
E' &= 180^\circ - D - F' = 180^\circ - 52^\circ - 115.8^\circ = 12.2^\circ
\end{aligned}$$

and finally, find the remaining side in each case:

$$\begin{aligned}
\frac{\sin E}{e} &= \frac{\sin D}{d} & \frac{\sin E'}{e'} &= \frac{\sin D}{d} \\
\frac{\sin 63.8^\circ}{e} &= \frac{\sin 52^\circ}{21} & \frac{\sin 12.2^\circ}{e'} &= \frac{\sin 52^\circ}{21} \\
e(\sin 52^\circ) &= 21 \sin 63.8^\circ & e'(\sin 52^\circ) &= 21 \sin 12.2^\circ \\
e &= \frac{21 \sin 63.8^\circ}{\sin 52^\circ} & e' &= \frac{21 \sin 12.2^\circ}{\sin 52^\circ} \\
e &= 23.91 & e' &= 5.63
\end{aligned}$$

All together, there are two triangles:

$\angle D = 52^\circ$	$\angle E = 63.8^\circ$	$\angle F = 64.2^\circ$
$d = 21$	$e = 23.91$	$f = 24$

$\angle D = 52^\circ$	$\angle E' = 12.2^\circ$	$\angle F' = 115.8^\circ$
$d = 21$	$e' = 5.63$	$f = 24$

4.4 Fall 2019 Final Exam

1. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\cos\frac{\pi}{6}$ b) $\tan 30°$ c) $\csc\frac{\pi}{3}$ d) $\cot 0$ e) $\sin 0$

2. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\cos 120°$ b) $\sin(-135°)$ c) $\cos 540°$ d) $\tan(-180°)$ e) $\sec 225°$

3. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\cos\frac{3\pi}{2}$ b) $\sin\frac{2\pi}{3}$ c) $\tan\frac{\pi}{4}$ d) $\cot\frac{11\pi}{4}$ e) $\cos\frac{7\pi}{4}$

4. $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) (3.6) $\sin^2 60°$
   b) (7.2) $\cos 75°$
   c) (3.6) $4\tan\frac{\pi}{3}\cdot 2$
   d) (3.5) $\cos 70° - \sin 20°$
   e) (3.6) $\csc\frac{\pi}{4}\cot\frac{-\pi}{3}$

5. $\boxed{\text{NC}}$ Suppose $\cos\theta = -\frac{1}{6}$ and $\sin\theta > 0$.

   a) (3.2) What quadrant is θ in?
   b) (6.1) Find $\sec\theta$.
   c) (3.5) Find $\cos(-\theta)$.
   d) (3.5) Find $\cos(180° - \theta)$.
   e) (3.4) Find $\sin\theta$.
   f) (3.4) Find $\tan\theta$.
   g) (7.3) Find $\tan 2\theta$.

6. $\boxed{\text{NC}}$ Sketch a crude graph of each function, labelling the graph appropriately:

   a) (5.3) $y = \sin x - 2$
   b) (5.3) $y = 4\cos x$
   c) (5.4) $y = \tan x$
   d) (5.3) $y = \sin 4x$

7. $\boxed{\text{NC}}$ In each problem, choose the picture A-U (the pictures are after the questions) which most closely describes what is given and what is being asked for in the problem.

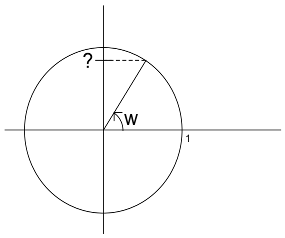
   a) (3.1) Find $\tan w$.
   b) (3.1) Find $\sin w$.
   c) (3.4) Suppose $\sin\theta = w$ and w is in Quadrant I. Find $\cos\theta$.

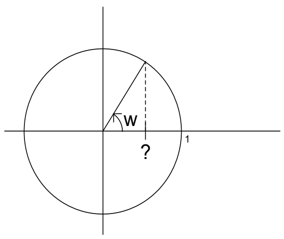
d) (3.7) Suppose $\cos\theta = w$. Find all possible values of θ.

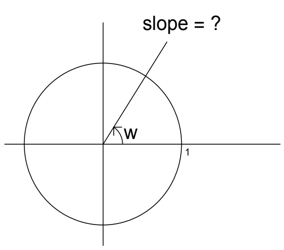
e) (3.7) Suppose $\tan\theta = w$. Find all possible values of θ.

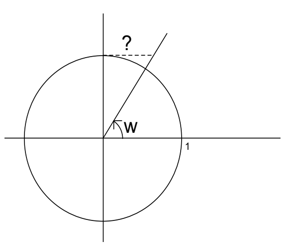
f) (3.7) Suppose $\tan\theta = w$ and $\sin\theta < 0$. Find all possible values of θ.

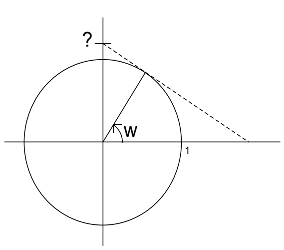
g) (3.4) Suppose $\tan\theta = w$ and $\cos\theta > 0$. Find all possible values of $\sin\theta$.

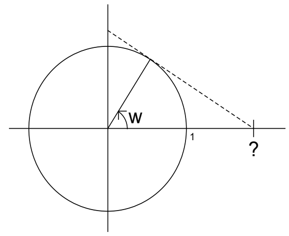
A. 

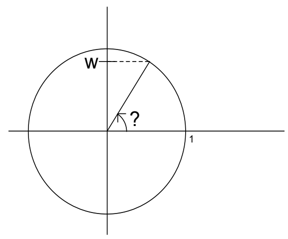
B. 

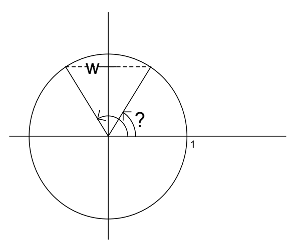
C. 

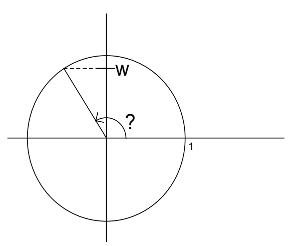
D. 

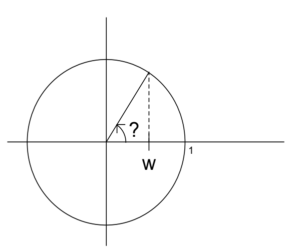
E. 

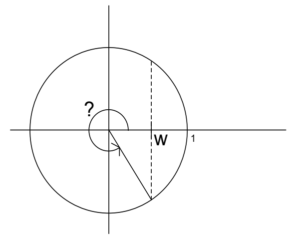
F. 

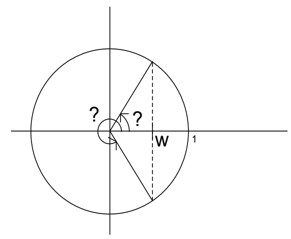
G. 

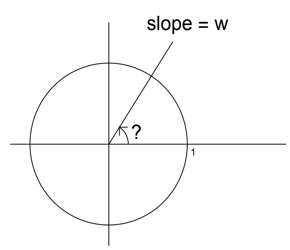
H. 

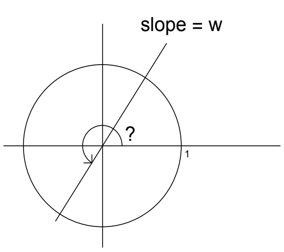
I. 

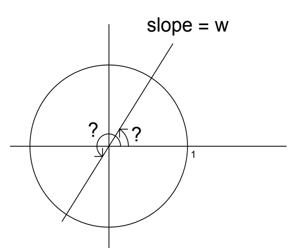
J. 

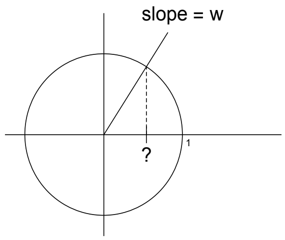
K. 

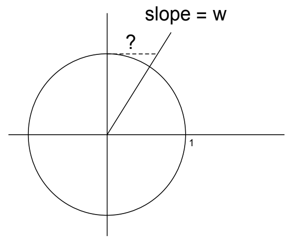
L. 

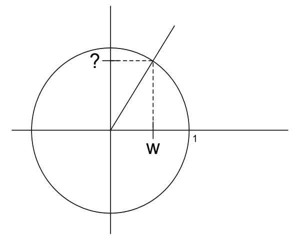
M. 

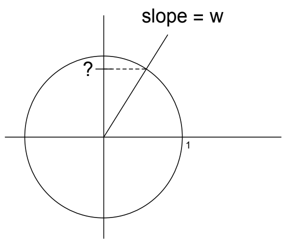
N. 

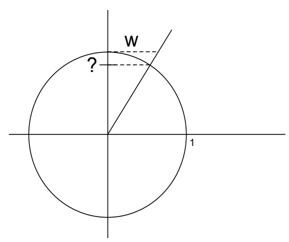
O. 

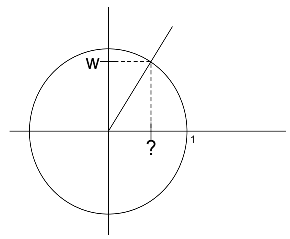
P. 

R. 

T. 

Q. 

S. 

U. 

8. (7.1) $\boxed{\text{NC}}$ Simplify the following expression using trig identities:

$$\frac{\sec(-\theta)\sin\theta}{\tan(-\theta)}$$

9. Throughout this problem, let $\mathbf{v} = \langle 6, 5\rangle$ and let $\mathbf{w} = \langle 3, 8\rangle$.

   a) (2.4) Compute $2\mathbf{v} - 3\mathbf{w}$.
   b) (2.4) Compute $\mathbf{v} \cdot \mathbf{w}$.
   c) (2.7) Compute $|\mathbf{v}|$.
   d) (4.4) Find the measure of the angle between $\mathbf{v}$ and $\mathbf{w}$.

10. a) (2.9) Convert $136.25°$ to radians, writing your answer as a decimal.
    b) (2.9) Convert 0.63 radians to degrees, writing your answer as a decimal.

11. (3.3, 6.1) Evaluate the following expressions using a calculator. Your answers can (and should) be written as decimals.

    a) $\tan 218°$
    b) $5 \sec 72°$
    c) $\cos 322° + \csc 66°$
    d) $\sin^2 118.5°$

12. (3.7) For each equation, find all angles θ between $0°$ and $360°$ satisfying the following equations. If the equation has no solution, say so.

    a) $\cos\theta = -.235$
    b) $\sin\theta = \frac{19}{11}$
    c) $\sin\theta = \frac{11}{19}$
    d) $\tan\theta = .1875$

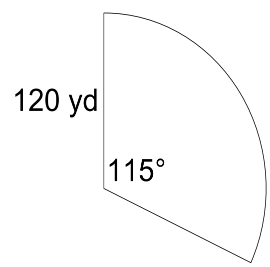
13. a) (4.1 or 4.3) A hot air balloon is launched and travels directly upward. A person standing 1300 feet away from where the balloon was launched looks up at the balloon. If his angle of elevation to the balloon is $10°$, and assuming the ground is flat, what is the elevation of the balloon?

    b) (3.3 or 4.2) Two kayakers, Aaron and Barbara, are paddling in a lake. Starting from a dock, Aaron paddles for $.85$ miles at an angle $24°$ west of north. Starting from the same dock, Barbara paddles for $.55$ miles at an angle $37°$ east of north. How far apart are Aaron and Barbara?

