Old Math 120 Exams

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Contents

Chapter 1

General comments on these exams

These are the exams I have given in a Math 120 (trigonometry) course at Ferris State University in Fall 2016 and Fall 2017. Each exam has two sections: on the first section, calculators are not allowed and exact answers must be given, but on the second section, calculators are permitted and decimal approximations are acceptable. Each exam, other than the Fall 2017 final exam, is followed by solutions (there may be minor errors in the solutions).

These exams correspond to the material in my lecture notes and the 7^{th} edition of the McKeague textbook as follows:

Exam 1: chapters 1 & 2 of my lecture notes / sections A.1, 1.1-1.2, 3.2, 3.4-3.5 of the McKeague text

Exam 2: chapter 3 of my lecture notes / sections 1.3-1.5, 2.1-2.4, 3.1, 3.3, 7.1-7.4 of the McKeague text

Exam 3: chapters 4 & 5 of my lecture notes / sections 1.3-2.5, 7.5-7.6 of the McKeague text

Final Exam: chapters 1-7 of my lecture notes / chapters 1-5 and 7 of the McKeague text

Chapter 2

Exams from Fall 2016

2.1 Fall 2016 Exam 1

No calculator allowed on these problems

1. Convert the following angles from radians to degrees, and draw them in standard position:

   a) $\frac{5\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{8\pi}{3}$ d) $\frac{-4\pi}{4}$

2. Let cat be the function defined by cat $x = x^2 + 1$. Compute each quantity:

   a) cat $2 + 3$ b) cat $2 \cdot 3$ c) cat(2) + 3 d) 2 cat 3

Calculators allowed on the rest of the exam

3. Parts (a), (b), (c) and (d) of this question are not related to one another.

   a) Find an angle measuring between $0°$ and $360°$ which is coterminal with $4375°$.
   b) Convert $222°$ to radians (write your answer as a decimal, rounded to two or more decimal places).
   c) Find the coordinates of a point on the unit circle which is on the terminal side of a $540°$ angle, drawn in standard position.
   d) Draw a picture of a triangle which is isoceles, but not equilateral.

4. Parts (a), (b), and (c) of this question are not related to one another.

   a) Find the distance between the points $(2, 8)$ and $(5, -3)$ (write your answer either as an exact answer or as a decimal, rounded to two or more decimal places).

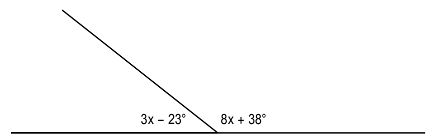
b) An angle measures $18°$ more than its complement. What is the measure of the angle?

c) The three angles of a triangle measure x, $x + 25°$ and $x - 40°$. Is this triangle an acute triangle, a right triangle, or an obtuse triangle?

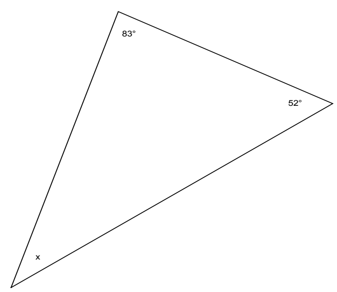
5. Parts (a) and (b) of this question are not related to one another.

a) A Ferris wheel makes $.065$ revolutions in a minute. If the radius of the Ferris wheel is 120 feet, what is the linear velocity of someone sitting in a bucket on the edge of the Ferris wheel?

b) Find the measure of each angle in this picture:

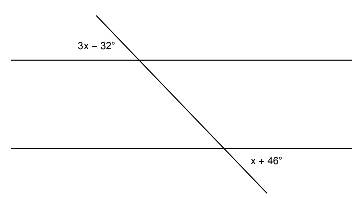


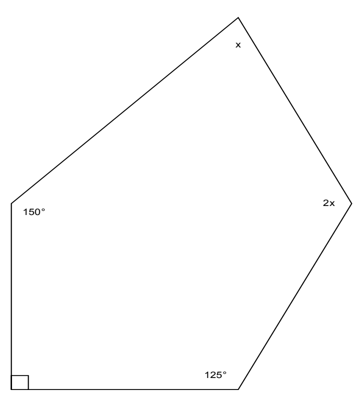
6. In each picture, find x:



a)

b) (in this picture, assume the horizontal lines are parallel)





c)

Exam 1 Solutions

1. a) $\frac{5\pi}{6} = 5 \cdot \frac{\pi}{6} = 5 \cdot 30° = 150°$. In standard position, this angle should point just north of the negative x-axis.

   b) $\frac{\pi}{2} = 90°$. In standard position, this angle points due north.

   c) $\frac{8\pi}{3} = 8 \cdot \frac{\pi}{3} = 8 \cdot 60° = 480°$. In standard position, this angle goes all the way around once, then halfway around again to end up on the negative x-axis.

   d) $\frac{-4\pi}{4} = -\pi = -180°$. In standard position, this angle is on the negative x-axis.

2. a) cat $2 + 3$ = cat$(2) + 3 = (2^2 + 1) + 3 = 5 + 3 = 8$.

   b) cat $2 \cdot 3 =$ cat $6 = 6^2 + 1 = 37$.

   c) cat$(2) + 3 = (2^2 + 1) + 3 = 5 + 3 = 8$.

   d) 2 cat $3 = 2(3^2 + 1) = 2(10) = 20$.

3. a) $4375 \div 360 = 12.15$ so the angle is $4375° - 12 \cdot 360° = 55°$.

   b) $222° \times \frac{\pi}{180°} = 3.87$ radians.

   c) $540° = 360° + 180°$ so the terminal side of this angle points on the negative x-axis. The point on the negative x-axis which is on the unit circle is $(-1, 0)$.

   d) The triangle should have two sides that are the same length, but not all three sides of the same length.

4. a) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5-2)^2 + (-3-8)^2} = \sqrt{3^2 + (-11)^2} = \sqrt{9 + 121} = \sqrt{130} \approx 11.4$.

   b) Let x be the angle; we have $x = 18° + (90° - x)$. Solve for x to get $x = 54°$.

   c) The three angles sum to $180°$, so $x + (x + 25°) + (x - 40°) = 180°$. Solve for x to get $x = 65°$. This makes the three angles $x = 65°$, $x + 25° = 90°$ and $x - 40° = 25°$. Since one of the angles is $90°$, the triangle is a right triangle.

5. a) The angular velocity is $\omega = .065 \cdot 2\pi = .408$ radians per minute. The linear velocity is therefore $v = r\omega = 120(.408) \approx 49$ feet per minute.

   b) The angles are supplementary, so $(3x - 23°) + (8x + 38°) = 180°$. Solve for x to get $x = 15°$; the angles are therefore $3(15°) - 23° = 22°$ and $8(15°) + 38° = 158°$.

6. a) The angles sum to $180°$: $x + 52° + 83° = 180°$. Solve for x to get $x = 45°$.

   b) The angles are equal, so $3x - 32° = x + 48°$. Solve for x to get $x = 39°$.

c) This shape has five sides, so its angles must add to $180°(5-2) = 540°$. Therefore $x + 2x + 125° + 90° + 150° = 540°$. Solve for x to get $x = \frac{175}{3}° \approx 58.33°$.

2.2 Fall 2016 Exam 2

No calculator allowed on these problems

1. Throughout this problem, assume $\sin\theta = \frac{-2}{3}$.

   a) What is $\sin(-\theta)$?

   b) If $\cos\theta > 0$, find $\cos\theta$.

   c) If $\cos\theta > 0$, what quadrant is θ in?

   d) What is $\sin(\theta + 360°)$?

   e) *(Bonus)* What is $\sin(\theta + 180°)$?

2. Find the exact value of each quantity:

   a) $\sin 45°$

   b) $\cos 120°$

   c) $\sin 180°$

   d) $\cos 300°$

   e) $\sin 570°$

   f) $\cos(-30°)$

3. Find the exact value of each quantity:

   a) $\sin \frac{5\pi}{6}$

   b) $\sin 8\pi$

   c) $\sin \frac{-3\pi}{4}$

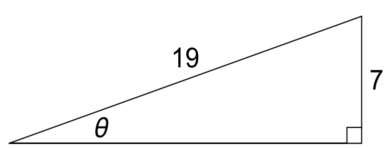
   d) $\cos \frac{10\pi}{3}$

   e) $\sin \frac{\pi}{2}$

   f) $\cos \frac{-7\pi}{2}$

Calculator allowed on the rest of the exam

4. Find $\cos\theta$, if θ is as in the following picture:



5. Suppose $\sin\theta = .55$. Find all possible values of $\cos\theta$.

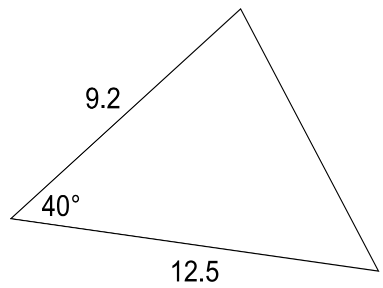
6. a) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sin\theta = .3$.

   b) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sin\theta = 1.3$.

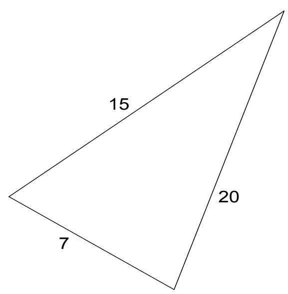
   c) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = .75$.

   d) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = 1$.

7. Find the area of each indicated triangle:

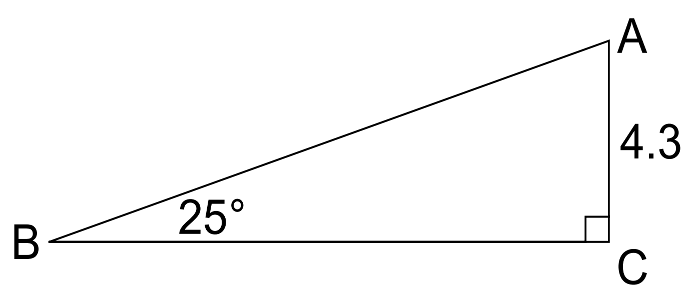


a)



b)

8. Solve triangle ABC, given the information in the picture below:



9. Solve triangle GHI if $g = 17$, $h = 25$ and $\angle I = 62°$.

10. Solve triangle ABC if $a = 17$, $b = 10$ and $\angle B = 40°$.

11. A person wants to measure the height of a flagpole. She walks 50 feet away from the base of the flagpole, turns around and notices that her angle of elevation to the top of the pole is $48°$. If her eyes are 5 feet above the ground, how tall is the flagpole?

Exam 2 Solutions

1. a) $\sin(-\theta) = -\sin\theta = \frac{2}{3}$.

   b) Start with the Pythagorean identity:

$$\begin{aligned}\cos^2\theta + \sin^2\theta &= 1\\ \cos^2\theta + \left(\frac{-2}{3}\right)^2 &= 1\\ \cos^2\theta + \frac{4}{9} &= 1\\ \cos^2\theta &= \frac{5}{9} \Rightarrow \cos\theta = \pm\sqrt{\frac{5}{9}}.\end{aligned}$$

   Since we are told $\cos\theta > 0$, $\cos\theta = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$.

   c) Since $\sin\theta < 0$ and $\cos\theta > 0$, θ must be in Quadrant IV.

   d) $\sin(\theta + 360°) = \sin\theta = \frac{-2}{3}$.

   e) $\theta + 180°$ is in Quadrant II, but has the same reference angle as θ, so $\sin(\theta + 180°) = \frac{2}{3}$.

