MATH 130
Exam 1 Study Guide

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Contents

Chapter 1

Exam 1 Information

1.1 Exam 1 content

Exam 1 covers Chapter 1 in the 2024 version of my MATH 130 lecture notes.

1.2 Tasks for Exam 1

NOTE: This guide is not meant to be an exact representation of exam material. I always reserve the right to ask some questions that use the course material in a creative way.

1. Answer questions involving course vocabulary
2. Classify statements as true or false
3. Solve simple inequalities
4. Perform arithmetic with fractions (add, subtract, multiply and divide them)
5. Evaluate expressions with exponents (including negative and rational exponents)
6. Rewrite and/or simplify expressions using exponent and/or radical rules
7. Distribute, FOIL and factor expressions
8. Compute the distance between two points in the coordinate plane

9. Convert between the graph of a circle, the equation of that circle, and the radius/center of the circle
10. Evaluate trigonometric functions of special angles

Chapter 2

Old MATH 130 Exam 1s

2.1 Spring 2024 Exam 1

1. (1.2) Rewrite each expression in the form $\square x^{\square}$, where the boxes are numbers:

   a) $\dfrac{10x^8}{20x^2}$

   b) $4\sqrt{4x^3}$

   c) $\dfrac{3}{5x^4}$

   d) $\dfrac{\sqrt{x}}{(x^3)^2}$

   e) $(5x^3)^2$

   f) $\sqrt{\sqrt{x}}$

2. Compute each quantity:

   a) (1.6) $\sin\dfrac{\pi}{4}$

   b) (1.6) $\cos\dfrac{4\pi}{3}$

   c) (1.6) $\tan\pi$

   d) (1.6) $\sin\dfrac{-3\pi}{2}$

   e) (1.6) $\sec\dfrac{5\pi}{6}$

   f) (1.3) $6\times 3\div 6\times 3$

   g) (1.2) $4^{18}4^{-15}$

   h) (1.2) $0^{1/5}$

   i) (1.2) $\dfrac{\sqrt{12}\sqrt{2}}{\sqrt{6}}$

   j) (1.2) $-81^{1/4}$

   k) (1.2) $5\div\dfrac{3}{4}$

   l) (1.2) $\left(\dfrac{4}{9}\right)^{-2}$

3. (1.4) Factor each expression completely:

   a) x^3-64x^2

   b) $x^2+4x-32$

   c) $2x^2-14x-88$

d) $4x^2 + 23x + 15$

4. (1.4) Perform the indicated operations and simplify completely:

   a) $3\sqrt{x}(\sqrt{x} + 1) - 5(x - 2)$

   b) $\left(\frac{1}{4}x + \frac{1}{3}\right)\left(\frac{1}{2}x - 1\right)$

5. a) (1.5) Sketch the graph of the circle whose equation is $(x-3)^2+(y+1)^2 = 4$.

   b) (1.5) Compute the distance between the points $(-2, 3)$ and $(1, 7)$; simplify your answer.

Solutions

1. a) $\dfrac{10x^8}{20x^2} = \dfrac{1}{2}x^{8-2} = \boxed{\dfrac{1}{2}x^6}$.

   b) $4\sqrt{4x^3} = 4\sqrt{4}\sqrt{x^3} = 4(2)x^{3/2} = \boxed{8x^{3/2}}$.

   c) $\dfrac{3}{5x^4} = \boxed{\dfrac{3}{5}x^{-4}}$.

   d) $\dfrac{\sqrt{x}}{(x^3)^2} = \dfrac{x^{1/2}}{x^6} = x^{1/2-6} = \boxed{x^{-11/2}}$.

   e) $(5x^3)^2 = \boxed{25x^6}$.

   f) $\sqrt{\sqrt{x}} = \sqrt{x^{1/2}} = \left(x^{1/2}\right)^{1/2} = x^{(1/2)(1/2)} = \boxed{x^{1/4}}$.

2. a) $\sin\dfrac{\pi}{4} = \boxed{\dfrac{\sqrt{2}}{2}}$ (Quadrant I; ref. angle $45°$)

   b) $\cos\dfrac{4\pi}{3} = \boxed{-\dfrac{1}{2}}$ (Quadrant III; ref. angle $60°$)

   c) $\tan\pi = \boxed{0}$ (quadrantal; point $(-1, 0)$ on the unit circle)

   d) $\sin\dfrac{-3\pi}{2} = \boxed{1}$ (quadrantal; point $(0, 1)$ on the unit circle)

   e) $\sec\dfrac{5\pi}{6} = \boxed{-\dfrac{2}{\sqrt{3}}}$ (Quadrant II; ref. angle $30°$)

   f) $6 \times 3 \div 6 \times 3 = 18 \div 6 \times 3 = 3 \times 3 = \boxed{9}$.

   g) $4^{18}4^{-15} = 4^{18-15} = 4^3 = \boxed{64}$.

   h) $0^{1/5} = \sqrt[5]{0} = \boxed{0}$.

   i) $\dfrac{\sqrt{12}\sqrt{2}}{\sqrt{6}} = \sqrt{\dfrac{12 \cdot 2}{6}} = \sqrt{4} = \boxed{2}$.

   j) $-81^{1/4} = -\sqrt[4]{81} = \boxed{-3}$.

   k) $5 \div \dfrac{3}{4} = 5 \cdot \dfrac{4}{3} = \boxed{\dfrac{20}{3}}$.

   l) $\left(\dfrac{4}{9}\right)^{-2} = \dfrac{1}{\left(\frac{4}{9}\right)^2} = \dfrac{1}{\frac{16}{81}} = \boxed{\dfrac{81}{16}}$.