14. (4.2) Suppose that the three sides of a triangle have lengths 18, 23 and 33 units. Find the measures of each of the three angles of the triangle.

15. (4.1) Solve triangle LMN, where $\angle L = 62°$, $l = 6.4$ and $n = 7.3$.

16. a) (2.10) A park is in the shape of a circular sector, as shown below.



    i. What is the area of the park, in square yards?

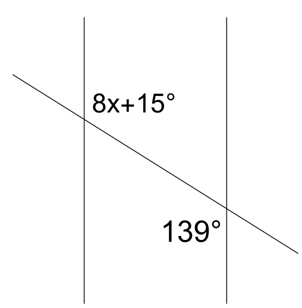
    ii. A bicycle path runs around the perimeter of the park. What is the total length (in yards) of the path?

    b) (2.10) A helicopter rotor turns at 460 revolutions per minute. Each of its rotor blades is 18 feet long.

    i. What is the angular velocity of the rotor, in radians per minute?

    ii. What is the linear velocity of a point on the end of one its rotor blades?

    c) (2.6) In the picture below, find the value of x. Assume that lines that appear parallel are, in fact, parallel.



Solutions

1. a) $\cos\frac{\pi}{6} = \boxed{\dfrac{\sqrt{3}}{2}}$

   b) $\tan 30^\circ = \boxed{\dfrac{1}{\sqrt{3}}}$

   c) $\csc\frac{\pi}{3} = \frac{1}{\sin 60^\circ} = \boxed{\dfrac{2}{\sqrt{3}}}$

   d) $\cot 0 = \frac{1}{\tan 0} = \frac{1}{0}$ which $\boxed{\text{DNE}}$.

   e) $\sin 0 = \boxed{0}$.

2. a) $\cos 120^\circ = \boxed{-\dfrac{1}{2}}$

   b) $\sin(-135^\circ) = \boxed{-\dfrac{\sqrt{2}}{2}}$

   c) $\cos 540^\circ = \boxed{-1}$

   d) $\tan(-180^\circ) = \boxed{0}$

   e) $\sec 225^\circ = \boxed{-\sqrt{2}}$

3. a) $\cos\frac{3\pi}{2} = \boxed{0}$

   b) $\sin\frac{2\pi}{3} = \boxed{\dfrac{\sqrt{3}}{2}}$

   c) $\tan\frac{\pi}{4} = \boxed{1}$

   d) $\cot\frac{11\pi}{4} = \boxed{-1}$

   e) $\cos\frac{7\pi}{4} = \boxed{\dfrac{\sqrt{2}}{2}}$

4. a) $\sin^2 60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 = \boxed{\dfrac{3}{4}}$

   b) Use the sum identity for cosine: $\cos 75^\circ = \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$.

   c) $4\tan\frac{\pi}{3}\cdot 2 = 4\tan\frac{2\pi}{3} = 4(-\sqrt{3}) = \boxed{-4\sqrt{3}}$.

   d) $\cos 70^\circ - \sin 20^\circ = \boxed{0}$ by the cofunction identity.

   e) $\csc\frac{\pi}{4}\cot\frac{-\pi}{3} = \sqrt{2}\cdot -\frac{1}{\sqrt{3}} = \boxed{-\dfrac{\sqrt{2}}{\sqrt{3}}}$.

5. a) Since $\cos\theta < 0$ and $\sin\theta > 0$, θ is in $\boxed{\text{Quadrant II}}$.

   b) $\sec\theta = \frac{1}{\cos\theta} = \boxed{-6}$.

   c) $\cos(-\theta) = \cos\theta = \boxed{-\dfrac{1}{6}}$.

   d) $\cos(180^\circ - \theta) = -\cos\theta = \boxed{\dfrac{1}{6}}$. (To see this, remember that $180^\circ - \theta$ is θ reflected across the y-axis, which in this case puts $180^\circ - \theta$ in Quadrant I.)

   e) Draw a triangle, labelling the hypotenuse as 6 and the adjacent side as 1. Solve for the opposite side to get $\sqrt{35}$. Then $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \boxed{\dfrac{\sqrt{35}}{6}}$.

f) Using the triangle from part (e), and the fact that the answer is negative since θ is in Quadrant II, $\tan\theta = -\frac{\text{opp}}{\text{adj}} = \boxed{-\sqrt{35}}$.

g) Use the double-angle identity for tangent to get $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot(-\sqrt{35})}{1-(-\sqrt{35})^2} = \frac{-2\sqrt{35}}{1-35} = \frac{-2\sqrt{35}}{-34} = \boxed{\frac{\sqrt{35}}{17}}$.

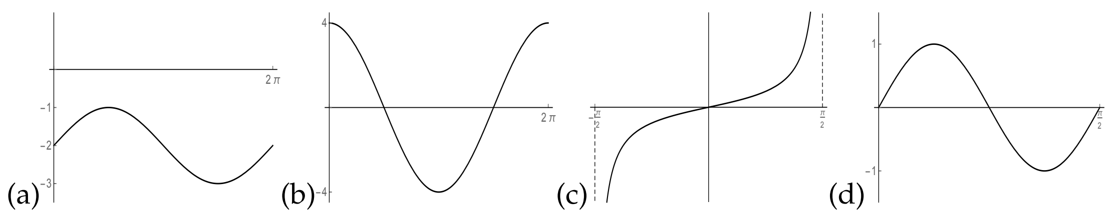
6. a) $y = \sin x - 2$ is the graph of $y = \sin x$, shifted down 2 units.

   b) $y = 4\cos x$ is the graph of $y = \cos x$, stretched vertically by a factor of 4.

   c) $y = \tan x$ looks like a cubic, with asymptotes at $x = \pm\frac{\pi}{2}$.

   d) $y = \sin 4x$ has period $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$.

   Here are the graphs:



7. a) C
   b) A
   c) U
   d) L
   e) O
   f) N
   g) Q

8. Write in terms of sines and cosines, and simplify using algebra:

$$\frac{\sec(-\theta)\sin\theta}{\tan(-\theta)} = \frac{\sec\theta\sin\theta}{-\tan\theta} = \frac{\frac{1}{\cos\theta}\cdot\sin\theta}{-\frac{\sin\theta}{\cos\theta}} = \frac{1}{\cos\theta}\cdot\frac{\sin\theta}{1}\cdot\frac{-\cos\theta}{\sin\theta} = \boxed{-1}.$$

9. a) $2\mathbf{v} - 3\mathbf{w} = 2\langle 6,5\rangle - 3\langle 3,8\rangle = \langle 12,10\rangle - \langle 9,24\rangle = \boxed{\langle 3,-14\rangle}$.

   b) $\mathbf{v}\cdot\mathbf{w} = 6\cdot 3 + 5\cdot 8 = \boxed{58}$.

   c) $||\mathbf{v}|| = \sqrt{6^2+5^2} = \boxed{\sqrt{61}} = \boxed{7.81}$.

   d) First, $||\mathbf{w}|| = \sqrt{3^2+8^2} = \sqrt{73} = 8.54$. Now, use the formula for angles:

$$\begin{aligned}
\mathbf{v}\cdot\mathbf{w} &= ||\mathbf{v}||\,||\mathbf{w}||\cos\theta\\
58 &= (7.81)(8.54)\cos\theta\\
.87 &= \cos\theta\\
\theta &= \arccos .87 = \boxed{29.6^\circ}.
\end{aligned}$$

10. a) $136.25^\circ \cdot \frac{\pi}{180^\circ} = \boxed{2.378}$.

    b) $0.63 \cdot \frac{180^\circ}{\pi} = \boxed{36.1^\circ}$.

11. a) $\tan 218° = \boxed{.781}$.

    b) $5\sec 72° = 5(3.236) = \boxed{16.18}$.

    c) $\cos 322° + \csc 66° = .788 + 1.094 = \boxed{1.882}$.

    d) $\sin^2 118.5° = (.8788)^2 = \boxed{.772}$.

12. a) The first angle is $\theta = \arccos -.235 = \boxed{103.6°}$. The second angle is $360° - \theta = \boxed{256.4°}$.

    b) Since $\frac{19}{11} > 1$, this equation has $\boxed{\text{no solution}}$.

    c) The first angle is $\theta = \arcsin \frac{11}{19} = \boxed{35.4°}$. The second angle is $180° - \theta = \boxed{144.6°}$.

    d) The first angle is $\theta = \arctan .1875 = \boxed{10.6°}$. The second angle is $180° + \theta = \boxed{190.6°}$.

13. a) Draw a right triangle; the adjacent side to $10°$ is 1300 and the opposite side is what we want. So we set up the equation

$$\begin{aligned} \tan 10° &= \frac{\text{opp}}{1300} \\ .176 &= \frac{\text{opp}}{1300} \\ 1300(.176) &= \text{opp} \\ \boxed{229.225 \text{ ft}} &= \text{opp}. \end{aligned}$$

    b) You could use the Law of Cosines, but I will solve this with vectors. Let vector $\mathbf{a}$ be Aaron's position; then since $\mathbf{a}$ has norm $.85$ and direction angle $90° + 24° = 114°$, we have $\mathbf{a} = \langle .85\cos 114°, .85\sin 114° \rangle = \langle -.346, .777 \rangle$. Let vector $\mathbf{b}$ be Barbara's position; this vector has norm $.55$ and direction angle $90° - 37° = 53°$, so $\mathbf{b} = \langle .55\cos 53°, .55\sin 53° \rangle = \langle .33, .44 \rangle$. Last, the vector from Aaron to Barbara is $\mathbf{b} - \mathbf{a} = \langle .33, .44 \rangle - \langle -.346, .777 \rangle = \langle .676, -.337 \rangle$; this vector has norm $\sqrt{(.676)2 + (-.337)^2} = \boxed{.755 \text{ mi}}$.

14. Let $a = 18$, $b = 23$ and $c = 33$. Then by the Law of Cosines,

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 18^2 &= 23^2 + 33^2 - 2(23)33\cos A \\ 324 &= 1618 - 1518\cos A \\ -1294 &= -1518\cos A \\ .852 &= \cos A \\ \boxed{31.5°} &= \angle A. \end{aligned}$$

Also (again using the Law of Cosines),

$$\begin{aligned}
b^2 &= a^2 + c^2 - 2ac\cos B \\
23^2 &= 18^2 + 33^2 - 2(18)33\cos B \\
529 &= 1413 - 1188\cos B \\
-884 &= -1188\cos B \\
.744 &= \cos B \\
\boxed{41.9^\circ} &= \angle B.
\end{aligned}$$

Last, $\angle C = 180 - \angle A - \angle B = 180^\circ - 31.5^\circ - 41.9^\circ = \boxed{106.6^\circ}$.