2. a) $\sin 45° = \frac{\sqrt{2}}{2}$.

   b) $\cos 120° = \frac{-1}{2}$ (Quadrant II; reference angle $60°$)

   c) $\sin 180° = 0$ (point on unit circle is $(-1, 0)$)

   d) $\cos 300° = \frac{1}{2}$ (Quadrant IV; reference angle $60°$)

   e) $\sin 570° = \sin 210° = \frac{-1}{2}$ (Quadrant II; reference angle $30°$)

   f) $\cos(-30°) = \cos 30° = \frac{\sqrt{3}}{2}$

3. a) $\sin\frac{5\pi}{6} = \sin 150° = \frac{1}{2}$ (Quadrant II; reference angle $30°$)

   b) $\sin 8\pi = \sin 0 = 0$

   c) $\sin\frac{-3\pi}{4} = \sin(-135°) = \frac{-\sqrt{2}}{2}$ (Quadrant III; reference angle $45°$)

   d) $\cos\frac{10\pi}{3} = \cos\frac{4\pi}{3} = \cos 240° = \frac{-1}{2}$ (Quadrant III; reference angle $60°$)

   e) $\sin\frac{\pi}{2} = \sin 90° = 1$ (point on unit circle is $(0, 1)$)

   f) $\cos\frac{-7\pi}{2} = \cos\frac{7\pi}{2} = \cos 270° = 0$ (point on unit circle is $(0, -1)$)

4. First, use the Pythagorean Theorem to find the adjacent side, which I'll call a:

$$a^2 + 7^2 = 19^2 \Rightarrow a = \sqrt{19^2 - 7^2} = \sqrt{312} \approx 17.66.$$

   Then $\cos\theta = \frac{\text{adjacent}}{\text{adjacent}} \approx \frac{17.66}{19} = .9297$.

5. Start with the Pythagorean identity:

$$\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1 \\
\cos^2\theta + (.55)^2 &= 1 \\
\cos^2\theta + .3025 &= 1 \\
\cos^2\theta &= .6975 \\
\cos\theta &= \pm\sqrt{.6975} \approx \pm .8352.
\end{aligned}$$

6. a) From a calculator, $\theta = \sin^{-1}(.3) \approx 17.46°$. A second angle which solves the equation is $180° - \theta \approx 162.54°$.

   b) There are no such angles (because $1.3 > 1$).

   c) From a calculator, $\theta = \cos^{-1}(.75) \approx 41.4°$. A second angle which solves the equation is $360° - \theta = 318.6°$.

   d) The only such angle is $\theta = 0$ (if you also included $\theta = 360°$, that's okay, but $360°$ is really the same angle as $0°$).

7. a) By the SAS Area Formula, $A = \frac{1}{2}ab\sin C = \frac{1}{2}(9.2)(12.5)\sin 40° \approx 36.96$ sq units.

   b) Use Heron's Formula (so first, find the semiperimeter):

$$s = \frac{a+b+c}{2} = \frac{7+15+20}{2} = 21.$$

   Then the area is

$$\begin{aligned}
A = \sqrt{s(s-a)(s-b)(s-c)} &= \sqrt{21(21-7)(21-15)(21-20)} \\
&= \sqrt{21(14)(6)(1)} \\
&= \sqrt{1764} \\
&= 42 \text{ sq units.}
\end{aligned}$$

8. First, the third angle is $A = 90° - 25° = 65°$.

   Second, find the hypotenuse c using a trig function (either sine or cosine):

$$\begin{aligned}
\sin 25° &= \frac{4.3}{c} \\
.4226 &= \frac{4.3}{c} \\
.4226c &= 4.3 \\
c &= \frac{4.3}{.4226} = 10.175.
\end{aligned}$$

Last, find the remaining side using the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + (4.3)^2 &= (10.175)^2 \\ a &= \sqrt{(10.175)^2 - (4.3)^2} = \sqrt{85.03} \approx 9.22. \end{aligned}$$

9. The given information is SAS, so start with the Law of Cosines to find side i:

$$\begin{aligned} i^2 &= g^2 + h^2 - 2gh\cos I \\ i^2 &= 17^2 + 25^2 - 2(17)(25)\cos 62^\circ \\ i^2 &= 289 + 625 - 850(.4695) \\ i^2 &= 514.949 \\ i &= \sqrt{514.949} \approx 22.69. \end{aligned}$$

Now use the Law of Sines (or the Law of Cosines) again to find $\angle G$ (you could find $\angle H$ first instead):

$$\begin{aligned} \frac{\sin G}{g} &= \frac{\sin I}{i} \\ \frac{\sin G}{17} &= \frac{\sin 62^\circ}{22.69} \\ \frac{\sin G}{17} &= \frac{.8829}{22.69} \\ \sin G &= \frac{17(.8829)}{22.69} = .6615 \\ G &= \sin^{-1}(.6615) = 41.4^\circ. \end{aligned}$$

Last, find $\angle H$: $H = 180^\circ - 62^\circ - 41.4^\circ = 76.6^\circ$.

10. The given information is SSA, which is the ambiguous case of the Law of Sines:

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin A}{17} &= \frac{\sin 40^\circ}{10} \\ 10\sin A &= 17(.6427) \\ \sin A &= \frac{17(.6427)}{10} = 1.092. \end{aligned}$$

This equation has no solution (because $1.092 > 1$), so there is no such triangle.

11. Draw a right triangle where the opposite side is the flagpole. We are given that the adjacent side is 50 feet and the angle to the horizontal is $48°$. That means, labelling the adjacent side as a, that

$$\begin{aligned}
\tan 48° &= \frac{\text{opposite}}{\text{adjacent}} \\
1.11 &= \frac{a}{50} \\
1.11(50) &= a \\
55.5 &= a.
\end{aligned}$$

Adding the 5 feet to account for the height of the viewer's eyes, we see that the height of the flagpole is $55.5 + 5 = 60.5$ feet.

2.3 Fall 2016 Exam 3

No calculators allowed on these problems

1. Throughout this problem, assume $\sec\theta = 4$.

   a) What is $\sec(-\theta)$?
   b) What is $\cos\theta$?
   c) What is $\sec(\theta - 360°)$?
   d) Which two quadrants might θ be in?
   e) If $\tan\theta < 0$, find $\cot\theta$.

2. Find the exact value of each quantity:

   a) $\sec 45°$
   b) $\tan 150°$
   c) $\cot 180°$
   d) $\csc(-120°)$
   e) $\sec 60°$

3. Find the exact value of each quantity:

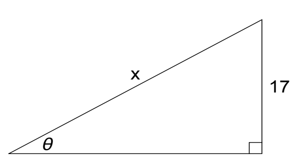
   a) $\tan 5\pi$
   b) $\csc \frac{\pi}{6}$
   c) $\tan \frac{3\pi}{4}$
   d) $\cos \frac{\pi}{2}$
   e) $\sin \frac{2\pi}{3}$

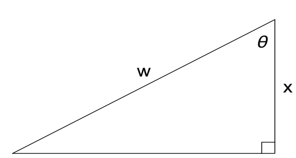
4. Find the exact value of each quantity:

   a) $\sin^2 125° + \cos^2 125°$
   b) $\cot(45° + 45°)$
   c) $\tan^2 \frac{5\pi}{6}$
   d) $3 \sin \frac{\pi}{2}$
   e) $\sin 3 \cdot \frac{\pi}{2}$

Calculators allowed on the rest of this exam

5. In each diagram below, write an equation for x in terms of the other given quantities in the picture.

   a) 

   b) 

6. Suppose that $\sin\theta = .614$ and that $\tan\theta > 0$. Find the values of all six trig functions of θ.

7. Use a calculator to compute these quantities:

a) $\csc 75°$

b) $\cot 40° + \cot 70°$

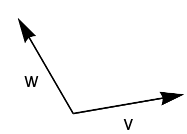
c) $\tan^2 125°$

d) $3 \sec 40°$

8. a) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\tan\theta = .72$.
   b) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sec\theta = 2.8$.
   c) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\csc\theta = .35$.

9. Throughout this question, assume that $\mathbf{u} = \langle -5, 3\rangle$, $\mathbf{v} = \langle -1, 8\rangle$ and that $\mathbf{w}$ is a vector whose magnitude is 7 and whose direction angle is $58°$.
   a) Compute $2\mathbf{u} + \mathbf{v}$.
   b) Compute $\mathbf{u} \cdot \mathbf{v}$.
   c) Compute the angle between $\mathbf{u}$ and $\mathbf{v}$.
   d) Compute the direction angle of $\mathbf{v}$.
   e) Compute $|\mathbf{u} + \mathbf{v}|$.
   f) Write $\mathbf{w}$ in component form.

10. Let $\mathbf{v}$ and $\mathbf{w}$ be the vectors indicated in the picture below.
    a) Sketch the vector $2\mathbf{v} + \mathbf{w}$ on the picture; label that vector "$2\mathbf{v} + \mathbf{w}$".
    b) Sketch the vector $\mathbf{w} - \mathbf{v}$ on the picture; label that vector "$\mathbf{w} - \mathbf{v}$".



11. A bird leaves its nest and flies 20 miles on a heading $37°$, measured east of north. The bird then flies 9 miles on a bearing $72°$, measured west of north.
    a) How far is the bird from its nest?
    b) If the bird wanted to return to its nest, would it be more accurate to say it should fly southeast, or southwest? Explain your answer.

Exam 3 Solutions

1. a) $\sec(-\theta) = \sec\theta = 4$.

   b) $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{4}$.

   c) $\sec(\theta - 360°) = \sec\theta = 4$.

   d) θ must be in Quadrant I or Quadrant IV.

   e) Since $\tan\theta < 0$, we now know θ is in Quadrant IV, so $\cot\theta < 0$. Sketch a triangle with hypotenuse 4 and adjacent side of length 1. From the Pythagorean Theorem, the opposite side is $\sqrt{4^2 - 1^2} = \sqrt{15}$, so $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\sqrt{15}}$.

2. a) $\sec 45° = \sqrt{2}$.

   b) $\tan 150° = \frac{-1}{\sqrt{3}}$ (reference angle $30°$, Quadrant II).

   c) $\cot 180°$ DNE.

   d) $\csc(-120°) = \frac{-2}{\sqrt{3}}$ (reference angle $60°$, Quadrant III).

   e) $\sec 60° = 2$.

3. a) $\tan 5\pi = 0$

   b) $\csc\frac{\pi}{6} = 2$

   c) $\tan\frac{3\pi}{4} = -1$ (reference angle $45°$, Quadrant II)

   d) $\cos\frac{\pi}{2} = 0$

   e) $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ (reference angle $60°$, Quadrant II)

4. a) $\sin^2 125° + \cos^2 125° = 1$ (by a Pythagorean identity)

   b) $\cot(45° + 45°) = \cot 90° = 0$

   c) $\tan^2\frac{5\pi}{6} = \left(\frac{-1}{\sqrt{3}}\right)^2 = \frac{1}{3}$

   d) $3\sin\frac{\pi}{2} = 3(1) = 3$

   e) $\sin 3\cdot\frac{\pi}{2} = -1$

5. a) $\frac{x}{17} = \frac{\text{hyp}}{\text{opp}} = \csc\theta$ so $x = 17\csc\theta$.

   b) $\frac{x}{w} = \frac{\text{adj}}{\text{hyp}} = \cos\theta$ so $x = w\cos\theta$.

