Fall 2016
Math 120 Exams

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Contents

0.1 General comments on these exams

These are the exams I gave in a Math 120 (trigonometry) course at Ferris State University in Fall 2016. Each exam has two sections: on the first section, calculators are not allowed and exact answers must be given, but on the second section, calculators are permitted and decimal approximations are acceptable. Each exam is followed by solutions (there may be minor errors in the solutions).

These exams correspond to the material in my lecture notes and the 10^{th} edition of the Lial textbook as follows:

Exam 1: sections 1-4 of my lecture notes / sections 1.1-1.2, 3.1-3.4 of the Lial text

Exam 2: sections 5-11 of my lecture notes / sections 1.3, 2.1-2.4, 3.3, 7.1-7.3 of the Lial text

Exam 3: sections 12-16 of my lecture notes / sections 1.1-2.4, 7.4-7.5 of the Lial text

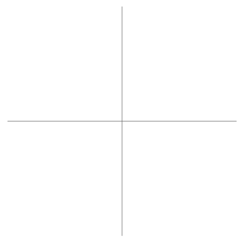
Final Exam: sections 1-21 of my lecture notes / chapters 1-5 and 7 of the Lial text

0.2 Fall 2016 Exam 1

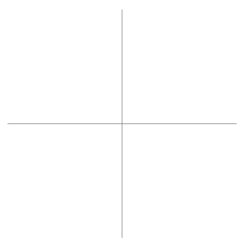
No calculator allowed on these problems

1. Convert the following angles from radians to degrees, and draw them in standard position on the provided axes:

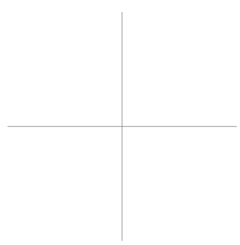
   a) $\frac{5\pi}{6}$



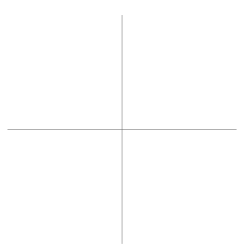
   b) $\frac{\pi}{2}$



   c) $\frac{8\pi}{3}$



   d) $\frac{-4\pi}{4}$



2. Let cat be the function defined by cat $x = x^2 + 1$. Compute each quantity:

   a) cat $2 + 3$

   b) cat $2 \cdot 3$

   c) cat(2) + 3

   d) 2 cat 3

Calculators allowed on the rest of the exam

3. Parts (a), (b), (c) and (d) of this question are not related to one another.

   a) Find an angle measuring between $0°$ and $360°$ which is coterminal with $4375°$.

   b) Convert $222°$ to radians (write your answer as a decimal, rounded to two or more decimal places).

   c) Find the coordinates of a point on the unit circle which is on the terminal side of a $540°$ angle, drawn in standard position.

   d) Draw a picture of a triangle which is isoceles, but not equilateral.

4. Parts (a), (b), and (c) of this question are not related to one another.

   a) Find the distance between the points $(2, 8)$ and $(5, -3)$ (write your answer either as an exact answer or as a decimal, rounded to two or more decimal places).

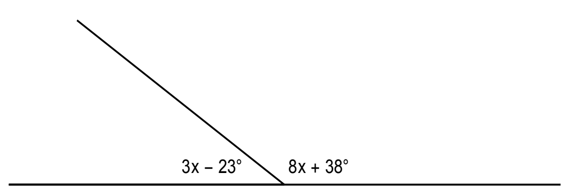
   b) An angle measures $18°$ more than its complement. What is the measure of the angle?

   c) The three angles of a triangle measure x, $x + 25°$ and $x - 40°$. Is this triangle an acute triangle, a right triangle, or an obtuse triangle?

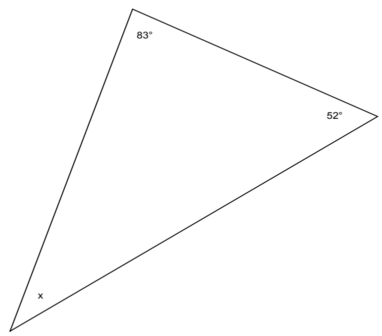
5. Parts (a) and (b) of this question are not related to one another.

   a) A Ferris wheel makes $.065$ revolutions in a minute. If the radius of the Ferris wheel is 120 feet, what is the linear velocity of someone sitting in a bucket on the edge of the Ferris wheel?

   b) Find the measure of each angle in this picture:

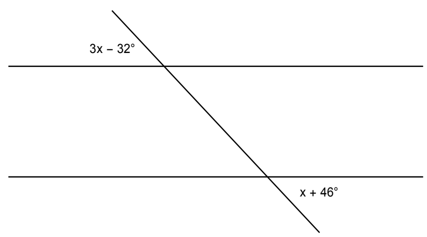


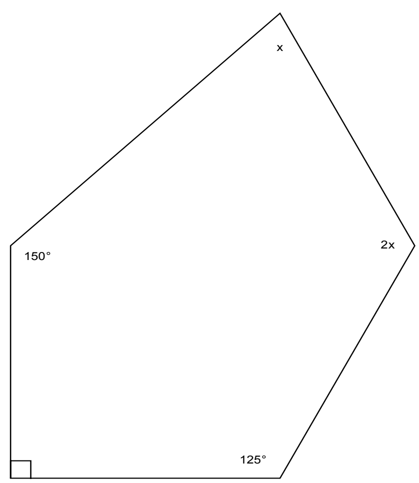
6. In each picture, find x:



a)

b) (in this picture, assume the horizontal lines are parallel)





c)

Exam 1 Solutions

1. a) $\frac{5\pi}{6} = 5 \cdot \frac{\pi}{6} = 5 \cdot 30° = 150°$. In standard position, this angle should point just north of the negative x-axis.

   b) $\frac{\pi}{2} = 90°$. In standard position, this angle points due north.

   c) $\frac{8\pi}{3} = 8 \cdot \frac{\pi}{3} = 8 \cdot 60° = 480°$. In standard position, this angle goes all the way around once, then halfway around again to end up on the negative x-axis.

   d) $\frac{-4\pi}{4} = -\pi = -180°$. In standard position, this angle is on the negative x-axis.

2. a) cat $2+3$ = cat$(2)+3 = (2^2+1)+3 = 5+3 = 8$.

   b) cat $2 \cdot 3 =$ cat $6 = 6^2+1 = 37$.

   c) cat$(2)+3 = (2^2+1)+3 = 5+3 = 8$.

   d) 2 cat $3 = 2(3^2+1) = 2(10) = 20$.