15. This is an SSA triangle, which suggests the ambiguous case of the Law of Sines:

$$\begin{aligned}
\frac{\sin L}{l} &= \frac{\sin N}{n} \\
\frac{\sin 38^\circ}{5.8} &= \frac{\sin N}{6.9} \\
\frac{.616}{5.8} &= \frac{\sin N}{6.9} \\
4.248 &= 5.8\sin N \\
.732 &= \sin N \\
\angle N &= \boxed{47.1^\circ}\ \underline{\text{or}}\ \angle N' = 180^\circ - 47.1^\circ = \boxed{132.9^\circ}.
\end{aligned}$$

Now find the third angle. In the first case,

$$\angle M = 180^\circ - 38^\circ - 47.1^\circ = \boxed{94.9^\circ}$$

and in the second case,

$$\angle M' = 180^\circ - 38^\circ - 132.9^\circ = \boxed{9.1^\circ}.$$

Since both these angle Ms are positive, there are two possible triangles. In each case, find m with the Law of Sines. In the first case, we get

$$\begin{aligned}
\frac{\sin L}{l} &= \frac{\sin M}{m} \\
\frac{\sin 38^\circ}{5.8} &= \frac{\sin 94.9^\circ}{m} \\
\frac{.616}{5.8} &= \frac{\sin 94.9^\circ}{m} \\
.616m &= 5.779 \\
m &= \boxed{9.38}.
\end{aligned}$$

In the second case, we get

$$\begin{aligned}
\frac{\sin L}{l} &= \frac{\sin M'}{m'} \\
\frac{\sin 38^\circ}{5.8} &= \frac{\sin 9.1^\circ}{m'} \\
\frac{.616}{5.8} &= \frac{\sin 9.1^\circ}{m'} \\
.616m' &= .917 \\
m' &= \boxed{1.49}.
\end{aligned}$$

In summary, there are two triangles:

$$\boxed{\begin{array}{lll} \angle L = 38^\circ & \angle M = 94.9^\circ & \angle N = 47.1^\circ \\ l = 5.8 & m = 9.38 & n = 6.9 \end{array}}$$

$$\boxed{\begin{array}{lll} \angle L = 38^\circ & \angle M' = 9.1^\circ & \angle N' = 132.9^\circ \\ l = 5.8 & m' = 1.49 & n = 6.9 \end{array}}$$

16. a) i. $A = \frac{1}{2}r^2\theta = \frac{1}{2}(120)^2\left(115^\circ \cdot \frac{\pi}{180^\circ}\right) = 7200(2.007) = \boxed{14450 \text{ sq yards}}$.

ii. The arc length of the curved part is $s = r\theta = 120\left(115^\circ \cdot \frac{\pi}{180^\circ}\right) = 120(2.007) = 240.84$ yards. The perimeter is this arc length plus the two straight segments: $240.84 + 120 + 120 = \boxed{480.84 \text{ yd}}$.

b) i. $\omega = 460\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi}{1\text{rev}} = \boxed{2890.27 \text{ rad/min}}$.

ii. $v = r\omega = 18 \text{ ft}(2890.27 \text{ rad/min}) = \boxed{52024.9 \text{ ft/min}}$.

c) The angles are equal, so we have $8x + 15^\circ = 139^\circ$. Solve for x to get $\boxed{x = 15.5^\circ}$.

4.5 Fall 2018 Final Exam

1. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\cos\dfrac{\pi}{4}$
   b) $\tan 90°$
   c) $\sin\dfrac{\pi}{3}$
   d) $\csc 0°$
   e) $\cos 0$

2. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\cot 240°$
   b) $\cos(-210°)$
   c) $\sin 315°$
   d) $\tan 180°$
   e) $\sec 660°$

3. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\cos\dfrac{5\pi}{3}$
   b) $\sin\dfrac{-7\pi}{6}$
   c) $\tan\dfrac{3\pi}{4}$
   d) $\sec\dfrac{5\pi}{4}$
   e) $\cot\dfrac{3\pi}{2}$

4. $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) (3.4) $\sin^2 200° + \cos^2 200°$
   b) (3.5) $\cos 25° - \sin 65°$
   c) (7.3) $\cos^2\dfrac{3\pi}{8} - \sin^2\dfrac{3\pi}{8}$
   d) (7.2) $\cos 20°\cos 40° - \sin 20°\sin 40°$
   e) (7.2) $\cos 195°$

5. (6.4) $\boxed{\text{NC}}$ Suppose $\cos\theta = -\dfrac{5}{6}$ and $\sin\theta > 0$. Find the exact values of all six trig functions of θ.

6. $\boxed{\text{NC}}$ Suppose $\sin\theta = \frac{3}{4}$ and $\cos\theta < 0$.

   a) (6.1) Find $\csc\theta$.
   b) (3.2) What quadrant is θ in?
   c) (3.2) Find $\sin(\theta - 360°)$.
   d) (3.5) Find $\sin(-\theta)$.
   e) (7.3) Find $\sin 2\theta$.
   f) (7.3) Find $\csc 2\theta$.

7. $\boxed{\text{NC}}$ Sketch a crude graph of each function, labelling the graph appropriately:

   a) (5.3) $y = \cos x + 3$
   b) (5.3) $y = 2\sin x$
   c) (5.4) $y = \tan x$
   d) (5.3) $y = \cos\frac{x}{2} - 1$

8. a) (3.1, 3.7) $\boxed{\text{NC}}$ Explain, in your own words, the difference between the mathematical statements "$\sin\theta = .5$" and "$\sin .5$".

b) (3.1, 7.2) $\boxed{\text{NC}}$ Explain, in your own words, the difference between the expressions "$2\sin\theta$" and "$\sin 2\theta$".

9. (7.1) $\boxed{\text{NC}}$ Simplify the following expression using trig identities: $\cot\theta\cos\theta + \sin\theta$

10. a) (2.9) Convert $75.8°$ to radians, writing your answer as a decimal.

    b) (2.9) Convert 5.32 radians to degrees, writing your answer as a decimal.

11. Let $\mathbf{v} = \langle 15.3, 5.8\rangle$ and let $\mathbf{w} = \langle 10.62, 23.57\rangle$.

    a) (2.4) Compute the magnitude of $\mathbf{v}$.

    b) (2.4) Compute $\mathbf{v}\cdot\mathbf{w}$.

    c) (4.4) Find the measure of the angle between $\mathbf{v}$ and $\mathbf{w}$.

12. (3.3, 6.1) Evaluate the following expressions using a calculator. Your answers can (and should) be written as decimals.

    a) $\sin 148°$

    b) $2\cos(36°)$

    c) $\csc 47° - \cot 35°$

    d) $\tan^2 231°$

    e) $2\sec 4 \cdot 10°$

13. (3.7, 6.2) For each equation, find all angles θ between $0°$ and $360°$ satisfying the following equations.

    a) $\cos\theta = .48$

    b) $\sin\theta = .702$

    c) $\tan\theta = -.275$

    d) $\csc\theta = 0$

    e) $\sec\theta = -3.735$

    f) $\cos\theta = -.823$

14. a) (4.1 or 4.3) A person standing on top of a 53-foot tall cliff sees a bear on the ground. If the person's angle of depression to the bear is $38°$, how far from the base of the cliff is the bear?

    b) (3.3) A bird leaves its nest and flies due south for 4 miles, then flies at an angle $42°$ north of east for 6.5 miles. How far from its nest is it?

15. (4.1) Solve triangle ABC, where $\angle A = 113°$, $\angle C = 35°$ and $a = 7$.

16. (4.2) Solve triangle XYZ, where $x = 18.3$, $y = 7.2$ and $\angle Z = 27°$.

17. Choose one of (a) or (b):

    a) (2.10) A large pizza is circular and has a diameter of 14 inches. If you eat three slices of pizza, and those three slices have central angles $10°$, $13°$ and $22°$, what is the area of the amount of pizza you ate?

b) (2.10) A turntable spins a record at a speed of 45 revolutions per minute. If the radius of the record is 8 inches, what is the linear velocity of a point on the edge of the record?

18. Find the area of triangle ABC, if $a = 7$, $b = 8$ and $c = 12$.

Solutions

1. These angles are all in Quadrant I:

a) $\cos \frac{\pi}{4} = \boxed{\dfrac{\sqrt{2}}{2}}$

b) $\tan 90^\circ$ $\boxed{\text{DNE}}$
(slope of vertical line)

c) $\sin \frac{\pi}{3} = \boxed{\dfrac{\sqrt{3}}{2}}$

d) $\csc 0^\circ = \frac{1}{\sin 0} = \frac{1}{0}$ which $\boxed{\text{DNE}}$

e) $\cos 0 = \boxed{1}$

2. a) $\cot 240^\circ = \boxed{\dfrac{1}{\sqrt{3}}}$ (Quadrant III; reference angle 60°).

b) $\cos(-210^\circ) = \cos 210^\circ = \boxed{-\dfrac{\sqrt{3}}{2}}$ (Quadrant III; reference angle 30°).

c) $\sin 315^\circ = \boxed{-\dfrac{\sqrt{2}}{2}}$ (Quadrant IV; reference angle 45°).

d) $\tan 180^\circ = \boxed{0}$ (slope of a horizontal line).

e) $\sec 660^\circ = \sec 300^\circ = \boxed{2}$ (Quadrant IV; reference angle 60°).

3. a) $\cos \frac{5\pi}{3} = \boxed{\dfrac{1}{2}}$ (Quadrant IV; reference angle $\frac{\pi}{3} = 60^\circ$).

   b) $\sin \frac{-7\pi}{6} = \boxed{\dfrac{1}{2}}$ (Quadrant II; reference angle $\frac{\pi}{6} = 30^\circ$).

   c) $\tan \frac{3\pi}{4} = \boxed{-1}$ (Quadrant II; reference angle $\frac{\pi}{4} = 45^\circ$).

   d) $\sec \frac{5\pi}{4} = \boxed{-\sqrt{2}}$ (Quadrant III; reference angle $\frac{\pi}{4} = 45^\circ$).

   e) $\cot \frac{3\pi}{2} = \frac{1}{\text{DNE}} = \boxed{0}$.