6. Since $\sin\theta > 0$ and $\tan\theta > 0$, θ is in Quadrant I, so all the trig functions are positive. We are given $\sin\theta = .614$, so $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{.614} = 1.629$. From the identity $\sin^2\theta + \cos^2\theta = 1$, we can solve for $\cos\theta$ to get $\cos\theta = .789$. Then $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{.789} = 1.267$. Last, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{.614}{.789} = .777$ and $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{.789}{.614} = 1.285$.

7. a) $\csc 75^\circ = 1.03528$.
   b) $\cot 40^\circ + \cot 70^\circ = 1.19175 + .36397 = 1.55572$.
   c) $\tan^2 125^\circ = (-1.42815)^2 = 2.03961$.
   d) $3\sec 40^\circ = 3(1.30541) = 3.91622$.

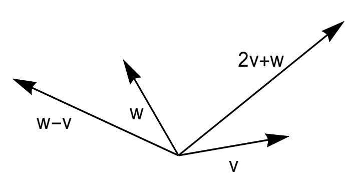
8. a) $\theta = \tan -1.72 = 35.7539^\circ$; a second angle is $\theta + 180^\circ = 215.7539^\circ$.
   b) If $\sec\theta = 2.8$, then $\cos\theta = \frac{1}{2.8}$ so $\theta = \cos^{-1}(\frac{1}{2.8}) = 69.07^\circ$. A second angle is $360^\circ - \theta = 290.93^\circ$.
   c) There are no such angles, because $\csc\theta$ cannot be between -1 and 1.

9. Throughout this question, assume that $\mathbf{u} = \langle -5, 3\rangle$, $\mathbf{v} = \langle -1, 8\rangle$ and that $\mathbf{w}$ is a vector whose magnitude is 7 and whose direction angle is 58°.
   a) $2\mathbf{u} + \mathbf{v} = \langle -10, 6\rangle + \langle -1, 8\rangle = \langle -11, 14\rangle$.
   b) $\mathbf{u}\cdot\mathbf{v} = (-5)(-1) + 3(8) = 5 + 24 = 29$.
   c) First, $|\mathbf{u}| = \sqrt{(-5)^2 + 3^2} = \sqrt{34} \approx 5.83$. Second, $|\mathbf{v}| = \sqrt{(-1)^2 + 8^2} = \sqrt{65} = 8.06$. Therefore,

$$\begin{aligned} \mathbf{u}\cdot\mathbf{v} &= |\mathbf{u}|\,|\mathbf{v}|\,\cos\theta \\ 29 &= (5.83)(8.06)\cos\theta \\ .6171 &= \cos\theta \\ 51.89^\circ &= \theta. \end{aligned}$$

   d) $\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{8}{-1} = \tan^{-1}(-8) = -82.875^\circ$. But this is in the wrong quadrant ($\mathbf{v}$ is in Quadrant II), so we need to add 180° to get 97.125°.
   e) $|\mathbf{u}+\mathbf{v}| = |\,\langle -6, 11\rangle\,| = \sqrt{(-6)^2 + 11^2} = \sqrt{36+121} = \sqrt{157} \approx 12.53$.
   f) $\mathbf{w} = \langle |\mathbf{w}|\cos\theta, |\mathbf{w}|\sin\theta\rangle = \langle 7\cos 58^\circ, 7\sin 58^\circ\rangle = \langle 3.709, 5.936\rangle$.

10.



11. a) Let $\mathbf{v}$ be the first part of the bird's journey; we have

$$\mathbf{v} = \langle |\mathbf{v}| \cos\theta, |\mathbf{v}| \sin\theta\rangle = \langle 20\cos 53°, 20\sin 53°\rangle = \langle 12.03, 15.97\rangle .$$

Let $\mathbf{w}$ be the second part of the bird's journey; we have

$$\mathbf{w} = \langle |\mathbf{w}| \cos\theta, |\mathbf{w}| \sin\theta\rangle = \langle 9\cos 162°, 9\sin 162°\rangle = \langle -8.559, 2.781\rangle .$$

Therefore the bird's position is $\mathbf{v} + \mathbf{w} = \langle 3.47, 18.75\rangle$. The distance from the bird to the nest is

$$|\mathbf{v} + \mathbf{w}| = |\,\langle 3.47, 18.75\rangle\,| = \sqrt{(3.47)^2 + (18.75)^2} = 19.06 \text{ mi}.$$

b) The first component of the bird's position is positive, so it has to fly southwest (although just barely west of south) to get back to its nest.

2.4 Fall 2016 Final Exam

No calculators allowed on these questions

1. Find the exact value of each quantity:

   a) $\tan 45°$ b) $\cos 60°$ c) $\sin 90°$ d) $\sec 30°$ e) $\cos 0°$

2. Find the exact value of each quantity:

   a) $\cot 240°$ b) $\cos(-150°)$ c) $\sin 315°$ d) $\csc 270°$ e) $\sec 450°$

3. Find the exact value of each quantity:

   a) $\sin \frac{11\pi}{6}$ b) $\sin \frac{-\pi}{3}$ c) $\cot \frac{3\pi}{2}$ d) $\tan \frac{5\pi}{6}$ e) $\tan \frac{11\pi}{3}$

4. Find the exact value of each quantity:

   a) $\sin^2 50° + \cos^2 50°$ b) $\cos^2 135°$ c) $\sin 75°$ d) $\tan 4 \cdot 30°$ e) $8 \sec \frac{\pi}{4}$

5. Suppose $\cos\theta = \frac{3}{5}$ and $\tan\theta < 0$. Find the exact values of all six trig functions of θ.

6. Suppose $\tan\theta = 3$.

   a) Find $\tan 2\theta$.
   b) Find $\cot\theta$.
   c) Find $\cot 2\theta$.
   d) What two quadrants might θ be in?
   e) Find $\tan(\theta + 720°)$.
   f) Find $\tan(-\theta)$.

7. Sketch the graph of $y = \tan x$.

8. a) Which of graphs A-K on the attached page (see page 21 of this pdf file) is the graph of $y = \sin\frac{x}{2}$?
   b) Which of graphs A-K is the graph of $y = 2\sin x$?
   c) Which of graphs A-K is the graph of $y = \sin x - 2$?
   d) Which of graphs A-K is the graph of $y = \sin(x + 2)$?

e) Which of graphs A-K is the graph of $y = \sin 2x$?

f) Which of graphs A-K is the graph of $y = \frac{\sin x}{2}$?

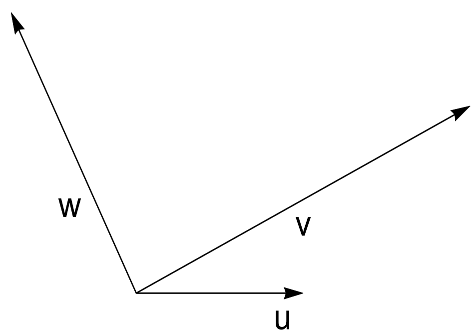
9. Vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are shown below. Sketch the following vectors on the same picture, being sure to label which vector is which:

a) $\mathbf{w} - \mathbf{u}$

b) $-\mathbf{w}$

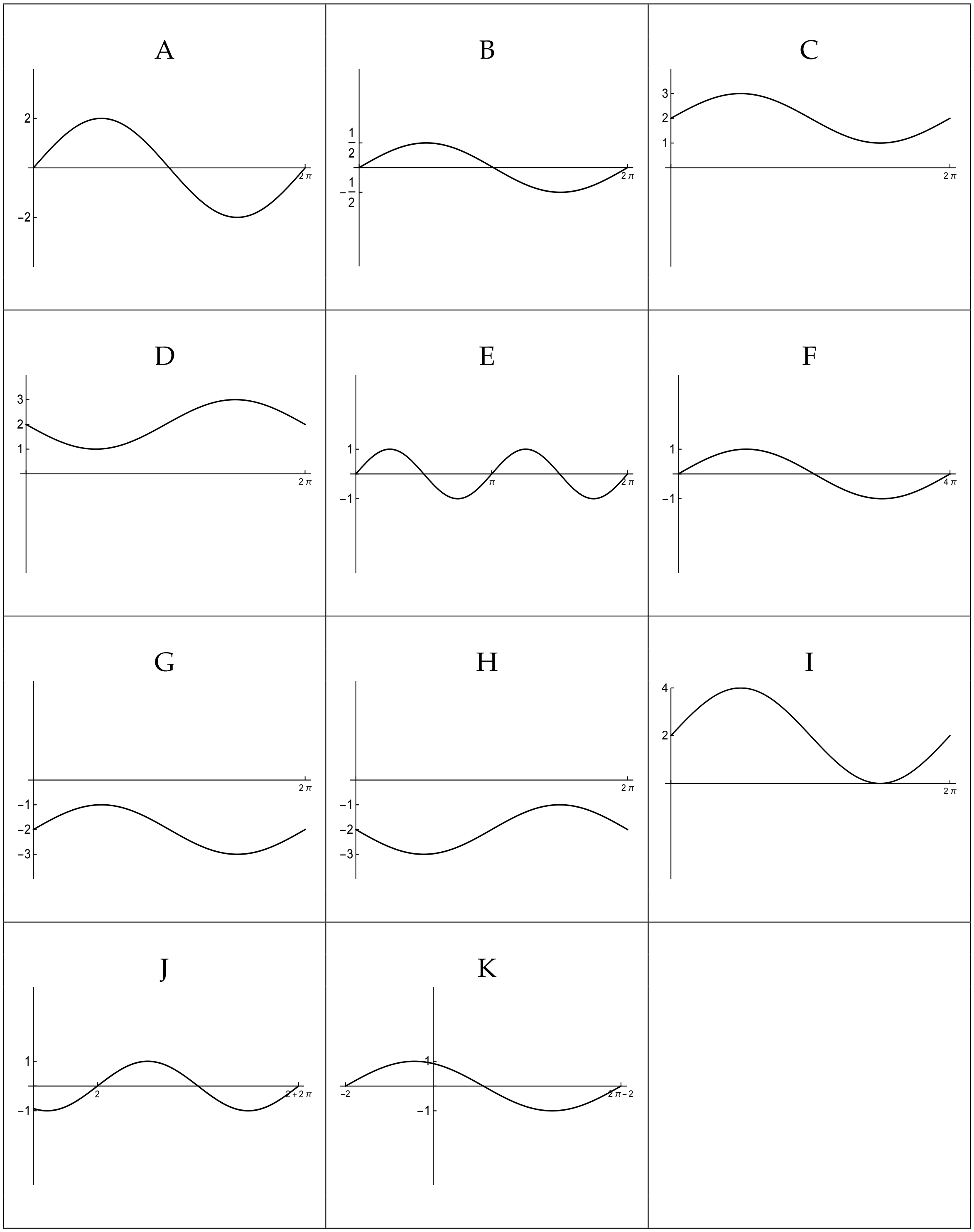
c) $\frac{1}{2}\mathbf{u}$

d) $\mathbf{u} + \mathbf{v} + \mathbf{w}$



Graphs for Problem 8

Use these graphs to answer the questions in Problem 8:



Calculators allowed on the rest of the exam

10. Evaluate the following expressions using a calculator. Your answers can (and should) be written as decimals.

    a) $\csc 52°$
    b) $\sin 25° + \sin 110°$
    c) $\sec(83° - 40°)$
    d) $\tan^2 117°$
    e) $4 \cot 286°$
    f) $\sin 100° \cos 44°$

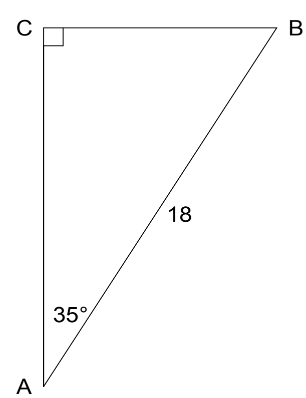
11. For each equation, find (decimal approximations of) all angles θ between $0°$ and $360°$ satisfying the following equations. If there are no such angles, say so.

    a) $\sin\theta = .28$
    b) $\cos\theta = 1.35$
    c) $\cot\theta = -.55$
    d) $\csc\theta = 1$

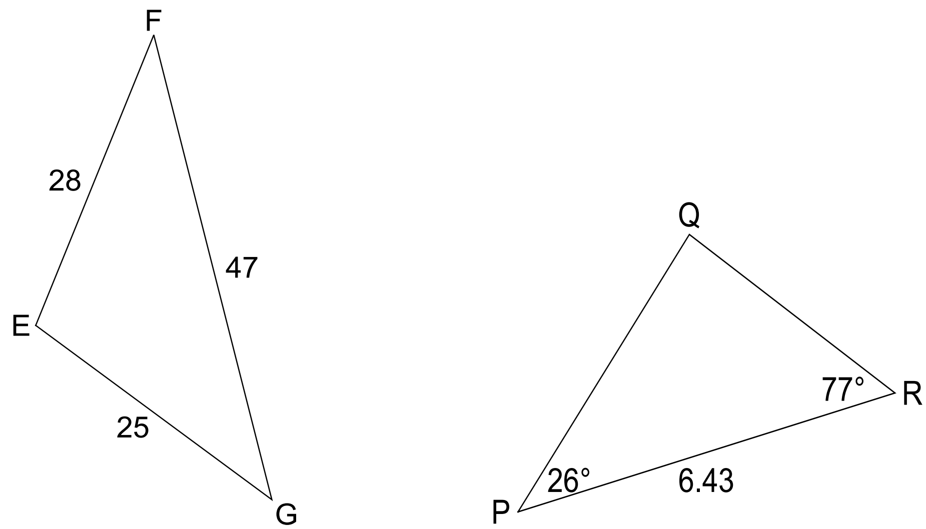
12. Suppose that θ is an angle in a right triangle whose hypotenuse has length 43.2 and whose side opposite θ has length 25.1. Find the six trig functions of angle θ.

13. a) Compute the norm of the vector $\langle -5, -4\rangle$.
    b) Compute the direction angle of the vector $\langle -25, 7\rangle$.
    c) Suppose $\mathbf{v} = \langle 6, -4\rangle$ and $\mathbf{w} = \langle 2, 11\rangle$. Compute $5\mathbf{v} + 2\mathbf{w}$.
    d) Compute $\mathbf{v} \cdot \mathbf{w}$, where $\mathbf{v}$ and $\mathbf{w}$ are as in part (c).
    e) Find the components of a vector which has norm 17 and direction angle $230°$.

14. Solve triangle ABC, which is pictured below:



15. Solve triangle EFG, which is pictured below at left:



16. Solve triangle PQR, which is pictured above at right.

17. Classify the following statements as true or false:

a) $\tan^2\theta - 1 = \sec^2\theta$

b) $\cos(-\theta) = -\cos\theta$

c) $\cos(90^\circ - \theta) = \sin\theta$

d) $1 - \cos^2\theta = \sin^2\theta$

e) $\tan\theta\cos\theta = \sin\theta$

18. Answer any two of the following five questions.

a) A wedge-shaped piece of pie taken from a pie with a $16''$ diameter has angle 50°. Find the area of the piece of pie.

b) A wheel of radius 18 feet rotates at a speed of 4 revolutions per minute. What is the linear velocity of a point on the edge of the wheel?

c) Find the area of triangle ABC, if $a = 30$, $b = 18$ and $\angle C = 48^\circ$.

d) A 16-foot ladder leans up against the side of the building. If the top of the ladder is 13.5 feet above the ground, what angle does the ladder make with the ground?

e) A boat leaves a harbor and sails 30 miles on a heading 40°, measured east of north, then sails 20 miles due east. How far is it from the harbor?

Final Exam Solutions

1. a) $\tan 45° = 1$
   b) $\cos 60° = \frac{1}{2}$
   c) $\sin 90° = 1$
   d) $\sec 30° = \frac{1}{\cos 30°} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$
   e) $\cos 0° = 1$

2. a) $\cot 240° = \frac{1}{\sqrt{3}}$ (quadrant III, reference angle $60°$)
   b) $\cos(-150°) = \cos 150° = \frac{-\sqrt{3}}{2}$ (quadrant II, reference angle $30°$)
   c) $\sin 315° = \frac{-\sqrt{2}}{2}$ (quadrant IV, reference angle $45°$)
   d) $\csc 270° = \frac{1}{-1} = -1$ (the point $(0,-1)$ on the unit circle)
   e) $\sec 450° = \frac{1}{\cos 90°} = \frac{1}{0}$ DNE.

3. a) $\sin \frac{11\pi}{6} = \frac{-1}{2}$ (quadrant IV, reference angle $30°$)
   b) $\sin \frac{-\pi}{3} = \frac{-\sqrt{3}}{2}$ (quadrant IV, reference angle $60°$)
   c) $\cot \frac{3\pi}{2} = \frac{\cos 270°}{\sin 270°} = \frac{0}{1} = 0$ (the point $(0,-1)$ on the unit circle)
   d) $\tan \frac{5\pi}{6} = \frac{-1}{\sqrt{3}}$ (quadrant II, reference angle $30°$)
   e) $\tan \frac{11\pi}{3} = -\sqrt{3}$ (quadrant IV, reference angle $60°$)

4. a) $\sin^2 50° + \cos^2 50° = 1$
   b) $\cos^2 135° = \left(\frac{-\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$.
   c) Use the addition identity for sine:

$$\begin{aligned}\sin 75° = \sin(30° + 45°) &= \sin 30° \cos 45° + \cos 30° \sin 45° \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4}.\end{aligned}$$

   d) $\tan 4 \cdot 30° = \tan(4 \cdot 30°) = \tan 120° = -\sqrt{3}$.
   e) $8 \sec \frac{\pi}{4} = 8\sqrt{2}$.

5. Since $\cos\theta > 0$ and $\tan\theta < 0$, θ must be in Quadrant IV. Therefore all the trig functions of θ are negative except for cosine and secant. Next, draw a triangle with adjacent side 3 and hypotenuse 5. Solve for the opposite side using the Pythagorean Theorem to get $\sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$. Therefore

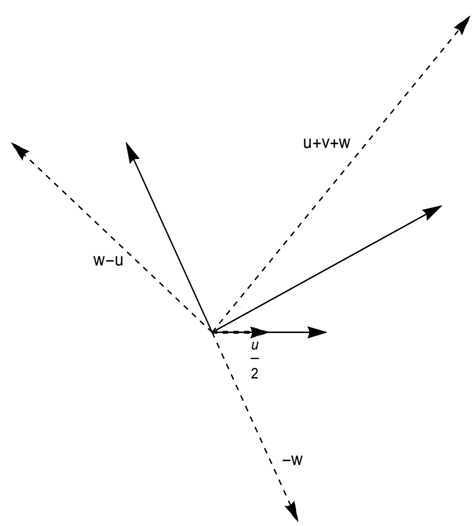
$$\cos\theta = \frac{3}{5}; \sec\theta = \frac{5}{3}; \sin\theta = \frac{-4}{5}; \csc\theta = \frac{-5}{4}; \tan\theta = \frac{-4}{3}; \cot\theta = \frac{-3}{4}.$$

6. a) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(3)}{1-3^2} = \frac{-6}{8} = \frac{-3}{4}$.
   b) $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{3}$.
   c) $\cot 2\theta = \frac{1}{\tan 2\theta} = \frac{-4}{3}$.
   d) Since $\tan\theta > 0$, θ is in Quadrant I or III.
   e) $\tan(\theta + 720°) = \tan\theta = 3$.
   f) $\tan(-\theta) = -\tan\theta = -3$.

7. The graph of $y = \tan x$ is on page 123 of my lecture notes.

8. a) This graph has period $\frac{2\pi}{1/2} = 4\pi$, so it is graph F.
   b) This graph has been stretched vertically by a factor of 2, so it is graph A.
   c) This graph is shifted down 2 units, so it is graph G.
   d) This graph is shifted left 2 units, so it is graph K.
   e) This graph has period $\frac{2\pi}{2} = \pi$, so it is graph E.
   f) This graph has been squashed vertically to be half as tall, so it is graph B.

9.



10. a) $\csc 52° = \frac{1}{\sin 52°} = 1.26902$.
   b) $\sin 25° + \sin 110° = .422618 + .939693 = 1.36231$.
   c) $\sec(83° - 40°) = \sec 43° = \frac{1}{\cos 43°} = 1.36733$.
   d) $\tan^2 117° = (-1.96261)^2 = 3.85184$.
   e) $4\cot 286° = 4\left(\frac{1}{\tan 286°}\right) = 4\frac{1}{-3.48741} = 4(-.286745) = -1.14698$.
   f) $\sin 100° \cos 44° = (.984808)(.71934) = .708411$.

11. a) $\theta = \sin^{-1} .28 = 16.26°$ and $180° - 16.26° = 163.74°$.

    b) $\cos\theta = 1.35$ has no solution since $1.35 > 1$.

    c) $\cot\theta = -.55$ means $\tan\theta = \frac{1}{-.55} = -1.81818$ so $\theta = \tan^{-1}(-1.81818) = 118.81°$ and $\theta = 118.81° + 180° = 298.81°$.

    d) $\csc\theta = 1$ means $\sin\theta = \frac{1}{1} = 1$ so $\theta = \sin^{-1} 1 = 90°$.