3. a) $4375 \div 360 = 12.15$ so the angle is $4375° - 12 \cdot 360° = 55°$.

   b) $222° \times \frac{\pi}{180°} = 3.87$ radians.

   c) $540° = 360° + 180°$ so the terminal side of this angle points on the negative x-axis. The point on the negative x-axis which is on the unit circle is $(-1, 0)$.

   d) The triangle should have two sides that are the same length, but not all three sides of the same length.

4. a) $D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(5-2)^2+(-3-8)^2} = \sqrt{3^2+(-11)^2} = \sqrt{9+121} = \sqrt{130} \approx 11.4$.

   b) Let x be the angle; we have $x = 18° + (90° - x)$. Solve for x to get $x = 54°$.

   c) The three angles sum to $180°$, so $x + (x+25°) + (x-40°) = 180°$. Solve for x to get $x = 65°$. This makes the three angles $x = 65°$, $x + 25° = 90°$ and $x - 40° = 25°$. Since one of the angles is $90°$, the triangle is a right triangle.

5. a) The angular velocity is $\omega = .065 \cdot 2\pi = .408$ radians per minute. The linear velocity is therefore $v = r\omega = 120(.408) \approx 49$ feet per minute.

   b) The angles are supplementary, so $(3x - 23°) + (8x + 38°) = 180°$. Solve for x to get $x = 15°$; the angles are therefore $3(15°) - 23° = 22°$ and $8(15°) + 38° = 158°$.

6. a) The angles sum to $180°$: $x + 52° + 83° = 180°$. Solve for x to get $x = 45°$.

   b) The angles are equal, so $3x - 32° = x + 48°$. Solve for x to get $x = 39°$.

c) This shape has five sides, so its angles must add to $180°(5-2) = 540°$. Therefore $x + 2x + 125° + 90° + 150° = 540°$. Solve for x to get $x = \frac{175}{3}° \approx 58.33°$.

0.3 Fall 2016 Exam 2

No calculator allowed on these problems

1. Throughout this problem, assume $\sin\theta = \frac{-2}{3}$.

   a) What is $\sin(-\theta)$?
   b) If $\cos\theta > 0$, find $\cos\theta$.
   c) If $\cos\theta > 0$, what quadrant is θ in?
   d) What is $\sin(\theta + 360°)$?
   e) *(Bonus)* What is $\sin(\theta + 180°)$?

2. Find the exact value of each quantity:

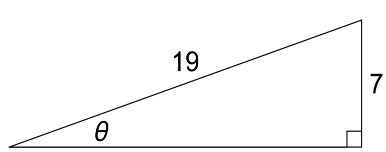
   a) $\sin 45°$
   b) $\cos 120°$
   c) $\sin 180°$
   d) $\cos 300°$
   e) $\sin 570°$
   f) $\cos(-30°)$

3. Find the exact value of each quantity:

   a) $\sin\frac{5\pi}{6}$
   b) $\sin 8\pi$
   c) $\sin\frac{-3\pi}{4}$
   d) $\cos\frac{10\pi}{3}$
   e) $\sin\frac{\pi}{2}$
   f) $\cos\frac{-7\pi}{2}$

Calculator allowed on the rest of the exam

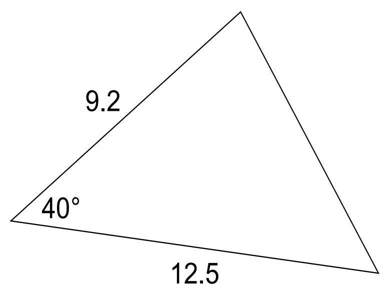
4. Find $\cos\theta$, if θ is as in the following picture:

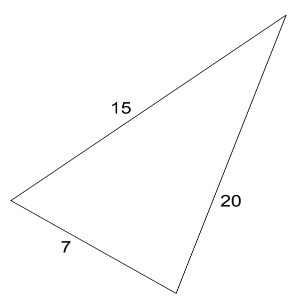


5. Suppose $\sin\theta = .55$. Find all possible values of $\cos\theta$.

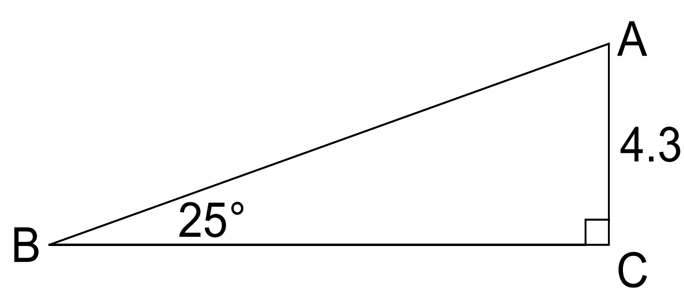
6. a) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sin\theta = .3$.
   b) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sin\theta = 1.3$.
   c) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = .75$.
   d) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = 1$.

7. Find the area of each indicated triangle:

a) 

b) 

8. Solve triangle ABC, given the information in the picture below:



9. Solve triangle GHI if $g = 17$, $h = 25$ and $\angle I = 62°$.

10. Solve triangle ABC if $a = 17$, $b = 10$ and $\angle B = 40°$.

11. A person wants to measure the height of a flagpole. She walks 50 feet away from the base of the flagpole, turns around and notices that her angle of elevation to the top of the pole is $48°$. If her eyes are 5 feet above the ground, how tall is the flagpole?

Exam 2 Solutions

1. a) $\sin(-\theta) = -\sin\theta = \frac{2}{3}$.

   b) Start with the Pythagorean identity:

$$\cos^2\theta + \sin^2\theta = 1$$
$$\cos^2\theta + \left(\frac{-2}{3}\right)^2 = 1$$
$$\cos^2\theta + \frac{4}{9} = 1$$
$$\cos^2\theta = \frac{5}{9} \Rightarrow \cos\theta = \pm\sqrt{\frac{5}{9}}.$$

   Since we are told $\cos\theta > 0$, $\cos\theta = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$.

   c) Since $\sin\theta < 0$ and $\cos\theta > 0$, θ must be in Quadrant IV.

   d) $\sin(\theta + 360°) = \sin\theta = \frac{-2}{3}$.

   e) $\theta + 180°$ is in Quadrant II, but has the same reference angle as θ, so $\sin(\theta + 180°) = \frac{2}{3}$.