4. a) $\sin^2 200^\circ + \cos^2 200^\circ = \boxed{1}$ (Pythagorean identity).

   b) Use the cofunction identity $\cos\theta = \sin(90^\circ - \theta)$ to see that $\cos 25^\circ - \sin 65^\circ = \sin 65^\circ - \sin 65^\circ = \boxed{0}$.

   c) This is the right-hand side of the double angle identity for cosine, with $\frac{3\pi}{8}$ plugged in for θ. Thus $\cos^2 \frac{3\pi}{8} - \sin^2 \frac{3\pi}{8} = \cos 2\left(\frac{3\pi}{8}\right) = \cos\frac{3\pi}{4} = \boxed{-\dfrac{\sqrt{2}}{2}}$.

   d) This is the right-hand side of the addition identity for cosine: $\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ = \cos(20^\circ + 40^\circ) = \cos 60^\circ = \boxed{\dfrac{1}{2}}$.

   e) Split up the angle and use the addition identity for cosine:

$$\begin{aligned}
\cos 195^\circ &= \cos(135^\circ + 60^\circ)\\
&= \cos 135^\circ \cos 60^\circ - \sin 135^\circ \sin 60^\circ\\
&= \frac{-\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\\
&= \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}.
\end{aligned}$$

5. Draw a right triangle and label the adjacent side 5 and the hypotenuse 6. Solve for the opposite side using the Pythagorean Theorem to get $\sqrt{6^2 - 5^2} = \sqrt{36 - 25} = \sqrt{11}$. That means:

   - $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \boxed{\dfrac{\sqrt{11}}{6}}$;
   - $\csc\theta = \frac{1}{\sin\theta} = \boxed{\dfrac{6}{\sqrt{11}}}$;
   - $\cos\theta = \boxed{-\dfrac{5}{6}}$ (given);
   - $\sec\theta = \frac{1}{\cos\theta} = \boxed{-\dfrac{6}{5}}$;
   - $\tan\theta = \frac{\text{opp}}{\text{adj}} = \boxed{-\dfrac{\sqrt{11}}{5}}$;
   - $\cot\theta = \frac{1}{\tan\theta} = \boxed{-\dfrac{5}{\sqrt{11}}}$.

Since $\cos\theta < 0$ and $\sin\theta > 0$, θ is in Quadrant II, justifying the signs of these answers.

6. a) $\csc\theta = \frac{1}{\sin\theta} = \boxed{\dfrac{4}{3}}$.

   b) Since $\sin\theta > 0$ and $\cos\theta < 0$, θ is in $\boxed{\text{Quadrant II}}$.

   c) $\sin(\theta - 360°) = \sin\theta = \boxed{\dfrac{3}{4}}$.

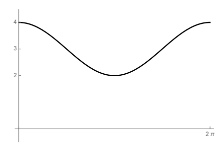
   d) $\sin(-\theta) = -\sin\theta = \boxed{-\dfrac{3}{4}}$.

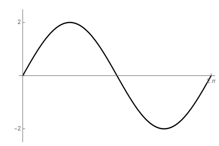
   e) First, we need to find $\cos\theta$. Draw a right triangle with opposite side 3 and hypotenuse 4; solve for the adjacent side using the Pythagorean Theorem to get $\sqrt{4^2 - 3^2} = \sqrt{7}$. Therefore $\cos\theta = \boxed{-\dfrac{\sqrt{7}}{4}}$.

   Now for the answer. Using the double angle identity for sine, $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{4}\right)\frac{-\sqrt{7}}{4} = \boxed{-\dfrac{3\sqrt{7}}{8}}$.

   f) $\csc 2\theta = \frac{1}{\sin 2\theta} = \boxed{-\dfrac{8}{3\sqrt{7}}}$.

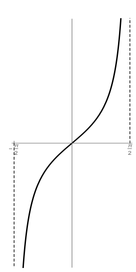
7. a) $y = \cos x + 3$ is the graph of $\cos x$, shifted up 3 units (shown below at left):

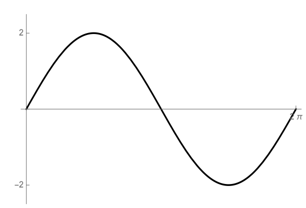




   b) $y = 2\sin x$ is the graph of $\sin x$, stretched vertically by a factor of 2 (shown above at right):

   c) $y = \tan x$ is shown below at left:





   d) $y = \cos\frac{x}{2} - 1$ is the graph of $\cos x$, with period $\frac{2\pi}{B} = \frac{2\pi}{1/2} = 4\pi$, shifted one unit down (shown above at right):

8. a) "$\sin\theta = .5$" means that θ is some angle whose sine is $.5$; "$\sin .5$" means you are taking the sine of an angle which measures $.5$ radians.

b) "$2\sin\theta$" means first take the sine of θ, then multiply by 2. "$\sin 2\theta$" means first multiply θ by 2, then take the sine of that angle.

9. Start by writing in terms of sines and cosines, then add the fractions by finding a common denominator:

$$\begin{aligned}\cot\theta\cos\theta+\sin\theta &= \frac{\cos\theta}{\sin\theta}\cos\theta+\sin\theta\\ &= \frac{\cos^2\theta}{\sin\theta}+\sin\theta\\ &= \frac{\cos^2\theta}{\sin\theta}+\frac{\sin^2\theta}{\sin\theta}\\ &= \frac{\cos^2\theta+\sin^2\theta}{\sin\theta}\\ &= \frac{1}{\sin\theta}=\boxed{\csc\theta}.\end{aligned}$$

10. a) $75.8^\circ\cdot\frac{\pi}{180^\circ}=\boxed{1.32296}$ radians.

b) $5.32\cdot\frac{180^\circ}{\pi}=\boxed{304.814^\circ}$.

11. a) $|\mathbf{v}|=\sqrt{15.3^2+5.8^2}=\sqrt{267.73}=\boxed{16.36}$.

b) $\mathbf{v}\cdot\mathbf{w}=(15.3)10.62+(5.8)23.57=\boxed{299.192}$.

c) First, $|\mathbf{w}|=\sqrt{10.62^2+23.57^2}=\sqrt{668.329}=25.852$. Then, use this formula and substitute, using previous parts of the problem:

$$\begin{aligned}\mathbf{v}\cdot\mathbf{w} &= |\mathbf{v}|\,|\mathbf{w}|\,\cos\theta\\ 299.192 &= 16.36(25.852)\cos\theta\\ .7074 &= \cos\theta\\ &\Rightarrow \theta=\arccos .7074=\boxed{44.97^\circ}.\end{aligned}$$

12. a) $\sin 148^\circ=\boxed{.529919}$.

b) $2\cos(36^\circ)=2(.809)=\boxed{1.618}$.

c) $\csc 47^\circ-\cot 35^\circ=\frac{1}{\sin 47^\circ}-\frac{1}{\tan 35^\circ}=1.367-1.42815=\boxed{-.0608}$.

d) $\tan^2 231^\circ=(1.2349)^2=\boxed{1.52497}$.

e) $2\sec 4\cdot 10^\circ=2\sec 40^\circ=2\cdot\frac{1}{\cos 40^\circ}=2\cdot 1.305=\boxed{2.61}$.

13. a) One angle is $\theta=\arccos .48=\boxed{61.31^\circ}$; a second angle is $360^\circ-\theta=\boxed{298.69^\circ}$.

b) One angle is $\theta=\arcsin .702=\boxed{44.59^\circ}$; a second angle is $180^\circ-\theta=\boxed{135.41^\circ}$.

c) One angle would be $\theta=\arctan -.275=-15.38^\circ$, but this is less than 0°. Add 180° to get $\boxed{164.62^\circ}$; add 180° again to get $\boxed{344.62^\circ}$.

d) $\boxed{\text{No such angle exists}}$, since 0 is between -1 and 1.

e) First, take the reciprocal of both sides to get $\cos\theta = -.2677$. Then, one angle is $\theta = \arccos -.2677 = \boxed{105.53^\circ}$; a second angle is $360^\circ - 105.53^\circ = \boxed{254.47^\circ}$.

f) One angle is $\theta = \arccos -.823 = \boxed{145.39^\circ}$; a second angle is $360^\circ - \theta = \boxed{214.61^\circ}$.

14. a) We have a triangle where the top of the cliff is A, the bottom of the cliff is C (this is the right angle), and the bear is at B. We are told that the angle of depression is 38°, so $\angle A = 90^\circ - 38^\circ = 52^\circ$ (since angles of depression are measured from the horizontal). We know the adjacent side to A is $b = 53$ and want to find the opposite side a, so we use tangent:

$$\tan 52^\circ = \frac{\text{opp}}{\text{adj}} = \frac{a}{53} \Rightarrow a = 53\tan 52^\circ = \boxed{67.83 \text{ ft}}.$$

b) *Solution # 1:* The first part of the bird's trip has length 4 and direction angle 270°, so it is the vector

$$\langle 4\cos 270^\circ, 4\sin 270^\circ\rangle = \langle 0, -4\rangle.$$

The second part of the trip has length 6.5 and direction angle 42°, so it is the vector

$$\langle 6.5\cos 42^\circ, 6.5\sin 42^\circ\rangle = \langle 4.83044, 4.34935\rangle.$$

Adding these, we get the bird's final position:

$$\langle 0 + 4.83044, -4 + 4.34935\rangle = \langle 4.83, .349\rangle.$$

Last, the distance from the nest is the magnitude of this last vector:

$$\sqrt{4.83^2 + .349^2} = \sqrt{23.45} = \boxed{4.843 \text{ mi}}.$$

Solution # 2: Use the Law of Cosines; with $a = 4$, $b = 6.5$ and $\angle C = 90^\circ - 42^\circ = 48^\circ$, we want to find c:

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ &= 4^2 + 6.5^2 - 2(4)6.5\cos 48^\circ \\ &= 23.45 \end{aligned}$$

Thus $c = \sqrt{23.45} = \boxed{4.843 \text{ mi}}$.

15. The given information is AAS, so start by finding the third angle and then use the Law of Sines.

$\angle B = 180^\circ - 113^\circ - 35^\circ = \boxed{32^\circ}$.

Next, $\frac{\sin A}{a} = \frac{\sin C}{c}$ so $\frac{\sin 113^\circ}{7} = \frac{\sin 35^\circ}{c}$ so $c = \frac{7\sin 35^\circ}{\sin 113^\circ} = \boxed{4.36}$.

Last, $\frac{\sin A}{a} = \frac{\sin B}{b}$ so $\frac{\sin 113^\circ}{7} = \frac{\sin 32^\circ}{b}$ so $b = \frac{7\sin 32^\circ}{\sin 113^\circ} = \boxed{4.03}$.

16. The given information is SAS, so start with the Law of Cosines:

$$\begin{aligned} z^2 &= x^2 + y^2 - 2xy\cos Z \\ z^2 &= 18.3^2 + 7.2^2 - 2(18.3)(7.2)\cos 27^\circ \\ z^2 &= 386.73 - 263.52(.891) \\ z^2 &= 151.934 \\ z &= \sqrt{151.934} = \boxed{12.33}. \end{aligned}$$

Then use the Law of Cosines again to find $\angle Y$:

$$\begin{aligned} y^2 &= x^2 + z^2 - 2xz\cos Y \\ 7.2^2 &= 18.3^2 + 12.33^2 - 2(18.3)12.33\cos Y \\ 51.84 &= 486.919 - 451.278\cos Y \\ -435.079 &= -451.278\cos Y \\ .964104 &= \cos Y \\ \Rightarrow Y &= \arccos .964104 = \boxed{15.4^\circ}. \end{aligned}$$

Finally, $\angle X = 180^\circ - 15.4^\circ - 27^\circ = \boxed{137.6^\circ}$.