12. First, the adjacent side is $\sqrt{43.2^2 - 25.1^2} = 35.16$. Therefore the six trig functions are

$$\sin\theta = \tfrac{25.1}{43.2} = .581 \qquad \csc\theta = \tfrac{43.2}{25.1} = 1.721$$

$$\tan\theta = \tfrac{25.1}{35.16} = .713 \quad \cot\theta = \tfrac{35.16}{25.1} = 1.4008$$

$$\cos\theta = \tfrac{35.16}{43.2} = .814 \quad \sec\theta = \tfrac{43.2}{35.16} = 1.229$$

13. a) $|\langle -5, -4\rangle| = \sqrt{5^2 + (-4)^2} = \sqrt{25 + 16} = \sqrt{41}$.

    b) $\theta = \tan^{-1} \frac{7}{-25} = \tan^{-2} -.28 = -15.64°$. Since θ must be in Quadrant II, add $180°$ to get $\theta = 164.36°$.

    c) $5\mathbf{v} + 2\mathbf{w} = \langle 30, -20\rangle + \langle 4, 22\rangle = \langle 34, 2\rangle$.

    d) Compute $\mathbf{v} \cdot \mathbf{w} = 6(2) + (-4)(11) = 12 - 44 = -32$.

    e) $\langle 17\cos 230°, 17\sin 230°\rangle = \langle -10.927, -13.0228\rangle$.

14. $\angle B = 180° - 90° - 35° = 55°$.

    $\sin 35° = \frac{a}{18}$ so $a = 18\sin 35° = 10.32$.

    $\cos 35° = \frac{b}{18}$ so $b = 18\cos 35° = 14.74$.

15. Start with the Law of Cosines to find angle E:

$$\begin{aligned}
e^2 &= f^2 + g^2 - 2fg\cos E \\
47^2 &= 25^2 + 28^2 - 2(25)(28)\cos E \\
2209 &= 625 + 784 - 1400\cos E \\
800 &= -1400\cos E \\
\frac{800}{-1400} &= \cos E \\
\cos^{-1}\frac{8}{-14} &= E \\
124.85° &= E
\end{aligned}$$

Then use the Law of Cosines (or the Law of Sines) to find angle G:

$$\begin{aligned} g^2 &= e^2 + f^2 - 2ef\cos G \\ 28^2 &= 25^2 + 47^2 - 2(25)(47)\cos G \\ 784 &= 625 + 2209 - 2350\cos G \\ -2050 &= -2350\cos G \\ \frac{-2050}{-2350} &= \cos G \\ \cos^{-1}\frac{205}{235} &= G \\ 29.27^\circ &= G \end{aligned}$$

Last, angle F is $180^\circ - 124.85^\circ - 29.27^\circ = 25.88^\circ$.

16. First, angle Q is $180^\circ - 26^\circ - 77^\circ = 77^\circ$.

    Second, since angles Q and R are equal, sides q and r are equal so $r = 6.43$.

    Last, use the Law of Sines to find p:

$$\begin{aligned} \frac{\sin P}{p} &= \frac{\sin R}{r} \\ \frac{\sin 26^\circ}{p} &= \frac{\sin 77^\circ}{6.43} \\ \frac{.438371}{p} &= \frac{.97437}{6.43} \\ .97437p &= (.438371)6.43 \\ p &= \frac{(.438371)6.43}{.97437} = 2.892. \end{aligned}$$

17. a) $\tan^2\theta - 1 = \sec^2\theta$ is FALSE (the $\tan^2$ and $\sec^2$ are backwards from what they are in the Pythagorean identity)
    b) $\cos(-\theta) = -\cos\theta$ is FALSE (the $-$ sign disappears)
    c) $\cos(90^\circ - \theta) = \sin\theta$ is TRUE (cofunction identity)
    d) $1 - \cos^2\theta = \sin^2\theta$ is TRUE (Pythagorean identity)
    e) $\tan\theta\cos\theta = \sin\theta$ is TRUE (write $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$ and then cancel the $\cos\theta$ terms on the left-hand side to see this)

18. a) The radius of the circle is $r = \frac{1}{2}(16) = 8$ inches; the angle in radians is $50^\circ \cdot \frac{\pi}{180^\circ} = .872665$. The area of the sector is therefore $A = \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2(.872665) = 27.9253$ square inches.
    b) The angular velocity is $\omega = 4 \cdot 2pi = 8\pi = 25.1327$ rad/sec, so the linear velocity is $v = r\omega = 18(25.1327) = 452.389$ feet per second.

c) By the SAS area formula, $A = \frac{1}{2}ab\sin C = \frac{1}{2}(30)18\sin 48^\circ = 200.649$ square units.

d) The opposite side to the angle is 13.5 and the hypotenuse is 16, so the angle is $\sin^{-1}\frac{13.5}{16} = 57.54^\circ$.

e) After the first heading, the boat is at position $\langle 30\cos 50^\circ, 30\sin 50^\circ\rangle = \langle 19.28, 22.98\rangle$ (the angle is 50° because $90^\circ - 40^\circ = 50^\circ$). After moving a further 20 miles east, the boat is at $\langle 19.28 + 20, 22.98\rangle = \langle 39.28, 22.98\rangle$. The distance from the boat to the harbor is the norm of this vector, which is $\sqrt{39.28^2 + 22.98^2} = 45.50$ miles.

Chapter 3

Exams from Fall 2017

3.1 Fall 2017 Exam 1

No calculator allowed on these problems

1. Convert the following angles from radians to degrees:

   a) $\frac{4\pi}{3}$ b) $\frac{3\pi}{2}$ c) $-\pi$

2. Draw the angle $250°$ in standard position.

3. Let "sub" be the function defined by sub $x = 2x - 3$. Compute each quantity:

   a) sub $4 - 3$ b) sub $4 \cdot 3$ c) sub $(4) \cdot 3$ d) 3 sub 4

Calculators allowed on the rest of the exam

4. Parts (a), (b), and (c) of this question are not related to one another.

   a) Find the measure of two different angles, one of which is less than $624°$ and one of which is greater than $624°$, both of which are coterminal with $624°$.

   b) Convert 5.6 radians to degrees (write your answer as an exact answer or as a decimal, rounded to two or more decimal places).

   c) Solve for x:
   $$\frac{x - 100}{3.25} = \frac{x}{4.8}$$

5. Parts (a), (b) and (c) of this question are not related to one another.

a) Find the distance between the points $(1, 3)$ and $(-2, 5)$ (write your answer either as an exact answer or as a decimal, rounded to two or more decimal places).

b) The point (x, y) is on the unit circle. If $x = -.275$, what are all possible values of y?

c) Find the area of a sector whose central angle is $53°$, taken from a circle whose diameter is 6 feet.

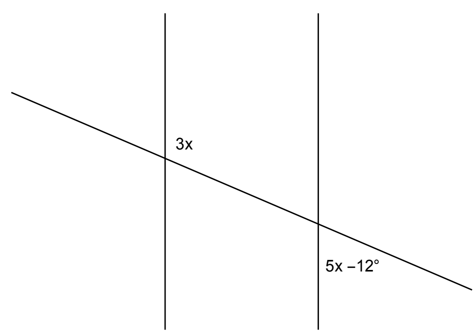
6. A dragster has two front tires of radius 16 inches, and two rear tires of radius 26 inches. If the dragster is travelling with all four tires on the ground, with no skidding or friction, and rear tires rotate at 35 revolutions per second, what is the angular velocity of the front tires (in radians per second)?

7. In each problem (a)-(c), find x:

a) Two complementary angles have measures $x + 34°$ and $3x - 20°$.

b) The three angles of a triangle are $x + 14°$, $3x - 35°$ and $7x + 3°$.

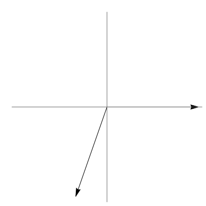
c) x is as in the picture below, where the vertical lines are parallel:



Solutions

1. a) $\frac{4\pi}{3} = 4 \cdot \frac{\pi}{3} = 4 \cdot 60° = 240°$.
   b) $\frac{3\pi}{2} = 3 \cdot \frac{\pi}{2} = 3 \cdot 90° = 270°$.
   c) $-\pi = -180°$.

2. Since $250°$ is a little less than $270°$, this angle should have its terminal side just west of south (in Quadrant III):



3. a) sub $4 - 3$ = sub$(4) - 3 = (2(4) - 3) - 3 = 5 - 3 = 2$.
   b) sub $4 \cdot 3 =$ sub $12 = 2(12) - 3 = 24 - 3 = 21$.
   c) sub$(4) \cdot 3 = (2(4) - 3) \cdot 3 = 5 \cdot 3 = 15$.
   d) 3 sub $4 = 3(2(4) - 3) = 3(5) = 15$. (This is the same as (b).)

4. a) For a smaller angle, subtract $360°$ from $624°$ to get $264°$; for a larger angle, add $360°$ to $624°$ to get $984°$.
   b) Multiply by $\frac{180°}{\pi}$ to get $320.85°$.
   c) Cross-multiply to get $4.8(x - 100) = 3.25x$. Distribute on the left-hand side to get $4.8x - 480 = 3.25x$; combine the like terms to get $1.55x = 480$; last, divide by 1.55 to get $x = 309.677$.

5. a) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-2 - 1)^2 + (5 - 3)^2} = \sqrt{(-3)^2 + (2)^2} = \sqrt{9 + 4} = \sqrt{13} \approx 3.6055$.
   b) From the unit circle formula $x^2 + y^2 = 1$, we see $x^2 + (-.275)^2 = 1$. Squaring out, this gives $x^2 + .0756 = 1$, i.e. $x^2 = 1 - .0756 = .9243$. Take the square root of both sides to get $x = \pm\sqrt{.9243} = \pm.9614$.
   c) The radius is half of the given diameter, i.e. $r = 3$. Convert the angle to radians by multiplying by $\frac{\pi}{180°}$ to get $\theta = .925$ radians. Then $A = \frac{1}{2}r^2\theta = \frac{1}{2}(3)^2(.925) = 4.1625$ sq ft.

6. The rear tires have angular velocity $\omega_{rear} = 35(2\pi) = 219.911$ rad/sec, so their linear velocity is $v_{rear} = r_{rear}\omega_{rear} = 26(219.911) = 5717.1$ in/sec. The front tires have the same linear velocity as the back tires, so $v_{front} = 5717.1$ in/sec as well. Finally, $\omega_{front} = \frac{v_{front}}{r_{front}} = \frac{5717.1}{16} = 357.356$ radians per second.

7. a) The angles are complementary, so $(x + 34°) + (3x - 20°) = 90°$. Combine the like terms to get $4x + 14° = 90°$, i.e. $4x = 76°$, i.e. $x = 19°$.

   b) The angles sum to $180°$: $(x+14°)+(3x-35°)+(7x+3°) = 180°$. Combine the like terms to get $11x - 18° = 180°$, i.e. $11x = 198°$, i.e. $x = 18°$.

   c) The angles are supplementary, so $3x + 5x - 12° = 180°$. This means $8x = 192°$, i.e. $x = 24°$.

3.2 Fall 2017 Exam 2

No calculator allowed on these problems

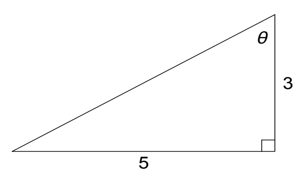
1. Throughout this problem, assume $\cos\theta = \frac{3}{4}$.
   a) What is $\cos(-\theta)$?
   b) What is $\cos(\theta - 360°)$?
   c) What are the two quadrants θ could lie in?
   d) If $\sin\theta < 0$, find $\sin\theta$.