2. a) $\sin 45° = \frac{\sqrt{2}}{2}$.

   b) $\cos 120° = \frac{-1}{2}$ (Quadrant II; reference angle $60°$)

   c) $\sin 180° = 0$ (point on unit circle is $(-1, 0)$)

   d) $\cos 300° = \frac{1}{2}$ (Quadrant IV; reference angle $60°$)

   e) $\sin 570° = \sin 210° = \frac{-1}{2}$ (Quadrant II; reference angle $30°$)

   f) $\cos(-30°) = \cos 30° = \frac{\sqrt{3}}{2}$

3. a) $\sin\frac{5\pi}{6} = \sin 150° = \frac{1}{2}$ (Quadrant II; reference angle $30°$)

   b) $\sin 8\pi = \sin 0 = 0$

   c) $\sin\frac{-3\pi}{4} = \sin(-135°) = \frac{-\sqrt{2}}{2}$ (Quadrant III; reference angle $45°$)

   d) $\cos\frac{10\pi}{3} = \cos\frac{4\pi}{3} = \cos 240° = \frac{-1}{2}$ (Quadrant III; reference angle $60°$)

   e) $\sin\frac{\pi}{2} = \sin 90° = 1$ (point on unit circle is $(0, 1)$)

   f) $\cos\frac{-7\pi}{2} = \cos\frac{7\pi}{2} = \cos 270° = 0$ (point on unit circle is $(0, -1)$)

4. First, use the Pythagorean Theorem to find the adjacent side, which I'll call a:

$$a^2 + 7^2 = 19^2 \Rightarrow a = \sqrt{19^2 - 7^2} = \sqrt{312} \approx 17.66.$$

   Then $\cos\theta = \frac{\text{adjacent}}{\text{adjacent}} \approx \frac{17.66}{19} = .9297$.

5. Start with the Pythagorean identity:

$$\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1 \\
\cos^2\theta + (.55)^2 &= 1 \\
\cos^2\theta + .3025 &= 1 \\
\cos^2\theta &= .6975 \\
\cos\theta &= \pm\sqrt{.6975} \approx \pm .8352.
\end{aligned}$$

6. a) From a calculator, $\theta = \sin^{-1}(.3) \approx 17.46°$. A second angle which solves the equation is $180° - \theta \approx 162.54°$.

   b) There are no such angles (because $1.3 > 1$).

   c) From a calculator, $\theta = \cos^{-1}(.75) \approx 41.4°$. A second angle which solves the equation is $360° - \theta = 318.6°$.

   d) The only such angle is $\theta = 0$ (if you also included $\theta = 360°$, that's okay, but $360°$ is really the same angle as $0°$).

7. a) By the SAS Area Formula, $A = \frac{1}{2}ab\sin C = \frac{1}{2}(9.2)(12.5)\sin 40° \approx 36.96$ sq units.

   b) Use Heron's Formula (so first, find the semiperimeter):

$$s = \frac{a+b+c}{2} = \frac{7+15+20}{2} = 21.$$

   Then the area is

$$\begin{aligned}
A = \sqrt{s(s-a)(s-b)(s-c)} &= \sqrt{21(21-7)(21-15)(21-20)} \\
&= \sqrt{21(14)(6)(1)} \\
&= \sqrt{1764} \\
&= 42 \text{ sq units.}
\end{aligned}$$

8. First, the third angle is $A = 90° - 25° = 65°$.

   Second, find the hypotenuse c using a trig function (either sine or cosine):

$$\begin{aligned}
\sin 25° &= \frac{4.3}{c} \\
.4226 &= \frac{4.3}{c} \\
.4226c &= 4.3 \\
c &= \frac{4.3}{.4226} = 10.175.
\end{aligned}$$

Last, find the remaining side using the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + (4.3)^2 &= (10.175)^2 \\ a &= \sqrt{(10.175)^2 - (4.3)^2} = \sqrt{85.03} \approx 9.22. \end{aligned}$$

9. The given information is SAS, so start with the Law of Cosines to find side i:

$$\begin{aligned} i^2 &= g^2 + h^2 - 2gh\cos I \\ i^2 &= 17^2 + 25^2 - 2(17)(25)\cos 62^\circ \\ i^2 &= 289 + 625 - 850(.4695) \\ i^2 &= 514.949 \\ i &= \sqrt{514.949} \approx 22.69. \end{aligned}$$

Now use the Law of Sines (or the Law of Cosines) again to find $\angle G$ (you could find $\angle H$ first instead):

$$\begin{aligned} \frac{\sin G}{g} &= \frac{\sin I}{i} \\ \frac{\sin G}{17} &= \frac{\sin 62^\circ}{22.69} \\ \frac{\sin G}{17} &= \frac{.8829}{22.69} \\ \sin G &= \frac{17(.8829)}{22.69} = .6615 \\ G &= \sin^{-1}(.6615) = 41.4^\circ. \end{aligned}$$

Last, find $\angle H$: $H = 180^\circ - 62^\circ - 41.4^\circ = 76.6^\circ$.

10. The given information is SSA, which is the ambiguous case of the Law of Sines:

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin A}{17} &= \frac{\sin 40^\circ}{10} \\ 10\sin A &= 17(.6427) \\ \sin A &= \frac{17(.6427)}{10} = 1.092. \end{aligned}$$

This equation has no solution (because $1.092 > 1$), so there is no such triangle.

11. Draw a right triangle where the opposite side is the flagpole. We are given that the adjacent side is 50 feet and the angle to the horizontal is $48°$. That means, labelling the adjacent side as a, that

$$\begin{aligned} \tan 48° &= \frac{\text{opposite}}{\text{adjacent}} \\ 1.11 &= \frac{a}{50} \\ 1.11(50) &= a \\ 55.5 &= a. \end{aligned}$$

Adding the 5 feet to account for the height of the viewer's eyes, we see that the height of the flagpole is $55.5 + 5 = 60.5$ feet.

0.4 Fall 2016 Exam 3

No calculators allowed on these problems

1. Throughout this problem, assume $\sec\theta = 4$.
   a) What is $\sec(-\theta)$?
   b) What is $\cos\theta$?
   c) What is $\sec(\theta - 360°)$?
   d) Which two quadrants might θ be in?
   e) If $\tan\theta < 0$, find $\cot\theta$.