17. a) The radius of the pizza is $r = \frac{1}{2}(14) = 7$. The angles of the pizza add to $10^\circ + 13^\circ + 22^\circ = 45^\circ = \frac{\pi}{4}$ radians. Thus the area of the pizza eaten is $A = \frac{1}{2}r^2\theta = \frac{1}{2}(7^2)\frac{\pi}{4} = \boxed{19.24}$ square inches.

    b) We have angular velocity $\omega = 45 \cdot 2\pi = 282.74$ radians per minute; thus the linear velocity is $v = r\omega = 8(282.74) = \boxed{\text{2261.95 inches per minute}}$.

18. Use Heron's formula: first, $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(7+8+12) = 13.5$. Then,

$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{13.5(6.5)(5.5)(1.5)} = \sqrt{723.93} = \boxed{\text{26.9 sq units}}.$$

4.6 Fall 2017 Final Exam

1. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\cos 30°$
   b) $\tan 90°$
   c) $\sin 45°$
   d) $\csc 90°$
   e) $\sin 0°$

2. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\tan 150°$
   b) $\cos(-225°)$
   c) $\sin 330°$
   d) $\cot 180°$
   e) $\sec 780°$

3. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\cos \frac{4\pi}{3}$
   b) $\sin \frac{-2\pi}{3}$
   c) $\tan \frac{3\pi}{4}$
   d) $\cot \frac{-\pi}{6}$
   e) $\csc \frac{5\pi}{2}$

4. $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) (3.4) $\sin^2 310° + \cos^2 310°$
   b) (3.2) $\cos 400° - \cos 40°$
   c) (6.4) $\tan^2 85° - \sec^2 85°$
   d) (7.2) $\sin 20° \cos 40° + \cos 20° \sin 40°$
   e) (7.2) $\cos 75°$

5. (6.4) $\boxed{\text{NC}}$ Suppose $\tan\theta = \frac{2}{3}$ and $\sin\theta < 0$. Find the exact values of all six trig functions of θ.

6. $\boxed{\text{NC}}$ Suppose $\sin\theta = \frac{1}{5}$ and $\cos\theta > 0$.

   a) (6.1) Find $\csc\theta$.
   b) (3.2) What quadrant is θ in?
   c) (3.2) Find $\sin(\theta - 360°)$.
   d) (3.5) Find $\sin(-\theta)$.
   e) (7.3) Find $\sin 2\theta$.
   f) (7.3) Find $\csc 2\theta$.

7. $\boxed{\text{NC}}$ Sketch a crude graph of each function, labelling the graph appropriately:

   a) (5.3) $y = \sin x - 4$
   b) (5.3) $y = 3\cos x$
   c) (6.5) $y = \cot x$
   d) (5.3) $y = \sin 2x + 1$

8. (7.1) $\boxed{\text{NC}}$ Simplify the following expression using trig identities:

$$\frac{1 - \sin^2\theta}{1 - \csc^2\theta}$$

9. (3.3, 6.1) Evaluate the following expressions using a calculator. Your answers can (and should) be written as decimals.

   a) $\sin 103°$
   b) $5\cos(-51°)$
   c) $\cos 133° - \cos 75°$
   d) $\csc(52° + 81°)$
   e) $\sin^2 231°$
   f) $\tan 40° \cot 108°$
   g) $2\sec 4 \cdot 83°$

10. (3.7, 6.2) For each equation, find (decimal approximations of) **<u>all</u>** angles θ between $0°$ and $360°$ satisfying the following equations. If the equation has no solution, say so.

    a) $\sin\theta = .683$
    b) $\cos\theta = .23$
    c) $\tan\theta = -3.21$
    d) $\sec\theta = 0$
    e) $\cos\theta = -.735$
    f) $\sin\theta = 1.04$

11. (3.2, 6.1) Find the six trig functions of the angle θ, if the point $(-2.7, 5.9)$ is on the terminal side of θ when it is drawn in standard position.

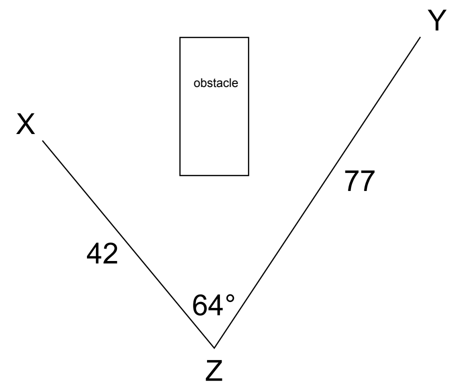
12. Let $\mathbf{v} = \langle 8, 1\rangle$ and let $\mathbf{w} = \langle 3, 7\rangle$.

    a) (2.4) Compute the magnitude of $\mathbf{v}$.
    b) (2.4) Compute $\mathbf{v} \cdot \mathbf{w}$.
    c) (4.4) Find the measure of the angle between $\mathbf{v}$ and $\mathbf{w}$.

13. (4.1 or 4.3) A chipmunk located 42 feet from a flagpole has to look up at an angle of $50°$ to see the top of the flagpole. How tall is the flagpole? (Assume the chipmunk's eyes are at ground level.)

14. (4.2) A surveyor is trying to determine the distance between points X and Y as shown below, but can't measure the distance directly because there is an obstacle in the way. So instead, the surveyor measures the distances from X and Y to a third point Z, and measures an angle at point Z as shown in the

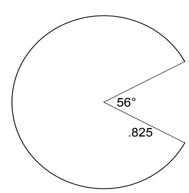
figure below. Find the distance from point X to point Y.

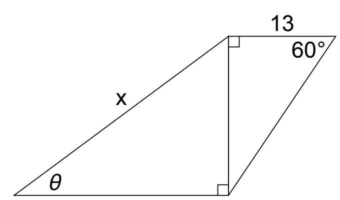


15. (4.1) Solve triangle PQR, where $\angle P = 75°$, $\angle Q = 58°$ and $p = 53$.

16. Choose any three of the following five questions.

    a) (2.10) The video game Pac-Man features a character that looks like the left-hand figure below. Find the area enclosed by Pac-Man.





    b) (2.7 and 4.3) Write an expression for x in terms of the other quantities given in the right-hand figure above. Simplify your answer.

    c) (2.10) A bicycle wheel of radius 16.5 inches rotates at a speed of 35 revolutions per minute. What is the linear velocity of a point on the edge of the wheel?

    d) (4.5) Find the area of triangle ABC, if $a = 6$, $b = 4$ and $c = 5$.

    e) (3.3) A bird leaves its nest and flies due west for 9 miles, then flies at an angle $57°$ north of east for 7 miles. How far from its nest is it?

Solutions

1. a) $\cos 30° = \boxed{\dfrac{\sqrt{3}}{2}}$.

   b) $\tan 90° = \frac{1}{0}$ which $\boxed{\text{DNE}}$.

   c) $\sin 45° = \boxed{\dfrac{\sqrt{2}}{2}}$.

   d) $\csc 90° = \frac{1}{1} = \boxed{1}$.

   e) $\sin 0° = \boxed{0}$.

2. a) $\tan 150° = \boxed{-\dfrac{\sqrt{3}}{3}}$ (quadrant II, reference angle $30°$)

   b) $\cos(-225°) = \boxed{-\dfrac{\sqrt{2}}{2}}$ (quadrant II, reference angle $45°$)

   c) $\sin 330° = \boxed{-\dfrac{1}{2}}$ (quadrant IV, reference angle $30°$)

   d) $\cot 180° = \frac{-1}{0}$ which $\boxed{\text{DNE}}$.

   e) $\sec 780° = \boxed{2}$ (quadrant I, reference angle $60°$)

3. a) $\cos \frac{4\pi}{3} = \boxed{-\dfrac{1}{2}}$ (quadrant III, reference angle $60°$)

   b) $\sin \frac{-2\pi}{3} = \boxed{-\dfrac{\sqrt{3}}{2}}$ (quadrant III, reference angle $60°$)

   c) $\tan \frac{3\pi}{4} = \boxed{-1}$ (quadrant II, reference angle $45°$)

   d) $\cot \frac{-\pi}{6} = \boxed{-\sqrt{3}}$ (quadrant IV, reference angle $30°$)

   e) $\csc \frac{5\pi}{2} = \csc 90° = \frac{1}{1} = \boxed{1}$.

4. a) $\sin^2 310° + \cos^2 310° = \boxed{1}$ (by the Pythagorean identity)

   b) $\cos 400° - \cos 40° = \cos 40° - \cos 40°$ (by periodicity) which simplifies to $\boxed{0}$.

   c) $\tan^2 85° - \sec^2 85° = \boxed{-1}$ (by a rewritten version of $\tan^2\theta + 1 = \sec^2\theta$)

   d) Use the addition identity for sine to get $\sin 20° \cos 40° + \cos 20° \sin 40° = \sin(20° + 40°) = \sin 60° = \boxed{\dfrac{\sqrt{3}}{2}}$

   e) Use the addition identity for cosine to get $\cos 75° = \cos(30° + 45°) = \cos 30° \cos 45° - \sin 30° \sin 45° = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$.

5. Draw a triangle, labelling the opposite side 2 and the adjacent side 3. Solve for the hypotenuse using the Pythagorean Theorem, obtaining $\sqrt{13}$. Since $\tan\theta > 0$ and $\sin\theta < 0$, the angle is in Quadrant III, so all trig functions other than tangent and cotangent are negative. Using the right triangle definitions of the trig functions, we get

$$\sin\theta = -\frac{2}{\sqrt{13}} \qquad \cos\theta = -\frac{3}{\sqrt{13}} \qquad \tan\theta = \frac{2}{3}$$
$$\csc\theta = -\frac{\sqrt{13}}{2} \qquad \sec\theta = -\frac{\sqrt{13}}{3} \qquad \cot\theta = \frac{3}{2}$$

6. a) $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{1/5} = \boxed{5}$.

   b) Since $\sin\theta$ and $\cos\theta$ are positive, θ is in $\boxed{\text{Quadrant I}}$.

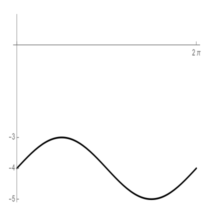
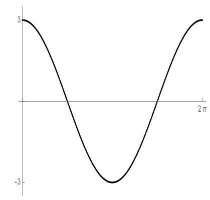
   c) $\sin(\theta - 360°) - \sin\theta = \boxed{\dfrac{1}{5}}$.

   d) $\sin(-\theta) = -\sin\theta = \boxed{-\dfrac{1}{5}}$.

   e) First, find $\cos\theta$: draw a triangle, label the opposite side 1 and the hypotenuse 5, and solve for the adjacent side using the Pythagorean Theorem to get $\sqrt{24}$. Since the angle is in Quadrant I, $\cos\theta = \frac{\sqrt{24}}{5}$. Next, $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{1}{5}\right)\left(\frac{\sqrt{24}}{5}\right) = \boxed{\dfrac{2\sqrt{24}}{25}}$.