2. Find the exact value of each quantity:

a) $\sin 180°$	c) $\sin -450°$	e) $\sin\frac{\pi}{4}$	g) $\sin\frac{-5\pi}{6}$
b) $\cos 60°$	d) $\cos 135°$	f) $\cos 3\pi$	h) $\cos\frac{\pi}{2}$

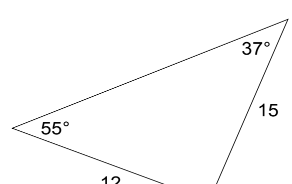
Calculators allowed on the rest of the exam

3. Find $\cos\theta$ and $\sin\theta$, if θ is as in the following picture. (Either an exact answer or a decimal is OK, but make sure to tell me which answer is $\sin\theta$ and which is $\cos\theta$.)



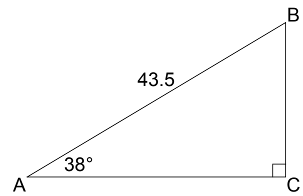
4. a) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sin\theta = -.265$.
   b) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = 2$.
   c) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\cos\theta = .43$.

5. Find the area of this triangle:



6. A 20-foot long ladder leans up against the side of a building. If the bottom of the ladder makes an angle of $70°$ with the ground, how high up the side of the building does the ladder go?

7. Solve triangle ABC, given the information in the picture below:



8. Solve triangle KLM if $k = 18$, $l = 13$ and $m = 10$.

9. Solve triangle ABC if $\angle A = 70°$, $\angle B = 58°$ and $a = 22$.

Solutions

1. a) $\cos(-\theta) = \cos\theta = \frac{3}{4}$.

   b) $\cos(\theta - 360°) = \cos\theta = \frac{3}{4}$.

   c) Since $\cos\theta > 0$, θ must be in Quadrants I or IV.

   d) Start with the Pythagorean identity:

$$\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1 \\
\left(\frac{3}{4}\right)^2 + \sin^2\theta &= 1 \\
\frac{9}{16} + \sin^2\theta &= 1 \\
\sin^2\theta &= \frac{7}{16} \Rightarrow \cos\theta = \pm\sqrt{\frac{7}{16}}.
\end{aligned}$$

   Since we are told $\sin\theta < 0$, $\sin\theta = -\sqrt{\frac{7}{16}} = -\frac{\sqrt{7}}{4}$.

2. a) $\sin 180° = 0$ (point on unit circle is $(-1, 0)$).

   b) $\cos 60° = \frac{1}{2}$

   c) $\sin -450° = \sin -90° = -\sin 90° = -1$ (point on unit circle is $(0, 1)$).

   d) $\cos 135° = \frac{-\sqrt{2}}{2}$ (Quadrant II; reference angle $45°$)

   e) $\sin\frac{\pi}{4} = \sin 45° = \frac{\sqrt{2}}{2}$ (Quadrant I)

   f) $\cos 3\pi = \cos 540° = \cos 180° = -1$ (point on unit circle is $(-1, 0)$)

   g) $\sin\frac{-5\pi}{6} = \sin(-150°) = \frac{-1}{2}$ (Quadrant III; reference angle $\frac{\pi}{6} = 30°$)

   h) $\cos\frac{\pi}{2} = \cos 90° = 0$ (point on unit circle is $(0, 1)$)

3. First, use the Pythagorean Theorem to find the hypotenuse, which I'll call c:

$$3^2 + 5^2 = c^2 \Rightarrow c = \sqrt{3^2 + 5^2} = \sqrt{34} \approx 5.83.$$

   Then $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \approx \frac{3}{5.83} = .5145$ and $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \approx \frac{5}{5.83} = .8575$.

4. a) From a calculator, $\theta = \sin^{-1}(-.265) \approx -15.37°$. Add $360°$ to this to get $344.63°$; take $180° -$ the calculator answer to get the second answer which is $196.37°$.

   b) There are no such angles (because $2 > 1$).

   c) From a calculator, $\theta = \cos^{-1}(.43) \approx 64.5°$. A second angle which solves the equation is $360° - \theta = 295.5°$.

5. First, the third angle is $180° - 55° - 37° = 88°$. Then, by the SAS Area Formula, $A = \frac{1}{2}ab\sin C = \frac{1}{2}(12)(15)\sin 88° \approx 89.94$ sq units.

6. In this problem, we have a right triangle where the hypotenuse is 20 and the angle from the hypotenuse to the ground is $70°$. We are asked to find the opposite side, which I'll call b. By SOHCAHTOA, we get

$$\sin 70° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{20}.$$

Solve for b by cross-multiplying to get $b = 20 \sin 70° = 18.79$ ft.

7. First, the third angle is $B = 90° - 38° = 52°$.

Second, find side a using a trig function (either sine or cosine):

$$\begin{aligned} \sin 38° &= \frac{a}{43.5} \\ .6157 &= \frac{a}{4}3.5 \\ 26.78 &= a. \end{aligned}$$

Last, find the remaining side using the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (26.78)^2 + b^2 &= (43.5)^2 \\ b &= \sqrt{(43.5)^2 - (26.78)^2} = \sqrt{1175} \approx 34.27. \end{aligned}$$

8. The given information is SSS, so start with the Law of Cosines to find any angle (say K):

$$\begin{aligned} k^2 &= l^2 + m^2 - 2lm \cos K \\ 18^2 &= 13^2 + 10^2 - 2(13)(10) \cos K \\ 324 &= 169 + 100 - 260 \cos K \\ 324 &= 269 - 260 \cos K \\ 55 &= -260 \cos K \\ -.2115 &= \cos K \\ 102.2° &= K \end{aligned}$$

Now use the Law of Cosines again (or the Law of Sines) to find a second angle (say L):

$$\begin{aligned} l^2 &= k^2 + m^2 - 2km\cos L \\ 13^2 &= 18^2 + 10^2 - 2(18)(10)\cos L \\ 169 &= 324 + 100 - 360\cos L \\ 169 &= 424 - 360\cos L \\ -255 &= -360\cos L \\ .708 &= \cos L \\ 44.9^\circ &= L \end{aligned}$$

Last, find the third angle (for me,, this is M): $M = 180^\circ - 102.2^\circ - 44.9^\circ = 32.9^\circ$.

9. The given information is SAA/AAS, which requires the Law of Sines.

   First, the third angle is $C = 180^\circ - 70^\circ - 58^\circ = 52^\circ$.

   Second, find side length b using the Law of Sines:

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 70^\circ}{22} &= \frac{\sin 58^\circ}{b} \\ b\sin 70^\circ &= 22\sin 58^\circ \\ .939b &= 18.65 \\ b &= 19.85. \end{aligned}$$

   Last, find side length c using the Law of Sines (you could use the Law of Cosines):

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin C}{c} \\ \frac{\sin 70^\circ}{22} &= \frac{\sin 52^\circ}{c} \\ c\sin 70^\circ &= 22\sin 52^\circ \\ .939c &= 17.33 \\ c &= 18.45 \end{aligned}$$

3.3 Fall 2017 Exam 3

No calculator allowed on these problems

1. Throughout this problem, assume $\cot\theta = \frac{5}{3}$.

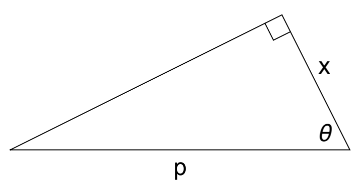
    a) What is $\cot(-\theta)$?
    b) What is $\tan\theta$?
    c) Which two quadrants might θ be in?
    d) If $\sin\theta < 0$, find $\cos\theta$.

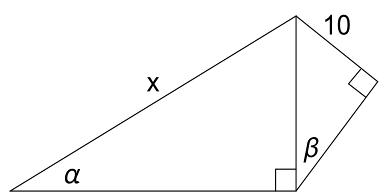
2. a) $\csc 135°$
    b) $\tan 60°$
    c) $\cot 180°$
    d) $\sec(-225°)$
    e) $\cot\frac{3\pi}{2}$
    f) $\sec\frac{\pi}{6}$
    g) $\tan\frac{3\pi}{4}$

3. Find the exact value of each quantity:

    a) $\sec^2\frac{4\pi}{3}$
    b) $8\sin\frac{\pi}{6}$
    c) $\sin^2 215° + \cos^2 215°$
    d) $\tan(2\cdot 90°)$
    e) $\cos 2\pi + 1$

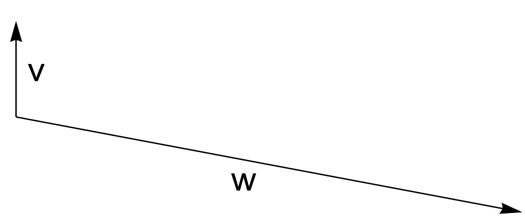
4. In each diagram below, write an equation for x in terms of the other given quantities in the picture. Your formula for x should not contain division in it.

a) 

b) 

5. Let **v** and **w** be the vectors indicated in the picture below.

    a) Sketch the vector $-\frac{1}{2}\mathbf{w}$ on the picture; label that vector “$-\frac{1}{2}\mathbf{w}$”.
    b) Sketch the vector $2\mathbf{v}+\mathbf{w}$ on the picture; label that vector “$2\mathbf{v}+\mathbf{w}$”.



Calculators allowed on the rest of the exam

6. Suppose that θ is some angle such that when drawn in standard position, $(-7, 12)$ is on the terminal side of θ. Find $\sec\theta$ and $\tan\theta$.

7. Use a calculator to compute decimal approximations of these quantities:

   a) $\csc 23°$

   b) $\tan 140° + \cot 140°$

   c) $\cos^2 213°$

8. a) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\tan\theta = 4.125$.

   b) Find <u>all</u> angles θ between $0°$ and $360°$ such that $\sec\theta = 3.38$.

9. a) Suppose $\mathbf{u} = \langle -3, 8\rangle$ and $\mathbf{v} = \langle 4, 5\rangle$. Find $3\mathbf{u} + \mathbf{v}$.

   b) Find $\mathbf{u} \cdot \mathbf{v}$, where $\mathbf{u}$ and $\mathbf{v}$ are as in part (a).

   c) Suppose $\mathbf{w}$ is a vector with magnitude 7 and direction angle $285°$. Find the components of $\mathbf{w}$.

   d) Find the magnitude of the vector $\mathbf{z} = \langle 6.3, 5.2\rangle$.

10. Nemo the fish swims away from his home on a trajectory $50°$ north of east for 2 miles, then changes course and swims due east for 1 mile. After that, Nemo changes course again and swims for 3 miles on a trajectory $35°$ south of east. At this point, what distance would Nemo have to swim if he wanted to swim directly home?

Solutions

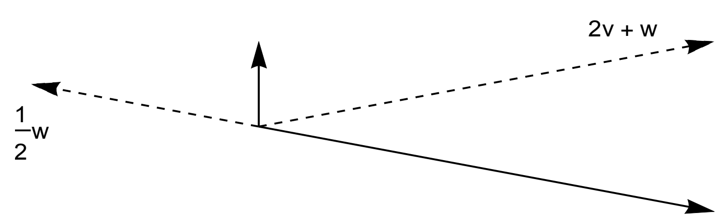
1. a) $\cot(-\theta) = -\cot\theta = -\frac{5}{3}$.
   b) $\tan\theta = \frac{1}{\cot\theta} = \frac{3}{5}$.
   c) $\cot\theta$ is positive in Quadrants I and III.
   d) Since $\cot\theta > 0$ and $\sin\theta < 0$, θ is in Quadrant III, so $\cos\theta$ will be negative. Now draw a triangle and label the adjacent side as 5 and the opposite side as 3. Solve for the hypotenuse using the Pythagorean Theorem to get $\sqrt{5^2+3^2} = \sqrt{34}$. Finally, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{-5}{\sqrt{34}}$.