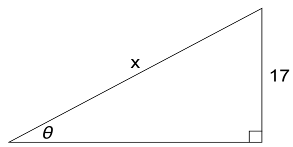
2. Find the exact value of each quantity:
   a) $\sec 45°$
   b) $\tan 150°$
   c) $\cot 180°$
   d) $\csc(-120°)$
   e) $\sec 60°$

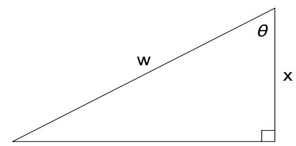
3. Find the exact value of each quantity:
   a) $\tan 5\pi$
   b) $\csc \frac{\pi}{6}$
   c) $\tan \frac{3\pi}{4}$
   d) $\cos \frac{\pi}{2}$
   e) $\sin \frac{2\pi}{3}$

4. Find the exact value of each quantity:
   a) $\sin^2 125° + \cos^2 125°$
   b) $\cot(45° + 45°)$
   c) $\tan^2 \frac{5\pi}{6}$
   d) $3\sin \frac{\pi}{2}$
   e) $\sin 3 \cdot \frac{\pi}{2}$

Calculators allowed on the rest of this exam

5. In each diagram below, write an equation for x in terms of the other given quantities in the picture.

a) 

b) 

6. Suppose that $\sin\theta = .614$ and that $\tan\theta > 0$. Find the values of all six trig functions of θ.

7. Use a calculator to compute these quantities:

   a) $\csc 75°$
   b) $\cot 40° + \cot 70°$
   c) $\tan^2 125°$
   d) $3 \sec 40°$

8. a) Find all angles θ between $0°$ and $360°$ such that $\tan\theta = .72$.
   b) Find all angles θ between $0°$ and $360°$ such that $\sec\theta = 2.8$.
   c) Find all angles θ between $0°$ and $360°$ such that $\csc\theta = .35$.

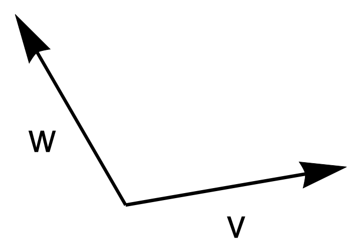
9. Throughout this question, assume that $\mathbf{u} = \langle -5, 3\rangle$, $\mathbf{v} = \langle -1, 8\rangle$ and that $\mathbf{w}$ is a vector whose magnitude is 7 and whose direction angle is $58°$.

   a) Compute $2\mathbf{u} + \mathbf{v}$.
   b) Compute $\mathbf{u} \cdot \mathbf{v}$.
   c) Compute the angle between $\mathbf{u}$ and $\mathbf{v}$.
   d) Compute the direction angle of $\mathbf{v}$.
   e) Compute $|\mathbf{u} + \mathbf{v}|$.
   f) Write $\mathbf{w}$ in component form.

10. Let $\mathbf{v}$ and $\mathbf{w}$ be the vectors indicated in the picture below.

    a) Sketch the vector $2\mathbf{v} + \mathbf{w}$ on the picture; label that vector "$2\mathbf{v} + \mathbf{w}$".

    b) Sketch the vector $\mathbf{w} - \mathbf{v}$ on the picture; label that vector "$\mathbf{w} - \mathbf{v}$".



11. A bird leaves its nest and flies 20 miles on a heading $37°$, measured east of north. The bird then flies 9 miles on a bearing $72°$, measured west of north.

    a) How far is the bird from its nest?

    b) If the bird wanted to return to its nest, would it be more accurate to say it should fly southeast, or southwest? Explain your answer.

Exam 3 Solutions

1. a) $\sec(-\theta) = \sec\theta = 4$.

   b) $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{4}$.

   c) $\sec(\theta - 360°) = \sec\theta = 4$.

   d) θ must be in Quadrant I or Quadrant IV.

   e) Since $\tan\theta < 0$, we now know θ is in Quadrant IV, so $\cot\theta < 0$. Sketch a triangle with hypotenuse 4 and adjacent side of length 1. From the Pythagorean Theorem, the opposite side is $\sqrt{4^2 - 1^2} = \sqrt{15}$, so $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\sqrt{15}}$.

2. a) $\sec 45° = \sqrt{2}$.

   b) $\tan 150° = \frac{-1}{\sqrt{3}}$ (reference angle $30°$, Quadrant II).

   c) $\cot 180°$ DNE.

   d) $\csc(-120°) = \frac{-2}{\sqrt{3}}$ (reference angle $60°$, Quadrant III).

   e) $\sec 60° = 2$.

3. a) $\tan 5\pi = 0$

   b) $\csc\frac{\pi}{6} = 2$

   c) $\tan\frac{3\pi}{4} = -1$ (reference angle $45°$, Quadrant II)

   d) $\cos\frac{\pi}{2} = 0$

   e) $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ (reference angle $60°$, Quadrant II)

4. a) $\sin^2 125° + \cos^2 125° = 1$ (by a Pythagorean identity)

   b) $\cot(45° + 45°) = \cot 90° = 0$

   c) $\tan^2\frac{5\pi}{6} = \left(\frac{-1}{\sqrt{3}}\right)^2 = \frac{1}{3}$

   d) $3\sin\frac{\pi}{2} = 3(1) = 3$

   e) $\sin 3 \cdot \frac{\pi}{2} = -1$

5. a) $\frac{x}{17} = \frac{\text{hyp}}{\text{opp}} = \csc\theta$ so $x = 17\csc\theta$.

   b) $\frac{x}{w} = \frac{\text{adj}}{\text{hyp}} = \cos\theta$ so $x = w\cos\theta$.

6. Since $\sin\theta > 0$ and $\tan\theta > 0$, θ is in Quadrant I, so all the trig functions are positive. We are given $\sin\theta = .614$, so $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{.614} = 1.629$. From the identity $\sin^2\theta + \cos^2\theta = 1$, we can solve for $\cos\theta$ to get $\cos\theta = .789$. Then $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{.789} = 1.267$. Last, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{.614}{.789} = .777$ and $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{.789}{.614} = 1.285$.

7. a) $\csc 75° = 1.03528$.
   b) $\cot 40° + \cot 70° = 1.19175 + .36397 = 1.55572$.
   c) $\tan^2 125° = (-1.42815)^2 = 2.03961$.
   d) $3 \sec 40° = 3(1.30541) = 3.91622$.