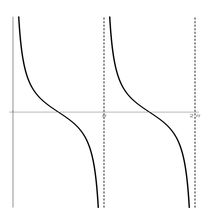
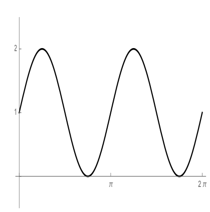
   f) $\csc 2\theta = \frac{1}{\sin 2\theta} = \frac{1}{2\sqrt{24}/25} = \boxed{\dfrac{25}{2\sqrt{24}}}$.

7. a) $y = \sin x - 4$ is the graph of $y = \sin x$, shifted down 4 units (shown below and to the left):

   b) $y = 3\cos x$ is the graph of $y = \cos x$, stretched vertically by a factor of 3 (shown above and to the right).

   c) $y = \cot x$ looks like the graph shown below at left:

   d) $y = \sin 2x + 1$ has period $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$, and is shifted up 1 unit (shown above at right).

8. Use Pythagorean identities, then a quotient identity, then divide the fractions:

$$\begin{aligned}\frac{1-\sin^2\theta}{1-\csc^2\theta} &= \frac{\cos^2\theta}{-\cot^2\theta}\\ &= \frac{\cos^2\theta}{\frac{-\cos^2\theta}{\sin^2\theta}}\\ &= \frac{\cos^2\theta}{1}\cdot\frac{\sin^2\theta}{-\cos^2\theta}\\ &= \frac{\sin^2\theta}{-1}\\ &= \boxed{-\sin^2\theta}.\end{aligned}$$

9. a) $\sin 103^\circ = \boxed{.97437}$.
   b) $5\cos(-51^\circ) = 5(.62932) = \boxed{3.1466}$.
   c) $\cos 133^\circ - \cos 75^\circ = -.681998 - .258819 = \boxed{-.940817}$.
   d) $\csc(52^\circ + 81^\circ) = \csc 133^\circ = \frac{1}{\sin 133^\circ} = \frac{1}{.731354} = \boxed{1.36733}$.
   e) $\sin^2 231^\circ = (\sin 231^\circ)^2 = (-.777146)^2 = \boxed{.603956}$.
   f) $\tan 40^\circ \cot 108^\circ = (.8391)\left(\frac{1}{\tan 108^\circ}\right) = (.8391)\left(\frac{1}{-3.07768}\right) = \boxed{-.27264}$.
   g) $2\sec 4\cdot 83^\circ = 2\left(\sec(332^\circ)\right) = 2\left(\frac{1}{\cos 332^\circ}\right) = 2\left(\frac{1}{.882948}\right) = \boxed{2.26514}$.

10. a) $\theta = \arcsin .683 = \boxed{43.1^\circ}$; the other angle is $180^\circ - \theta = \boxed{136.9^\circ}$.
    b) $\theta = \arccos .23 = \boxed{76.7^\circ}$; the other angle is $360^\circ - \theta = \boxed{283.3^\circ}$.
    c) $\theta = \arctan -3.21 = -72.7^\circ$. Add 360° to get $\boxed{287.3^\circ}$; the other angle is $\theta + 180^\circ = \boxed{107.3^\circ}$.
    d) $\sec\theta = 0$ has $\boxed{\text{no solution}}$ because $-1 < 0 < 1$.
    e) $\theta = \arccos -.735 = \boxed{137.3^\circ}$; the other angle is $360^\circ - \theta = \boxed{222.7^\circ}$.
    f) $\sin\theta = 1.04$ has $\boxed{\text{no solution}}$ because $1.04 > 1$.

11. We are given $x = -2.7$, $y = 5.9$ and need to find r. By the Pythagorean Theorem, $r = \sqrt{x^2+y^2} = \sqrt{(-2.7)^2 + 5.9^2} = 6.488$. Then:

$$\sin\theta = \frac{y}{r} = \frac{5.9}{6.488} = \boxed{.909} \qquad \cos\theta = \frac{x}{r} = \frac{-2.7}{6.488} = \boxed{-.416} \qquad \tan\theta = \frac{y}{x} = \frac{5.9}{-2.7} = \boxed{-2.185}$$

$$\csc\theta = \frac{r}{y} = \frac{6.488}{5.9} = \boxed{1.099} \qquad \sec\theta = \frac{r}{x} = \frac{6.488}{-2.7} = \boxed{-2.402} \qquad \cot\theta = \frac{x}{y} = \frac{-2.7}{5.9} = \boxed{-.416}.$$

12. a) $||\mathbf{v}|| = \sqrt{8^2+1^2} = \boxed{\sqrt{65}}$.
    b) $\mathbf{v}\cdot\mathbf{w} = 8(3) + 1(7) = 24 + 7 = \boxed{31}$.

c) Start with the formula $\mathbf{v} \cdot \mathbf{w} = ||\mathbf{v}|| \, ||\mathbf{w}|| \cos\theta$ and solve for θ. From parts (a) and (b), we know $\mathbf{v} \cdot \mathbf{w}$ and $||\mathbf{v}||$, so we need to find $||\mathbf{w}|| = \sqrt{3^2 + 7^2} = \sqrt{9 + 49} = \sqrt{58}$. Now from the formula, we have

$$\begin{aligned}
\mathbf{v} \cdot \mathbf{w} &= ||\mathbf{v}|| \, ||\mathbf{w}|| \cos\theta \\
31 &= \sqrt{65} \cdot \sqrt{58} \cos\theta \\
\frac{31}{\sqrt{65}\sqrt{58}} &= \cos\theta \\
.504883 &= \cos\theta \\
\arccos .504883 &= \theta \\
\boxed{59.67^\circ} &= \theta.
\end{aligned}$$

13. Draw a right triangle, labelling the angle to the horizontal with 50°. We are given that the adjacent side is 42 and need to find the opposite side, so use tangent:

$$\tan 50^\circ = \frac{h}{42} \Rightarrow h = 42 \tan 50^\circ = \boxed{50.05 \text{ feet}}.$$

14. Use the Law of Cosines to find z, the distance between X and Y:

$$\begin{aligned}
z^2 &= x^2 + y^2 - 2xy \cos Z \\
z^2 &= 42^2 + 77^2 - 2(42)(77) \cos 64^\circ \\
z^2 &= 1764 + 5929 - 2(42)(77)(.438371) \\
z^2 &= 1764 + 5929 - 2835.38 \\
z^2 &= 4857.62 \\
z &= \sqrt{4857.62} = \boxed{69.6966}.
\end{aligned}$$

15. First, the third angle is $180^\circ - 75^\circ - 58^\circ = \boxed{47^\circ}$.

Next, use the Law of Sines to find q:

$$\begin{aligned}
\frac{\sin P}{p} &= \frac{\sin Q}{q} \\
\frac{\sin 75^\circ}{53} &= \frac{\sin 58^\circ}{q} \\
\frac{.965926}{53} &= \frac{.848048}{q} \\
.965926q &= 53(.84808) = 44.9465 \\
q &= \frac{44.9465}{.965926} = \boxed{46.5321}.
\end{aligned}$$

Last, use the Law of Sines to find r:

$$\begin{aligned}\frac{\sin P}{p} &= \frac{\sin R}{r}\\ \frac{\sin 75°}{53} &= \frac{\sin 47°}{r}\\ \frac{.965926}{53} &= \frac{.731354}{q}\\ .965926q &= 53(.731354) = 38.7617\\ q &= \frac{38.7617}{.965926} = \boxed{40.1291}.\end{aligned}$$

16. a) The angle enclosed by Pac-Man is $360° - 56° = 304° = 304° \cdot \frac{\pi}{180°} = 5.3058$ radians. Now by the area formula for sectors, $A = \frac{1}{2}r^2\theta = \frac{1}{2}(.825)^2(5.3058) = \boxed{1.80563 \text{ square units}}$.

b) Let w be the vertical line segment; using the right-hand triangle we have $\frac{w}{13} = \tan 60° = \sqrt{3}$, so $w = 13\sqrt{3}$. Now from the left-hand triangle, we have $\frac{x}{w} = \csc\theta$ so $x = w\csc\theta$. Substituting in for w, we get $\boxed{x = 13\sqrt{3}\csc\theta}$.

c) $\omega = 35(2\pi) = 219.911$ radians per minute. Then the linear velocity is $v = r\omega = 16.5(219.911) = \boxed{3628.54 \text{ inches per minute}}$.

d) The semiperimeter is $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(6+4+5) = 7.5$; then by Heron's formula the area is

$$\begin{aligned}A &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{7.5(7.5-6)(7.5-4)(7.5-5)}\\ &= \sqrt{98.4375}\\ &= \boxed{9.92157 \text{ square units}}.\end{aligned}$$

e) The first part of the bird's journey is the vector $\mathbf{v} = \langle -9, 0\rangle$; the second part has angle $57°$ and magnitude 7 so its components are $\mathbf{w} = \langle 7\cos 57°, 7\sin 57°\rangle = \langle 3.81247, 5.87069\rangle$. Therefore the bird has travelled $\mathbf{v}+\mathbf{w} = \langle -5.18753, 5.87069\rangle$; the distance from its nest is $||\mathbf{v}+\mathbf{w}|| = \sqrt{(-5.18753)^2 + 5.87069^2} = \sqrt{61.3754} = \boxed{7.83425 \text{ miles}}$.

4.7 Fall 2016 Final Exam

No calculators allowed on these questions

1. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\tan 45°$
   b) $\cos 60°$
   c) $\sin 90°$
   d) $\sec 30°$
   e) $\cos 0°$

2. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\cot 240°$
   b) $\cos(-150°)$
   c) $\sin 315°$
   d) $\csc 270°$
   e) $\sec 450°$

3. (3.6, 6.3) $\boxed{\text{NC}}$ Find the exact value of each quantity:

   a) $\sin \frac{11\pi}{6}$
   b) $\sin \frac{-\pi}{3}$
   c) $\cot \frac{3\pi}{2}$
   d) $\tan \frac{5\pi}{6}$
   e) $\tan \frac{11\pi}{3}$

4. Find the exact value of each quantity:

   a) (3.4) $\sin^2 50° + \cos^2 50°$
   b) (3.6) $\cos^2 135°$
   c) (7.2) $\sin 75°$
   d) (3.6) $\tan 4 \cdot 30°$
   e) (6.3) $8 \sec \frac{\pi}{4}$

5. (6.4) Suppose $\cos\theta = \frac{3}{5}$ and $\tan\theta < 0$. Find the exact values of all six trig functions of θ.

6. Suppose $\tan\theta = 3$.

   a) (7.3) Find $\tan 2\theta$.
   b) (6.1) Find $\cot\theta$.
   c) (7.3) Find $\cot 2\theta$.
   d) (3.2) What quadrant(s) might θ be in?
   e) (3.2) Find $\tan(\theta + 720°)$.
   f) (3.5) Find $\tan(-\theta)$.