2. a) $\csc 135° = \sqrt{2}$.
   b) $\tan 60° = \sqrt{3}$.
   c) $\cot 180°$ DNE.
   d) $\sec(-225°) = \sec 225° = -\sqrt{2}$.
   e) $\cot\frac{3\pi}{2} = 0$.
   f) $\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$.
   g) $\tan\frac{3\pi}{4} = -1$.

3. a) $\sec^2\frac{4\pi}{3} = (-2)^2 = 4$.
   b) $8\sin\frac{\pi}{6} = 8\cdot\frac{1}{2} = 4$.
   c) $\sin^2 215° + \cos^2 215° = 1$.
   d) $\tan(2\cdot 90°) = \tan 180° = 0$.
   e) $\cos 2\pi + 1 = 1 + 1 = 2$.

4. a) $\frac{x}{p} = \frac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta$, so $x = p\cos\theta$.
   b) Label the vertical segment common to both triangles as w. Then $w = 10\csc\beta$ and $x = w\csc\alpha$, so by substitution $x = 10\csc\beta\csc\alpha$.

5. To get $-\frac{1}{2}\mathbf{w}$, sketch a vector in the opposite direction as $\mathbf{w}$, but half as long. To get $2\mathbf{v}+\mathbf{w}$, first draw $2\mathbf{v}$ (twice as long as $\mathbf{v}$, in the same direction) and then add by drawing a parallelogram. You end up with the following:



6. We have $x = -7$ and $y = 12$ so $r = \sqrt{x^2 + y^2} = \sqrt{(-7)^2 + 12^2} = \sqrt{49 + 144} = \sqrt{193}$. Therefore

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{193}}{-7} \approx -1.984 \qquad \text{and} \qquad \tan\theta = \frac{y}{x} = \frac{12}{-7} \approx -1.714.$$

7. a) $\csc 23° = \frac{1}{\sin 23°} = 2.5593$.
   b) $\tan 140° + \cot 140° = -.8391 + (-1.1917) = -2.0308$.
   c) $\cos^2 213° = (-.8386)^2 = -.7033$.

8. a) $\theta = \tan^{-1} 4.125 = 76.37°$. The other angle is $180° + \theta = 256.37°$.
   b) $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{3.38} = .2958$. Therefore $\theta = \cos^{-1} .2958 = 72.79°$. The other angle is $360° - \theta = 278.21°$.

9. a) $3\mathbf{u} + \mathbf{v} = \langle -9, 24\rangle + \langle 4, 5\rangle = \langle -5, 29\rangle$.
   b) $\mathbf{u} \cdot \mathbf{v} = (-3)4 + 8(5) = 28$.
   c) $\mathbf{w} = \langle 7\cos 285°, 7\sin 285°\rangle = \langle 1.811, -6.761\rangle$.
   d) $|\mathbf{z}| = \sqrt{6.3^2 + 5.2^2} = \sqrt{66.73} = 8.16884$.

10. Nemo's journey can be described by three vectors: the first part of his trip is

$$\mathbf{a} = \langle 2\cos 50°, 2\sin 50°\rangle = \langle 1.285, 1.532\rangle\,;$$

the second part of his trip is

$$\mathbf{b} = \langle 1\cos 0°, 1\sin 0°\rangle = \langle 1, 0\rangle\,;$$

the last part of his trip is

$$\mathbf{c} = \langle 3\cos 325°, 3\sin 325°\rangle = \langle 2.457, -1.72\rangle\,.$$

(The angle is $360° - 35° = 325°$ because the trajectory is <u>south</u> of east.) That means Nemo's eventual position is

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \langle 1.285 + 1 + 2.457, 1.532 + 0 - 1.72\rangle = \langle 4.742, -.188\rangle\,.$$

The magnitude of this vector, which is the distance from Nemo to his home, is

$$\sqrt{4.742^2 + (-.188)^2} = \sqrt{22.5219} \approx 4.745 \text{ miles.}$$

3.4 Fall 2017 Final Exam

No calculator allowed on these problems

1. Find the exact value of each quantity:

   a) $\cos 30^\circ$
   b) $\tan 90^\circ$
   c) $\sin 45^\circ$
   d) $\csc 90^\circ$
   e) $\sin 0^\circ$

2. Find the exact value of each quantity:

   a) $\tan 150^\circ$
   b) $\cos(-225^\circ)$
   c) $\sin 330^\circ$
   d) $\cot 180^\circ$
   e) $\sec 780^\circ$

3. Find the exact value of each quantity:

   a) $\cos \frac{4\pi}{3}$
   b) $\sin \frac{-2\pi}{3}$
   c) $\tan \frac{3\pi}{4}$
   d) $\cot \frac{-\pi}{6}$
   e) $\csc \frac{5\pi}{2}$

4. Find the exact value of each quantity:

   a) $\sin^2 310^\circ + \cos^2 310^\circ$
   b) $\cos 400^\circ - \cos 40^\circ$
   c) $\tan^2 85^\circ - \sec^2 85^\circ$
   d) $\sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ$
   e) $\cos 75^\circ$

5. Suppose $\tan\theta = \frac{2}{3}$ and $\sin\theta < 0$. Find the exact values of all six trig functions of θ.

6. Suppose $\sin\theta = \frac{1}{5}$ and $\cos\theta > 0$.

   a) Find $\csc\theta$.
   b) What quadrant is θ in?
   c) Find $\sin(\theta - 360^\circ)$.
   d) Find $\sin(-\theta)$.
   e) Find $\sin 2\theta$.
   f) Find $\csc 2\theta$.

7. Sketch a crude graph of each function, labelling the graph appropriately:

   a) $y = \sin x - 4$
   b) $y = 3\cos x$
   c) $y = \cot x$
   d) $y = \sin 2x + 1$

8. Simplify the following expression using trig identities:

$$\frac{1 - \sin^2\theta}{1 - \csc^2\theta}$$

Calculators allowed on the rest of the exam

9. Evaluate the following expressions using a calculator. Your answers can (and should) be written as decimals.

   a) $\sin 103°$
   b) $5\cos(-51°)$
   c) $\cos 133° - \cos 75°$
   d) $\csc(52° + 81°)$
   e) $\sin^2 231°$
   f) $\tan 40° \cot 108°$
   g) $2\sec 4 \cdot 83°$

10. For each equation, find (decimal approximations of) **all** angles θ between $0°$ and $360°$ satisfying the following equations. If the equation has no solution, say so.

    a) $\sin\theta = .683$
    b) $\cos\theta = .23$
    c) $\tan\theta = -3.21$
    d) $\sec\theta = 0$
    e) $\cos\theta = -.735$
    f) $\sin\theta = 1.04$

11. Find the six trig functions of the angle θ, if the point $(-2.7, 5.9)$ is on the terminal side of θ when it is drawn in standard position.

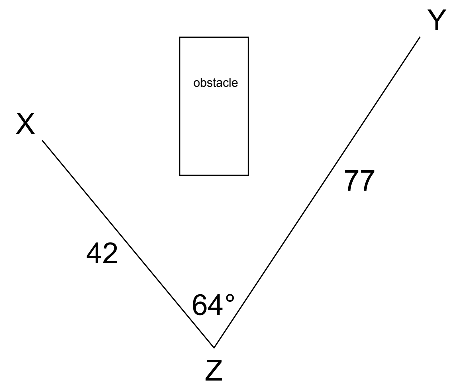
12. Let $\mathbf{v} = \langle 8, 1\rangle$ and let $\mathbf{w} = \langle 3, 7\rangle$.

    a) Compute the norm of $\mathbf{v}$.
    b) Compute $\mathbf{v} \cdot \mathbf{w}$.
    c) Find the measure of the angle between $\mathbf{v}$ and $\mathbf{w}$.

13. A chipmunk located 42 feet from a flagpole has to look up at an angle of $50°$ to see the top of the flagpole. How tall is the flagpole? (Assume the chipmunk's eyes are at ground level.)

14. A surveyor is trying to determine the distance between points X and Y as shown below, but can't measure the distance directly because there is an obstacle in the way. So instead, the surveyor measures the distances from X and Y to a third point Z, and measures an angle at point Z as shown in the

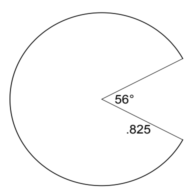
figure below. Find the distance from point X to point Y.

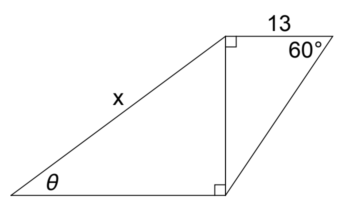


15. Solve triangle PQR, where $\angle P = 75°$, $\angle Q = 58°$ and $p = 53$.

16. Choose any three of the following five questions.

    a) The video game Pac-Man features a character that looks like the left-hand figure below. Find the area enclosed by Pac-Man.





    b) Write an expression for x in terms of the other quantities given in the right-hand figure above. Simplify your answer.

    c) A bicycle wheel of radius 16.5 inches rotates at a speed of 35 revolutions per minute. What is the linear velocity of a point on the edge of the wheel?

    d) Find the area of triangle ABC, if $a = 6$, $b = 4$ and $c = 5$.

    e) A bird leaves its nest and flies due west for 9 miles, then flies at an angle $57°$ north of east for 7 miles. How far from its nest is it?

Solutions

1. a) $\cos 30° = \frac{\sqrt{3}}{2}$.
   b) $\tan 90° = \frac{1}{0}$ which DNE.
   c) $\sin 45° = \frac{\sqrt{2}}{2}$.
   d) $\csc 90° = \frac{1}{1} = 1$.
   e) $\sin 0° = 0$.

2. a) $\tan 150° = \frac{-\sqrt{3}}{3}$ (quadrant II, reference angle $30°$)
   b) $\cos(-225°) = \frac{-\sqrt{2}}{2}$ (quadrant II, reference angle $45°$)
   c) $\sin 330° = \frac{-1}{2}$ (quadrant IV, reference angle $30°$)
   d) $\cot 180° = \frac{-1}{0}$ which DNE.
   e) $\sec 780° = 2$ (quadrant I, reference angle $60°$)

3. a) $\cos \frac{4\pi}{3} = \frac{-1}{2}$ (quadrant III, reference angle $60°$)
   b) $\sin \frac{-2\pi}{3} = \frac{-\sqrt{3}}{2}$ (quadrant III, reference angle $60°$)
   c) $\tan \frac{3\pi}{4} = -1$ (quadrant II, reference angle $45°$)
   d) $\cot \frac{-\pi}{6} = -\sqrt{3}$ (quadrant IV, reference angle $30°$)
   e) $\csc \frac{5\pi}{2} = \csc 90° = \frac{1}{1} = 1$.