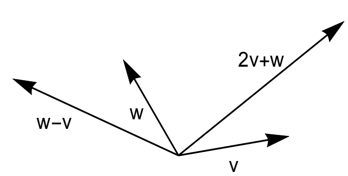
8. a) $\theta = \tan -1.72 = 35.7539°$; a second angle is $\theta + 180° = 215.7539°$.
   b) If $\sec\theta = 2.8$, then $\cos\theta = \frac{1}{2.8}$ so $\theta = \cos^{-1}(\frac{1}{2.8}) = 69.07°$. A second angle is $360° - \theta = 290.93°$.
   c) There are no such angles, because $\csc\theta$ cannot be between -1 and 1.

9. Throughout this question, assume that $\mathbf{u} = \langle -5, 3\rangle$, $\mathbf{v} = \langle -1, 8\rangle$ and that $\mathbf{w}$ is a vector whose magnitude is 7 and whose direction angle is $58°$.
   a) $2\mathbf{u} + \mathbf{v} = \langle -10, 6\rangle + \langle -1, 8\rangle = \langle -11, 14\rangle$.
   b) $\mathbf{u} \cdot \mathbf{v} = (-5)(-1) + 3(8) = 5 + 24 = 29$.
   c) First, $|\mathbf{u}| = \sqrt{(-5)^2 + 3^2} = \sqrt{34} \approx 5.83$. Second, $|\mathbf{v}| = \sqrt{(-1)^2 + 8^2} = \sqrt{65} = 8.06$. Therefore,

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= |\mathbf{u}|\,|\mathbf{v}|\,\cos\theta \\ 29 &= (5.83)(8.06)\cos\theta \\ .6171 &= \cos\theta \\ 51.89° &= \theta. \end{aligned}$$

   d) $\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{8}{-1} = \tan^{-1}(-8) = -82.875°$. But this is in the wrong quadrant ($\mathbf{v}$ is in Quadrant II), so we need to add $180°$ to get $97.125°$.
   e) $|\mathbf{u} + \mathbf{v}| = |\langle -6, 11\rangle| = \sqrt{(-6)^2 + 11^2} = \sqrt{36 + 121} = \sqrt{157} \approx 12.53$.
   f) $\mathbf{w} = \langle |\mathbf{w}|\cos\theta, |\mathbf{w}|\sin\theta\rangle = \langle 7\cos 58°, 7\sin 58°\rangle = \langle 3.709, 5.936\rangle$.

10.



11. a) Let $\mathbf{v}$ be the first part of the bird's journey; we have

$$\mathbf{v} = \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta\rangle = \langle 20\cos 53°, 20\sin 53°\rangle = \langle 12.03, 15.97\rangle\,.$$

Let $\mathbf{w}$ be the second part of the bird's journey; we have

$$\mathbf{w} = \langle |\mathbf{w}|\cos\theta, |\mathbf{w}|\sin\theta\rangle = \langle 9\cos 162°, 9\sin 162°\rangle = \langle -8.559, 2.781\rangle\,.$$

Therefore the bird's position is $\mathbf{v}+\mathbf{w} = \langle 3.47, 18.75\rangle$. The distance from the bird to the nest is

$$|\mathbf{v}+\mathbf{w}| = |\,\langle 3.47, 18.75\rangle\,| = \sqrt{(3.47)^2 + (18.75)^2} = 19.06 \text{ mi.}$$

b) The first component of the bird's position is positive, so it has to fly southwest (although just barely west of south) to get back to its nest.

0.5 Fall 2016 Final Exam

No calculators allowed on these questions

1. Find the exact value of each quantity:

   a) $\tan 45°$
   b) $\cos 60°$
   c) $\sin 90°$
   d) $\sec 30°$
   e) $\cos 0°$

2. Find the exact value of each quantity:

   a) $\cot 240°$
   b) $\cos(-150°)$
   c) $\sin 315°$
   d) $\csc 270°$
   e) $\sec 450°$

3. Find the exact value of each quantity:

   a) $\sin \frac{11\pi}{6}$
   b) $\sin \frac{-\pi}{3}$
   c) $\cot \frac{3\pi}{2}$
   d) $\tan \frac{5\pi}{6}$
   e) $\tan \frac{11\pi}{3}$

4. Find the exact value of each quantity:

   a) $\sin^2 50° + \cos^2 50°$
   b) $\cos^2 135°$
   c) $\sin 75°$
   d) $\tan 4 \cdot 30°$
   e) $8 \sec \frac{\pi}{4}$

5. Suppose $\cos\theta = \frac{3}{5}$ and $\tan\theta < 0$. Find the exact values of all six trig functions of θ.

6. Suppose $\tan\theta = 3$.

   a) Find $\tan 2\theta$.
   b) Find $\cot\theta$.
   c) Find $\cot 2\theta$.
   d) What two quadrants might θ be in?
   e) Find $\tan(\theta + 720°)$.
   f) Find $\tan(-\theta)$.

7. Sketch the graph of $y = \tan x$.

8. a) Which of graphs A-K on the attached page (see page 22 of this pdf file) is the graph of $y = \sin\frac{x}{2}$?
   b) Which of graphs A-K is the graph of $y = 2\sin x$?
   c) Which of graphs A-K is the graph of $y = \sin x - 2$?
   d) Which of graphs A-K is the graph of $y = \sin(x + 2)$?

e) Which of graphs A-K is the graph of $y = \sin 2x$?

f) Which of graphs A-K is the graph of $y = \frac{\sin x}{2}$?