7. (5.4) Sketch the graph of $y = \tan x$.

8. a) (5.3) Which of graphs A-K below is the graph of $y = \sin \frac{x}{2}$?

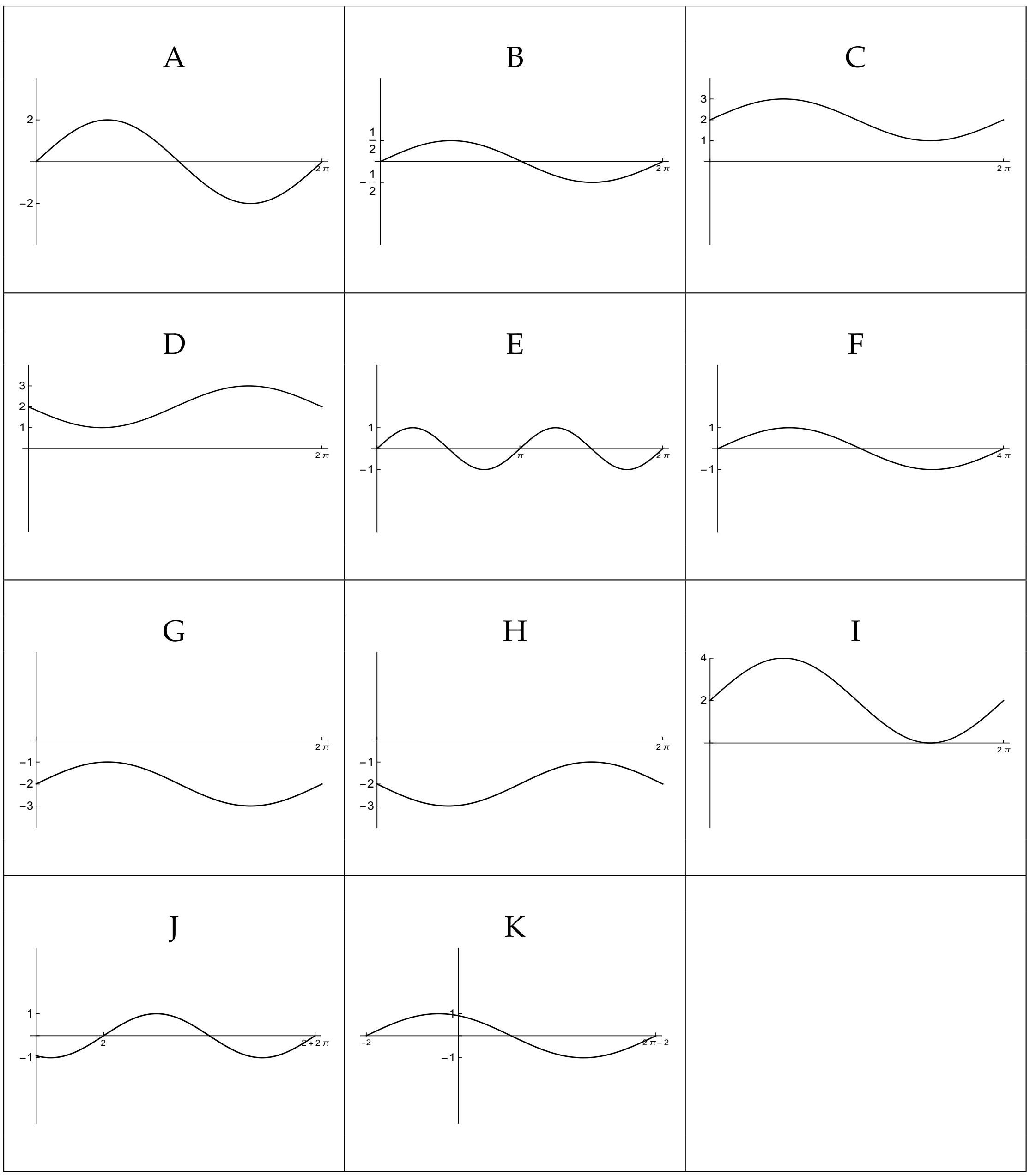
b) (5.3) Which of graphs A-K is the graph of $y = 2\sin x$?

c) (5.3) Which of graphs A-K is the graph of $y = \sin x - 2$?

d) (5.3) Which of graphs A-K is the graph of $y = \sin(x + 2)$?

e) (5.3) Which of graphs A-K is the graph of $y = \sin 2x$?

f) (5.3) Which of graphs A-K is the graph of $y = \dfrac{\sin x}{2}$?



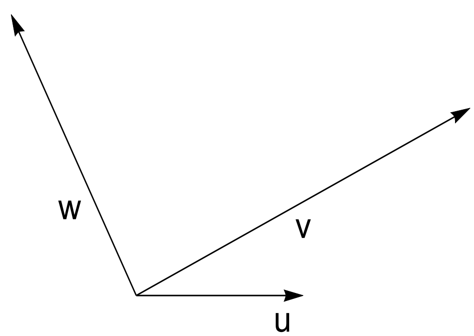
9. (2.4) Vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are shown below. Sketch the following vectors on the same picture, being sure to label which vector is which:

   a) $\mathbf{w} - \mathbf{u}$

   b) $-\mathbf{w}$

   c) $\frac{1}{2}\mathbf{u}$

   d) $\mathbf{u} + \mathbf{v} + \mathbf{w}$



10. (3.3, 6.1) Evaluate the following expressions using a calculator.

   a) $\csc 52°$

   b) $\sin 25° + \sin 110°$

   c) $\sec(83° - 40°)$

   d) $\tan^2 117°$

   e) $4 \cot 286°$

   f) $\sin 100° \cos 44°$

11. For each equation, find (decimal approximations of) all angles θ between $0°$ and $360°$ satisfying the following equations. If there are no such angles, say so.

   a) (3.7) $\sin\theta = .28$

   b) (3.7) $\cos\theta = 1.35$

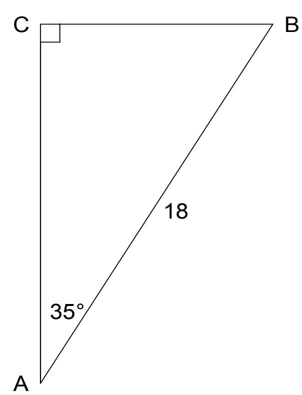
   c) (6.2) $\cot\theta = -.55$

   d) (6.2) $\csc\theta = 1$

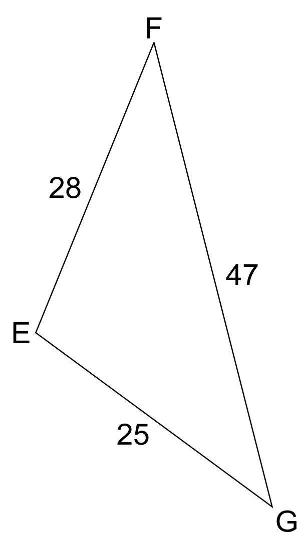
12. (6.1) Suppose that θ is an angle in a right triangle whose hypotenuse has length 43.2 and whose side opposite θ has length 25.1. Find the six trig functions of angle θ.

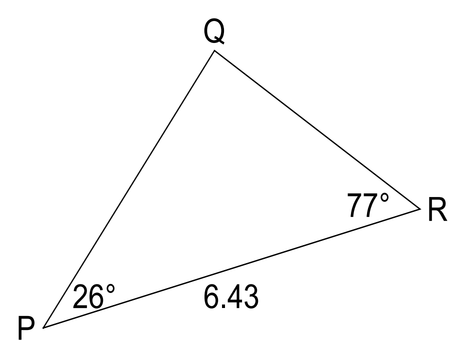
13. a) (2.4) Compute the magnitude of the vector $\langle -5, -4 \rangle$.
    b) (3.7) Compute the direction angle of the vector $\langle -25, 7 \rangle$.
    c) (2.4) Suppose $\mathbf{v} = \langle 6, -4 \rangle$ and $\mathbf{w} = \langle 2, 11 \rangle$. Compute $5\mathbf{v} + 2\mathbf{w}$.
    d) (2.4) Compute $\mathbf{v} \cdot \mathbf{w}$, where $\mathbf{v}$ and $\mathbf{w}$ are as in part (c).
    e) (3.3) Find the components of a vector which has norm 17 and direction angle $230°$.

14. (4.1 or 4.3) Solve triangle ABC, which is pictured below:



15. (4.2) Solve triangle EFG, which is pictured below at left:





16. (4.1) Solve triangle PQR, which is pictured above at right.

17. Classify the following statements as true or false:

a) (6.4) $\tan^2\theta - 1 = \sec^2\theta$

b) (3.5) $\cos(-\theta) = -\cos\theta$

c) (3.5) $\cos(90° - \theta) = \sin\theta$

d) (3.4) $1 - \cos^2\theta = \sin^2\theta$

e) (7.1) $\tan\theta\cos\theta = \sin\theta$

18. Answer any two of the following five questions.

    a) (2.10) A wedge-shaped piece of pie taken from a pie with a $16''$ diameter has angle $50°$. Find the area of the piece of pie.

    b) (2.10) A wheel of radius 18 feet rotates at a speed of 4 revolutions per minute. What is the linear velocity of a point on the edge of the wheel?

    c) (4.5) Find the area of triangle ABC, if $a = 30$, $b = 18$ and $\angle C = 48°$.

    d) (4.1 or 4.3) A 16-foot ladder leans up against the side of the building. If the top of the ladder is 13.5 feet above the ground, what angle does the ladder make with the ground?

    e) (3.3) A boat leaves a harbor and sails 30 miles on a heading $40°$, measured east of north, then sails 20 miles due east. How far is it from the harbor?

Solutions

1. a) $\tan 45° = \boxed{1}$

   b) $\cos 60° = \boxed{\frac{1}{2}}$

   c) $\sin 90° = \boxed{1}$

   d) $\sec 30° = \frac{1}{\cos 30°} = \frac{1}{\sqrt{3}/2} = \boxed{\frac{2}{\sqrt{3}}}$

   e) $\cos 0° = \boxed{1}$

2. a) $\cot 240° = \boxed{\frac{1}{\sqrt{3}}}$ (quadrant III, reference angle $60°$)

b) $\cos(-150°) = \cos 150° = \boxed{-\dfrac{\sqrt{3}}{2}}$ (quadrant II, reference angle $30°$)

c) $\sin 315° = \boxed{-\dfrac{\sqrt{2}}{2}}$ (quadrant IV, reference angle $45°$)

d) $\csc 270° = \frac{1}{-1} = \boxed{-1}$ (the point $(0,-1)$ on the unit circle)

e) $\sec 450° = \frac{1}{\cos 90°} = \frac{1}{0}$ $\boxed{\text{DNE}}$.