4. a) $\sin^2 310° + \cos^2 310° = 1$ (by the Pythagorean identity)
   b) $\cos 400° - \cos 40° = \cos 40° - \cos 40°$ (by periodicity) which simplifies to 0.
   c) $\tan^2 85° - \sec^2 85° = -1$ (by a rewritten version of $\tan^2\theta + 1 = \sec^2\theta$)
   d) Use the addition identity for sine to get $\sin 20° \cos 40° + \cos 20° \sin 40° = \sin(20° + 40°) = \sin 60° = \frac{\sqrt{3}}{2}$
   e) Use the addition identity for cosine to get $\cos 75° = \cos(30° + 45°) = \cos 30° \cos 45° - \sin 30° \sin 45° = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$.

5. Draw a triangle, labelling the opposite side 2 and the adjacent side 3. Solve for the hypotenuse using the Pythagorean Theorem, obtaining $\sqrt{13}$. Since $\tan\theta > 0$ and $\sin\theta < 0$, the angle is in Quadrant III, so all trig functions other than tangent and cotangent are negative. Using the right triangle definitions of the trig functions, we get

$$\sin\theta = -\frac{2}{\sqrt{13}} \qquad \cos\theta = -\frac{3}{\sqrt{13}} \qquad \tan\theta = \frac{2}{3}$$

$$\csc\theta = -\frac{\sqrt{13}}{2} \qquad \sec\theta = -\frac{\sqrt{13}}{3} \qquad \cot\theta = \frac{3}{2}$$

6. a) $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{1/5} = 5$.

   b) Since $\sin\theta$ and $\cos\theta$ are positive, θ is in Quadrant I.

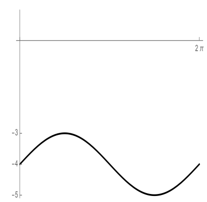
   c) $\sin(\theta - 360°) - \sin\theta = \frac{1}{5}$.

   d) $\sin(-\theta) = -\sin\theta = \frac{-1}{5}$.

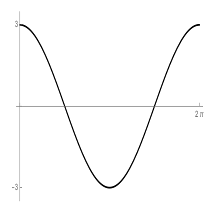
   e) First, find $\cos\theta$: draw a triangle, label the opposite side 1 and the hypotenuse 5, and solve for the adjacent side using the Pythagorean Theorem to get $\sqrt{24}$. Since the angle is in Quadrant I, $\cos\theta = \frac{\sqrt{24}}{5}$. Next, $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{1}{5}\right)\left(\frac{\sqrt{24}}{5}\right) = \frac{2\sqrt{24}}{25}$.

   f) $\csc 2\theta = \frac{1}{\sin 2\theta} = \frac{1}{2\sqrt{24}/25} = \frac{25}{2\sqrt{24}}$.

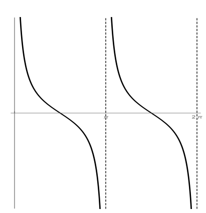
7. a) $y = \sin x - 4$ is the graph of $y = \sin x$, shifted down 4 units:



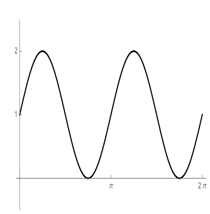
   b) $y = 3\cos x$ is the graph of $y = \cos x$, stretched vertically by a factor of 3:



   c) $y = \cot x$ looks like this:



   d) $y = \sin 2x + 1$ has period $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$, and is shifted up 1 unit:



8. Use Pythagorean identities, then a quotient identity, then divide the fractions:

$$\begin{aligned}\frac{1-\sin^2\theta}{1-\csc^2\theta} &= \frac{\cos^2\theta}{-\cot^2\theta}\\ &= \frac{\cos^2\theta}{\frac{-\cos^2\theta}{\sin^2\theta}}\\ &= \frac{\cos^2\theta}{1}\cdot\frac{\sin^2\theta}{-\cos^2\theta}\\ &= \frac{\sin^2\theta}{-1}\\ &= -\sin^2\theta.\end{aligned}$$

9. a) $\sin 103° = .97437$.
   b) $5\cos(-51°) = 5(.62932) = 3.1466$.
   c) $\cos 133° - \cos 75° = -.681998 - .258819 = -.940817$.
   d) $\csc(52° + 81°) = \csc 133° = \frac{1}{\sin 133°} = \frac{1}{.731354} = 1.36733$.
   e) $\sin^2 231° = (\sin 231°)^2 = (-.777146)^2 = .603956$.
   f) $\tan 40° \cot 108° = (.8391)\left(\frac{1}{\tan 108°}\right) = (.8391)\left(\frac{1}{-3.07768}\right) = -.27264$.
   g) $2\sec 4\cdot 83° = 2\left(\sec(332°)\right) = 2\left(\frac{1}{\cos 332°}\right) = 2\left(\frac{1}{.882948}\right) = 2.26514$.

10. a) $\theta = \sin^{-1} .683 = 43.1°$; the other angle is $180° - \theta = 136.9°$.
    b) $\theta = \cos^{-1} .23 = 76.7°$; the other angle is $360° - \theta = 283.3°$.
    c) $\theta = \tan^{-1} -3.21 = -72.7°$. Add $360°$ to get $287.3°$; the other angle is $\theta + 180° = 107.3°$.
    d) $\sec\theta = 0$ has solution because $-1 < 0 < 1$.
    e) $\theta = \cos^{-1} -.735 = 137.3°$; the other angle is $360° - \theta = 222.7°$.
    f) $\sin\theta = 1.04$ has no solution because $1.04 > 1$.

11. We are given $x = -2.7$, $y = 5.9$ and need to find r. By the Pythagorean Theorem, $r = \sqrt{x^2+y^2} = \sqrt{(-2.7)^2 + 5.9^2} = 6.488$. Then:

$$\sin\theta = \frac{y}{r} = \frac{5.9}{6.488} = .909 \qquad \cos\theta = \frac{x}{r} = \frac{-2.7}{6.488} = -.416 \qquad \tan\theta = \frac{y}{x} = \frac{5.9}{-2.7} = -2.185$$

$$\csc\theta = \frac{r}{y} = \frac{6.488}{5.9} = 1.099 \qquad \sec\theta = \frac{r}{x} = \frac{6.488}{-2.7} = -2.402 \qquad \cot\theta = \frac{x}{y} = \frac{-2.7}{5.9} = -.416.$$

12. a) $||\mathbf{v}|| = \sqrt{8^2+1^2} = \sqrt{65}$.
    b) $\mathbf{v}\cdot\mathbf{w} = 8(3) + 1(7) = 24 + 7 = 31$.

c) Find the measure of the angle between $\mathbf{v}$ and $\mathbf{w}$.

Start with the formula $\mathbf{v} \cdot \mathbf{w} = ||\mathbf{v}||\,||\mathbf{w}|| \cos\theta$ and solve for θ. From parts (a) and (b), we know $\mathbf{v}\cdot\mathbf{w}$ and $||\mathbf{v}||$, so we need to find $||\mathbf{w}|| = \sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}$. Now from the formula, we have

$$\begin{aligned}
\mathbf{v} \cdot \mathbf{w} &= ||\mathbf{v}||\,||\mathbf{w}|| \cos\theta \\
31 &= \sqrt{65} \cdot \sqrt{58} \cos\theta \\
\frac{31}{\sqrt{65}\sqrt{58}} &= \cos\theta \\
.504883 &= \cos\theta \\
\cos^{-1}(.504883) &= \theta \\
59.67^\circ &= \theta.
\end{aligned}$$

13. Draw a right triangle, labelling the angle to the horizontal with 50°. We are given that the adjacent side is 42 and need to find the opposite side, so use tangent:

$$\tan 50^\circ = \frac{h}{42} \Rightarrow h = 42\tan 50^\circ = 50.05 \text{ feet.}$$

14. Use the Law of Cosines to find z, the distance between X and Y:

$$\begin{aligned}
z^2 &= x^2 + y^2 - 2xy\cos Z \\
z^2 &= 42^2 + 77^2 - 2(42)(77)\cos 64^\circ \\
z^2 &= 1764 + 5929 - 2(42)(77)(.438371) \\
z^2 &= 1764 + 5929 - 2835.38 \\
z^2 &= 4857.62 \\
z &= \sqrt{4857.62} = 69.6966.
\end{aligned}$$

15. First, the third angle is $180^\circ - 75^\circ - 58^\circ = 47^\circ$.

Next, use the Law of Sines to find q:

$$\begin{aligned}
\frac{\sin P}{p} &= \frac{\sin Q}{q} \\
\frac{\sin 75^\circ}{53} &= \frac{\sin 58^\circ}{q} \\
\frac{.965926}{53} &= \frac{.848048}{q} \\
.965926q &= 53(.84808) = 44.9465 \\
q &= \frac{44.9465}{.965926} = 46.5321.
\end{aligned}$$

Last, use the Law of Sines to find r:

$$\begin{aligned}
\frac{\sin P}{p} &= \frac{\sin R}{r} \\
\frac{\sin 75^\circ}{53} &= \frac{\sin 47^\circ}{r} \\
\frac{.965926}{53} &= \frac{.731354}{q} \\
.965926q &= 53(.731354) = 38.7617 \\
q &= \frac{38.7617}{.965926} = 40.1291.
\end{aligned}$$

16. a) The angle enclosed by Pac-Man is $360^\circ - 56^\circ = 304^\circ = 304^\circ \cdot \frac{\pi}{180^\circ} = 5.3058$ radians. Now by the area formula for sectors, $A = \frac{1}{2}r^2\theta = \frac{1}{2}(.825)^2(5.3058) = 1.80563$ square units.

    b) Let w be the vertical line segment; using the right-hand triangle we have $\frac{w}{13} = \tan 60^\circ = \sqrt{3}$, so $w = 13\sqrt{3}$. Now from the left-hand triangle, we have $\frac{x}{w} = \csc\theta$ so $x = w\csc\theta$. Substituting in for w, we get $x = 13\sqrt{3}\csc\theta$.

    c) $\omega = 35(2\pi) = 219.911$ radians per minute. Then the linear velocity is $v = r\omega = 16.5(219.911) = 3628.54$ inches per minute.

    d) The semiperimeter is $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(6+4+5) = 7.5$; then by Heron's formula the area is

$$\begin{aligned}
A &= \sqrt{s(s-a)(s-b)(s-c)} \\
&= \sqrt{7.5(7.5-6)(7.5-4)(7.5-5)} \\
&= \sqrt{98.4375} \\
&= 9.92157 \text{ square units.}
\end{aligned}$$

    e) The first part of the bird's journey is the vector $\mathbf{v} = \langle -9, 0\rangle$; the second part has angle 57° and magnitude 7 so its components are $\mathbf{w} = \langle 7\cos 57^\circ, 7\sin 57^\circ\rangle = \langle 3.81247, 5.87069\rangle$. Therefore the bird has travelled $\mathbf{v} + \mathbf{w} = \langle -5.18753, 5.87069\rangle$; the distance from its nest is $||\mathbf{v} + \mathbf{w}|| = \sqrt{(-5.18753)^2 + 5.87069^2} = \sqrt{61.3754} = 7.83425$ miles.