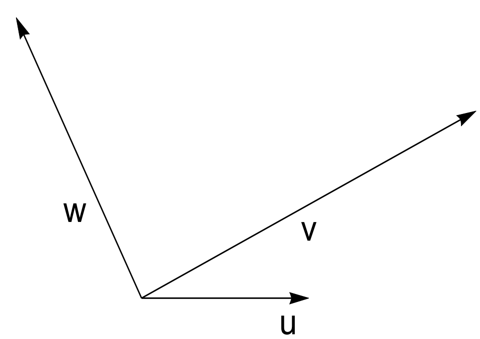
9. Vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are shown below. Sketch the following vectors on the same picture, being sure to label which vector is which:

   a) $\mathbf{w} - \mathbf{u}$

   b) $-\mathbf{w}$

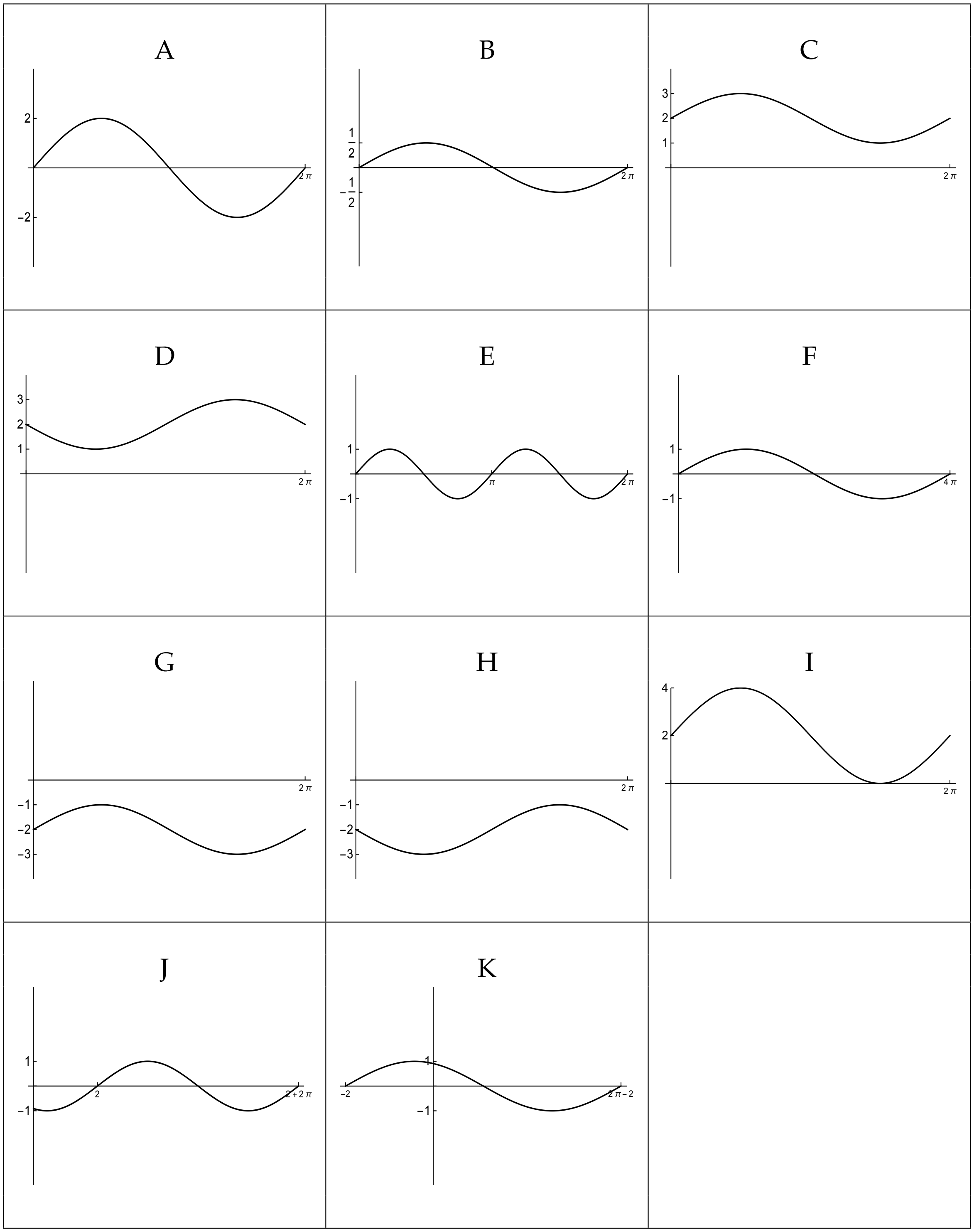
   c) $\frac{1}{2}\mathbf{u}$

   d) $\mathbf{u} + \mathbf{v} + \mathbf{w}$



Graphs for Problem 8

Use these graphs to answer the questions in Problem 8:



Calculators allowed on the rest of the exam

10. Evaluate the following expressions using a calculator. Your answers can (and should) be written as decimals.

    a) $\csc 52°$
    b) $\sin 25° + \sin 110°$
    c) $\sec(83° - 40°)$
    d) $\tan^2 117°$
    e) $4 \cot 286°$
    f) $\sin 100° \cos 44°$

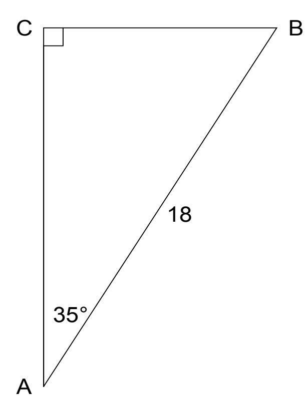
11. For each equation, find (decimal approximations of) all angles θ between $0°$ and $360°$ satisfying the following equations. If there are no such angles, say so.

    a) $\sin \theta = .28$
    b) $\cos \theta = 1.35$
    c) $\cot \theta = -.55$
    d) $\csc \theta = 1$

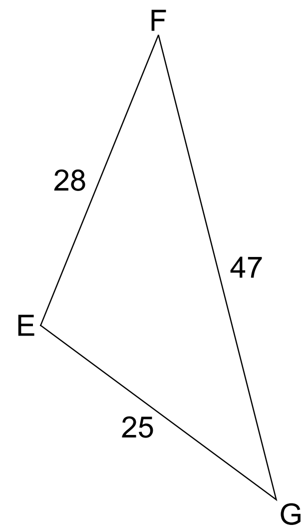
12. Suppose that θ is an angle in a right triangle whose hypotenuse has length 43.2 and whose side opposite θ has length 25.1. Find the six trig functions of angle θ.

13. a) Compute the norm of the vector $\langle -5, -4 \rangle$.
    b) Compute the direction angle of the vector $\langle -25, 7 \rangle$.
    c) Suppose $\mathbf{v} = \langle 6, -4 \rangle$ and $\mathbf{w} = \langle 2, 11 \rangle$. Compute $5\mathbf{v} + 2\mathbf{w}$.
    d) Compute $\mathbf{v} \cdot \mathbf{w}$, where $\mathbf{v}$ and $\mathbf{w}$ are as in part (c).
    e) Find the components of a vector which has norm 17 and direction angle $230°$.