3. a) $\sin \frac{11\pi}{6} = \boxed{-\dfrac{1}{2}}$ (quadrant IV, reference angle $30°$)

b) $\sin \frac{-\pi}{3} = \boxed{-\dfrac{\sqrt{3}}{2}}$ (quadrant IV, reference angle $60°$)

c) $\cot \frac{3\pi}{2} = \frac{\cos 270°}{\sin 270°} = \frac{0}{1} = \boxed{0}$ (the point $(0,-1)$ on the unit circle)

d) $\tan \frac{5\pi}{6} = \boxed{-\dfrac{1}{\sqrt{3}}}$ (quadrant II, reference angle $30°$)

e) $\tan \frac{11\pi}{3} = \boxed{-\sqrt{3}}$ (quadrant IV, reference angle $60°$)

4. a) $\sin^2 50° + \cos^2 50° = \boxed{1}$

b) $\cos^2 135° = \left(\frac{-\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \boxed{\dfrac{1}{2}}$.

c) Use the addition identity for sine:

$$\begin{aligned}\sin 75° = \sin(30° + 45°) &= \sin 30° \cos 45° + \cos 30° \sin 45° \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}.\end{aligned}$$

d) $\tan 4 \cdot 30° = \tan(4 \cdot 30°) = \tan 120° = \boxed{-\sqrt{3}}$.

e) $8 \sec \frac{\pi}{4} = \boxed{8\sqrt{2}}$.

5. Since $\cos\theta > 0$ and $\tan\theta < 0$, θ must be in Quadrant IV. Therefore all the trig functions of θ are negative except for cosine and secant. Next, draw a triangle with adjacent side 3 and hypotenuse 5. Solve for the opposite side using the Pythagorean Theorem to get $\sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$. Therefore

$$\boxed{\cos\theta = \frac{3}{5}; \sec\theta = \frac{5}{3}; \sin\theta = \frac{-4}{5}; \csc\theta = \frac{-5}{4}; \tan\theta = \frac{-4}{3}; \cot\theta = \frac{-3}{4}}.$$

6. a) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2(3)}{1-3^2} = \dfrac{-6}{8} = \boxed{-\dfrac{3}{4}}$.

b) $\cot\theta = \frac{1}{\tan\theta} = \boxed{\frac{1}{3}}$.

c) $\cot 2\theta = \frac{1}{\tan 2\theta} = \boxed{-\frac{4}{3}}$.

d) Since $\tan\theta > 0$, θ is in $\boxed{\text{Quadrant I or III}}$.

e) $\tan(\theta + 720°) = \tan\theta = \boxed{3}$.

f) $\tan(-\theta) = -\tan\theta = \boxed{-3}$.

7. The graph of $y = \tan x$ is Section 5.4 of my lecture notes.

8. a) This graph has period $\frac{2\pi}{1/2} = 4\pi$, so it is graph $\boxed{\text{F}}$.

   b) This graph has been stretched vertically by a factor of 2, so it is graph $\boxed{\text{A}}$.

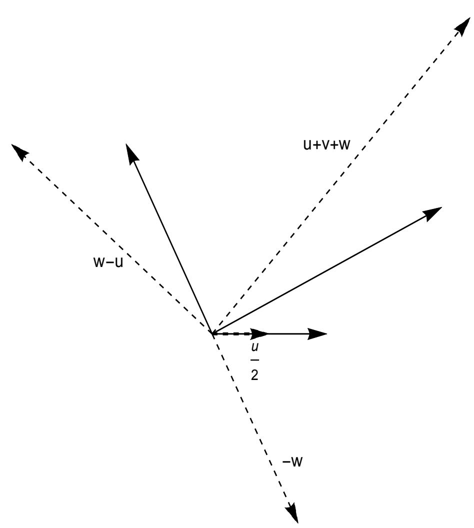
   c) This graph is shifted down 2 units, so it is graph $\boxed{\text{G}}$.

   d) This graph is shifted left 2 units, so it is graph $\boxed{\text{K}}$.

   e) This graph has period $\frac{2\pi}{2} = \pi$, so it is graph $\boxed{\text{E}}$.

   f) This graph has been squashed vertically to be half as tall, so it is graph $\boxed{\text{B}}$.

9.



10. a) $\csc 52° = \frac{1}{\sin 52°} = \boxed{1.26902}$.

   b) $\sin 25° + \sin 110° = .422618 + .939693 = \boxed{1.36231}$.

   c) $\sec(83° - 40°) = \sec 43° = \frac{1}{\cos 43°} = \boxed{1.36733}$.

   d) $\tan^2 117° = (-1.96261)^2 = \boxed{3.85184}$.

   e) $4\cot 286° = 4\left(\frac{1}{\tan 286°}\right) = 4\frac{1}{-3.48741} = 4(-.286745) = \boxed{-1.14698}$.

   f) $\sin 100° \cos 44° = (.984808)(.71934) = \boxed{.708411}$.

11. a) $\theta = \arcsin .28 = \boxed{16.26°}$ and $180° - 16.26° = \boxed{163.74°}$.

    b) $\cos\theta = 1.35$ has $\boxed{\text{no solution}}$ since $1.35 > 1$.

    c) $\cot\theta = -.55$ means $\tan\theta = \frac{1}{-.55} = -1.81818$ so $\theta = \arctan -1.81818 = \boxed{118.81°}$ and $\theta = 118.81° + 180° = \boxed{298.81°}$.

    d) $\csc\theta = 1$ means $\sin\theta = \frac{1}{1} = 1$ so $\theta = \arcsin 1 = \boxed{90°}$.

12. First, the adjacent side is $\sqrt{43.2^2 - 25.1^2} = 35.16$. Therefore the six trig functions are

$$\boxed{\begin{array}{cc} \sin\theta = \frac{25.1}{43.2} = .581 & \csc\theta = \frac{43.2}{25.1} = 1.721 \\ \tan\theta = \frac{25.1}{35.16} = .713 & \cot\theta = \frac{35.16}{25.1} = 1.4008 \\ \cos\theta = \frac{35.16}{43.2} = .814 & \sec\theta = \frac{43.2}{35.16} = 1.229 \end{array}}$$

13. a) $|\langle -5, -4\rangle| = \sqrt{5^2 + (-4)^2} = \sqrt{25+16} = \boxed{\sqrt{41}}$.

    b) $\theta = \arctan\frac{7}{-25} = \tan^{-2} -.28 = -15.64°$. Since θ must be in Quadrant II, add $180°$ to get $\theta = \boxed{164.36°}$.

    c) $5\mathbf{v} + 2\mathbf{w} = \langle 30, -20\rangle + \langle 4, 22\rangle = \boxed{\langle 34, 2\rangle}$.

    d) $\mathbf{v}\cdot\mathbf{w} = 6(2) + (-4)(11) = 12 - 44 = \boxed{-32}$.

    e) $\langle 17\cos 230°, 17\sin 230°\rangle = \boxed{\langle -10.927, -13.0228\rangle}$.

14. $\angle B = 180° - 90° - 35° = \boxed{55°}$.

    $\sin 35° = \frac{a}{18}$ so $a = 18\sin 35° = \boxed{10.32}$.

    $\cos 35° = \frac{b}{18}$ so $b = 18\cos 35° = \boxed{14.74}$.

15. Start with the Law of Cosines to find angle E:

$$\begin{aligned} e^2 &= f^2 + g^2 - 2fg\cos E \\ 47^2 &= 25^2 + 28^2 - 2(25)(28)\cos E \\ 2209 &= 625 + 784 - 1400\cos E \\ 800 &= -1400\cos E \\ \frac{800}{-1400} &= \cos E \\ \arccos\frac{8}{-14} &= E \\ \boxed{124.85°} &= E \end{aligned}$$

Then use the Law of Cosines (or the Law of Sines) to find angle G:

$$\begin{aligned}
g^2 &= e^2 + f^2 - 2ef\cos G \\
28^2 &= 25^2 + 47^2 - 2(25)(47)\cos G \\
784 &= 625 + 2209 - 2350\cos G \\
-2050 &= -2350\cos G \\
\frac{-2050}{-2350} &= \cos G \\
\arccos\frac{205}{235} &= G \\
\boxed{29.27^\circ} &= G
\end{aligned}$$

Last, angle F is $180^\circ - 124.85^\circ - 29.27^\circ = \boxed{25.88^\circ}$.

16. First, angle Q is $180^\circ - 26^\circ - 77^\circ = \boxed{77^\circ}$.

    Second, since angles Q and R are equal, sides q and r are equal so $\boxed{r = 6.43}$.

    Last, use the Law of Sines to find p:

$$\begin{aligned}
\frac{\sin P}{p} &= \frac{\sin R}{r} \\
\frac{\sin 26^\circ}{p} &= \frac{\sin 77^\circ}{6.43} \\
\frac{.438371}{p} &= \frac{.97437}{6.43} \\
.97437p &= (.438371)6.43 \\
p &= \frac{(.438371)6.43}{.97437} = \boxed{2.892}.
\end{aligned}$$

17. a) $\tan^2\theta - 1 = \sec^2\theta$ is $\boxed{\text{FALSE}}$ (the $\tan^2$ and $\sec^2$ are backwards from what they are in the Pythagorean identity)

    b) $\cos(-\theta) = -\cos\theta$ is $\boxed{\text{FALSE}}$ (the $-$ sign disappears)

    c) $\cos(90^\circ - \theta) = \sin\theta$ is $\boxed{\text{TRUE}}$ (cofunction identity)

    d) $1 - \cos^2\theta = \sin^2\theta$ is $\boxed{\text{TRUE}}$ (Pythagorean identity)

    e) $\tan\theta\cos\theta = \sin\theta$ is $\boxed{\text{TRUE}}$ (write $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$ and then cancel the $\cos\theta$ terms on the left-hand side to see this)

18. a) The radius of the circle is $r = \frac{1}{2}(16) = 8$ inches; the angle in radians is $50^\circ \cdot \frac{\pi}{180^\circ} = .872665$. The area of the sector is therefore $A = \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2(.872665) = \boxed{27.9253 \text{ sq in}}$.

    b) The angular velocity is $\omega = 4 \cdot 2pi = 8\pi = 25.1327$ rad/sec, so the linear velocity is $v = r\omega = 18(25.1327) = \boxed{452.389 \text{ feet per second}}$.

c) By the SAS area formula, $A = \frac{1}{2}ab\sin C = \frac{1}{2}(30)18\sin 48^\circ = \boxed{200.649 \text{ square units}}$.

d) The opposite side to the angle is 13.5 and the hypotenuse is 16, so the angle is $\arcsin\frac{13.5}{16} = \boxed{57.54^\circ}$.

e) After the first heading, the boat is at position $\langle 30\cos 50^\circ, 30\sin 50^\circ\rangle = \langle 19.28, 22.98\rangle$ (the angle is 50° because $90^\circ - 40^\circ = 50^\circ$). After moving a further 20 miles east, the boat is at $\langle 19.28 + 20, 22.98\rangle = \langle 39.28, 22.98\rangle$. The distance from the boat to the harbor is the norm of this vector, which is $\sqrt{39.28^2 + 22.98^2} = \boxed{45.50 \text{ miles}}$.