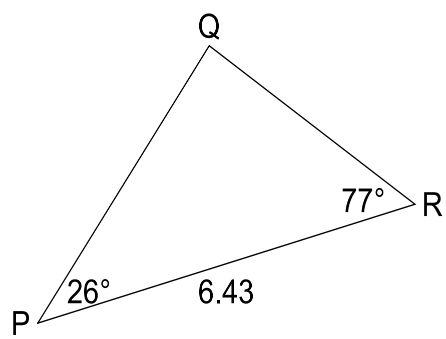
14. Solve triangle ABC, which is pictured below:



15. Solve triangle EFG, which is pictured below:



16. Solve triangle PQR, which is pictured below:



17. Classify the following statements as true or false:

    a) $\tan^2\theta - 1 = \sec^2\theta$
    b) $\cos(-\theta) = -\cos\theta$
    c) $\cos(90^\circ - \theta) = \sin\theta$
    d) $1 - \cos^2\theta = \sin^2\theta$
    e) $\tan\theta\cos\theta = \sin\theta$

18. Answer any two of the following five questions.

    a) A wedge-shaped piece of pie taken from a pie with a $16''$ diameter has angle 50°. Find the area of the piece of pie.

b) A wheel of radius 18 feet rotates at a speed of 4 revolutions per minute. What is the linear velocity of a point on the edge of the wheel?

c) Find the area of triangle ABC, if $a = 30$, $b = 18$ and $\angle C = 48^\circ$.

d) A 16-foot ladder leans up against the side of the building. If the top of the ladder is 13.5 feet above the ground, what angle does the ladder make with the ground?

e) A boat leaves a harbor and sails 30 miles on a heading 40°, measured east of north, then sails 20 miles due east. How far is it from the harbor?

Final Exam Solutions

1. a) $\tan 45^\circ = 1$
   b) $\cos 60^\circ = \frac{1}{2}$
   c) $\sin 90^\circ = 1$
   d) $\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$
   e) $\cos 0^\circ = 1$

2. a) $\cot 240^\circ = \frac{1}{\sqrt{3}}$ (quadrant III, reference angle 60°)
   b) $\cos(-150^\circ) = \cos 150^\circ = \frac{-\sqrt{3}}{2}$ (quadrant II, reference angle 30°)
   c) $\sin 315^\circ = \frac{-\sqrt{2}}{2}$ (quadrant IV, reference angle 45°)
   d) $\csc 270^\circ = \frac{1}{-1} = -1$ (the point $(0, -1)$ on the unit circle)
   e) $\sec 450^\circ = \frac{1}{\cos 90^\circ} = \frac{1}{0}$ DNE.

3. a) $\sin \frac{11\pi}{6} = \frac{-1}{2}$ (quadrant IV, reference angle 30°)
   b) $\sin \frac{-\pi}{3} = \frac{-\sqrt{3}}{2}$ (quadrant IV, reference angle 60°)
   c) $\cot \frac{3\pi}{2} = \frac{\cos 270^\circ}{\sin 270^\circ} = \frac{0}{1} = 0$ (the point $(0, -1)$ on the unit circle)
   d) $\tan \frac{5\pi}{6} = \frac{-1}{\sqrt{3}}$ (quadrant II, reference angle 30°)
   e) $\tan \frac{11\pi}{3} = -\sqrt{3}$ (quadrant IV, reference angle 60°)

4. a) $\sin^2 50^\circ + \cos^2 50^\circ = 1$
   b) $\cos^2 135^\circ = \left(\frac{-\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$.
   c) Use the addition identity for sine:

$$\begin{aligned}\sin 75^\circ = \sin(30^\circ + 45^\circ) &= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4}.\end{aligned}$$

   d) $\tan 4 \cdot 30^\circ = \tan(4 \cdot 30^\circ) = \tan 120^\circ = -\sqrt{3}$.
   e) $8 \sec \frac{\pi}{4} = 8\sqrt{2}$.

5. Since $\cos\theta > 0$ and $\tan\theta < 0$, θ must be in Quadrant IV. Therefore all the trig functions of θ are negative except for cosine and secant. Next, draw a triangle with adjacent side 3 and hypotenuse 5. Solve for the opposite side using the Pythagorean Theorem to get $\sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$. Therefore

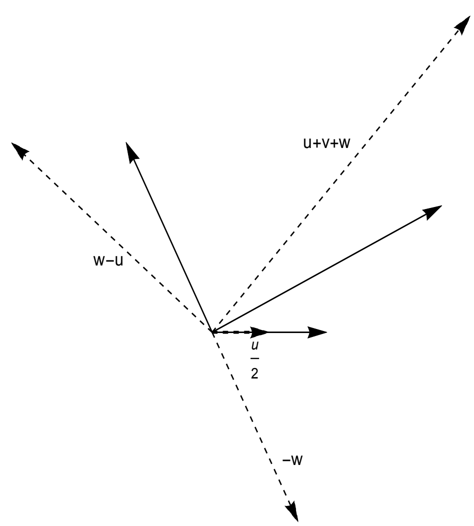
$$\cos\theta = \frac{3}{5}; \sec\theta = \frac{5}{3}; \sin\theta = \frac{-4}{5}; \csc\theta = \frac{-5}{4}; \tan\theta = \frac{-4}{3}; \cot\theta = \frac{-3}{4}.$$

6. a) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(3)}{1-3^2} = \frac{-6}{8} = \frac{-3}{4}$.
   b) $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{3}$.
   c) $\cot 2\theta = \frac{1}{\tan 2\theta} = \frac{-4}{3}$.
   d) Since $\tan\theta > 0$, θ is in Quadrant I or III.
   e) $\tan(\theta + 720°) = \tan\theta = 3$.
   f) $\tan(-\theta) = -\tan\theta = -3$.

7. The graph of $y = \tan x$ is on page 123 of my lecture notes.

8. a) This graph has period $\frac{2\pi}{1/2} = 4\pi$, so it is graph F.
   b) This graph has been stretched vertically by a factor of 2, so it is graph A.
   c) This graph is shifted down 2 units, so it is graph G.
   d) This graph is shifted left 2 units, so it is graph K.
   e) This graph has period $\frac{2\pi}{2} = \pi$, so it is graph E.
   f) This graph has been squashed vertically to be half as tall, so it is graph B.

9.



10. a) $\csc 52° = \frac{1}{\sin 52°} = 1.26902$.
   b) $\sin 25° + \sin 110° = .422618 + .939693 = 1.36231$.
   c) $\sec(83° - 40°) = \sec 43° = \frac{1}{\cos 43°} = 1.36733$.
   d) $\tan^2 117° = (-1.96261)^2 = 3.85184$.
   e) $4\cot 286° = 4\left(\frac{1}{\tan 286°}\right) = 4\frac{1}{-3.48741} = 4(-.286745) = -1.14698$.
   f) $\sin 100° \cos 44° = (.984808)(.71934) = .708411$.

11. a) $\theta = \sin^{-1} .28 = 16.26^\circ$ and $180^\circ - 16.26^\circ = 163.74^\circ$.
    b) $\cos\theta = 1.35$ has no solution since $1.35 > 1$.
    c) $\cot\theta = -.55$ means $\tan\theta = \frac{1}{-.55} = -1.81818$ so $\theta = \tan^{-1}(-1.81818) = 118.81^\circ$ and $\theta = 118.81^\circ + 180^\circ = 298.81^\circ$.
    d) $\csc\theta = 1$ means $\sin\theta = \frac{1}{1} = 1$ so $\theta = \sin^{-1} 1 = 90^\circ$.

12. First, the adjacent side is $\sqrt{43.2^2 - 25.1^2} = 35.16$. Therefore the six trig functions are

$$\sin\theta = \tfrac{25.1}{43.2} = .581 \qquad \csc\theta = \tfrac{43.2}{25.1} = 1.721$$

$$\tan\theta = \tfrac{25.1}{35.16} = .713 \quad \cot\theta = \tfrac{35.16}{25.1} = 1.4008$$

$$\cos\theta = \tfrac{35.16}{43.2} = .814 \quad \sec\theta = \tfrac{43.2}{35.16} = 1.229$$

13. a) $|\langle -5, -4\rangle| = \sqrt{5^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$.
    b) $\theta = \tan^{-1} \frac{7}{-25} = \tan^{-2} -.28 = -15.64^\circ$. Since θ must be in Quadrant II, add 180° to get $\theta = 164.36^\circ$.
    c) $5\mathbf{v} + 2\mathbf{w} = \langle 30, -20\rangle + \langle 4, 22\rangle = \langle 34, 2\rangle$.
    d) Compute $\mathbf{v}\cdot\mathbf{w} = 6(2) + (-4)(11) = 12 - 44 = -32$.
    e) $\langle 17\cos 230^\circ, 17\sin 230^\circ\rangle = \langle -10.927, -13.0228\rangle$.

14. $\angle B = 180^\circ - 90^\circ - 35^\circ = 55^\circ$.
    $\sin 35^\circ = \frac{a}{18}$ so $a = 18\sin 35^\circ = 10.32$.
    $\cos 35^\circ = \frac{b}{18}$ so $b = 18\cos 35^\circ = 14.74$.

15. Start with the Law of Cosines to find angle E:

$$\begin{aligned}
e^2 &= f^2 + g^2 - 2fg\cos E \\
47^2 &= 25^2 + 28^2 - 2(25)(28)\cos E \\
2209 &= 625 + 784 - 1400\cos E \\
800 &= -1400\cos E \\
\frac{800}{-1400} &= \cos E \\
\cos^{-1}\frac{8}{-14} &= E \\
124.85^\circ &= E
\end{aligned}$$

Then use the Law of Cosines (or the Law of Sines) to find angle G:

$$\begin{aligned} g^2 &= e^2 + f^2 - 2ef\cos G \\ 28^2 &= 25^2 + 47^2 - 2(25)(47)\cos G \\ 784 &= 625 + 2209 - 2350\cos G \\ -2050 &= -2350\cos G \\ \frac{-2050}{-2350} &= \cos G \\ \cos^{-1}\frac{205}{235} &= G \\ 29.27^\circ &= G \end{aligned}$$

Last, angle F is $180^\circ - 124.85^\circ - 29.27^\circ = 25.88^\circ$.

16. First, angle Q is $180^\circ - 26^\circ - 77^\circ = 77^\circ$.

    Second, since angles Q and R are equal, sides q and r are equal so $r = 6.43$.

    Last, use the Law of Sines to find p:

$$\begin{aligned} \frac{\sin P}{p} &= \frac{\sin R}{r} \\ \frac{\sin 26^\circ}{p} &= \frac{\sin 77^\circ}{6.43} \\ \frac{.438371}{p} &= \frac{.97437}{6.43} \\ .97437p &= (.438371)6.43 \\ p &= \frac{(.438371)6.43}{.97437} = 2.892. \end{aligned}$$

17. a) $\tan^2\theta - 1 = \sec^2\theta$ is FALSE (the $\tan^2$ and $\sec^2$ are backwards from what they are in the Pythagorean identity)
    b) $\cos(-\theta) = -\cos\theta$ is FALSE (the $-$ sign disappears)
    c) $\cos(90^\circ - \theta) = \sin\theta$ is TRUE (cofunction identity)
    d) $1 - \cos^2\theta = \sin^2\theta$ is TRUE (Pythagorean identity)
    e) $\tan\theta\cos\theta = \sin\theta$ is TRUE (write $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$ and then cancel the $\cos\theta$ terms on the left-hand side to see this)

18. a) The radius of the circle is $r = \frac{1}{2}(16) = 8$ inches; the angle in radians is $50^\circ \cdot \frac{\pi}{180^\circ} = .872665$. The area of the sector is therefore $A = \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2(.872665) = 27.9253$ square inches.
    b) The angular velocity is $\omega = 4 \cdot 2pi = 8\pi = 25.1327$ rad/sec, so the linear velocity is $v = r\omega = 18(25.1327) = 452.389$ feet per second.

c) By the SAS area formula, $A = \frac{1}{2}ab\sin C = \frac{1}{2}(30)18\sin 48° = 200.649$ square units.

d) The opposite side to the angle is 13.5 and the hypotenuse is 16, so the angle is $\sin^{-1}\frac{13.5}{16} = 57.54°$.

e) After the first heading, the boat is at position $\langle 30\cos 50°, 30\sin 50°\rangle = \langle 19.28, 22.98\rangle$ (the angle is $50°$ because $90° - 40° = 50°$). After moving a further 20 miles east, the boat is at $\langle 19.28 + 20, 22.98\rangle = \langle 39.28, 22.98\rangle$. The distance from the boat to the harbor is the norm of this vector, which is $\sqrt{39.28^2 + 22.98^2} = 45.50$ miles.