3. a) $x^3 - 64x^2 = \boxed{x^2(x-64)}$

   b) $x^2 + 4x - 32 = \boxed{(x+8)(x-4)}$

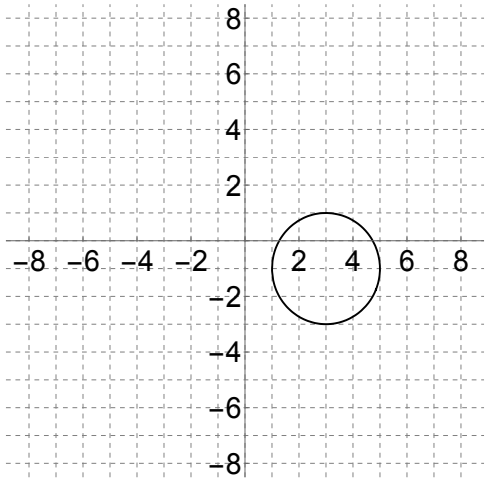
c) $2x^2 - 14x - 88 = 2(x^2 - 7x + 44) = \boxed{2(x-11)(x+4)}$

d) $4x^2 + 23x + 15 = \boxed{(4x+3)(x+5)}$

4. a) $3\sqrt{x}(\sqrt{x}+1) - 5(x-2) = 3x + 3\sqrt{x} - 5x + 10 = \boxed{3\sqrt{x} - 2x + 10}$

b) $\left(\frac{1}{4}x + \frac{1}{3}\right)\left(\frac{1}{2}x - 1\right) = \frac{1}{8}x^2 - \frac{1}{4}x + \frac{1}{6}x - \frac{1}{3} = \frac{1}{8}x^2 - \frac{3}{12}x + \frac{2}{12}x - \frac{1}{3} = \boxed{\frac{1}{8}x^2 - \frac{1}{12}x - \frac{1}{3}}$.

5. a) The circle has center $(3, -1)$ and radius $\sqrt{4} = 2$, so it looks like this:



b) Use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1 - (-2))^2 + (7 - 3)^2}$$
$$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = \boxed{5}.$$

2.2 Relevant exam questions from Spring 2018

1. Rewrite each expression in the form $\square^{\square}$, where your answer contains no radicals nor fractions in the base of any exponent:

   a) $\dfrac{1}{p^7}$ b) $\sqrt[3]{p^4}$ c) $x^2\sqrt{x}$ d) $\dfrac{1}{(x^3)^{-2/3}}$

2. Perform the indicated operations and simplify:

   a) $(2x-1)^2 - x(x+3)$

Solutions

1. a) $\dfrac{1}{p^7} = \boxed{p^{-7}}$.

   b) $\sqrt[3]{p^4} = \boxed{p^{4/3}}$.

   c) $x^2\sqrt{x} = x^2x^{1/2} = x^{2+1/2} = \boxed{x^{5/2}}$.

   d) $\dfrac{1}{(x^3)^{-2/3}} = \dfrac{1}{x^{3(-2/3)}} = \dfrac{1}{x^{-2}} = x^{-(-2)} = \boxed{x^2}$.

2. a) $(2x-1)^2 - x(x+3) = (2x-1)(2x-1) - x(x+3) = 4x^2 - 2x - 2x + 1 - (x^2+3x) = 4x^2 - 4x + 1 - x^2 - 3x = \boxed{3x^2 - 7x + 1}$.

Chapter 3

Additional Practice Exam 1s

3.1 Practice Exam A

A1. Compute each quantity

a) $\dfrac{\frac{2}{3}}{7}$

b) $\sqrt{-16}$

c) 5^{-3}

d) $\left(\sqrt{6}\right)^4$

e) $\sin\dfrac{\pi}{3}$

f) $\cos\dfrac{3\pi}{4}$

g) $\tan\dfrac{-5\pi}{6}$

h) $\cos\dfrac{3\pi}{2}$

A2. Rewrite each equation so that the left-hand side is zero, and then simplify the left-hand side:

a) $3x^2 + 4x = x^2 - 2x - 7$

b) $4(x^3 - 2xy) = 5yx + 2$

A3. Rewrite each expression in the form $\square x^{\square}$, where the boxes are numbers:

a) $2(x^5)^3$

b) $(2x^5)^3$

c) $x^5(2x)^3$

d) $x^5 2x^3$

e) $\sqrt[5]{x^2}$

f) $\dfrac{-2}{3x\sqrt[3]{x}}$

A4. Perform the indicated operations and simplify completely:

a) $(x-2)^2$ b) $(2x-3)(x+4)-(x+2)x$

A5. Factor each expression completely:

a) $x^4-4x^3-32x^2$ c) $10x^2-70$
b) $6x^2+18x-108$ d) $6x^2-x-15$

A6. Solve each inequality for x:

a) $8-3x>-7$ b) $2x\geq 6$

A7. Compute the distance between the points $(3,-2)$ and $(6,-1)$.

A8. Find the center and radius of the circle $x^2+(y-5)^2=14$.

Practice Exam A Solutions

A1. a) $\dfrac{\frac{2}{3}}{7}=\dfrac{2}{3}\div 7=\dfrac{2}{3}\cdot\dfrac{1}{7}=\boxed{\dfrac{2}{21}}$.

b) $\sqrt{-16}$ $\boxed{\text{DNE}}$.

c) $5^{-3}=\dfrac{1}{5^3}=\boxed{\dfrac{1}{125}}$.

d) $\left(\sqrt{6}\right)^4=\left(6^{1/2}\right)^4=6^{(1/2)\cdot 4}=6^2=\boxed{36}$.

e) $\sin\dfrac{\pi}{3}=\boxed{\dfrac{\sqrt{3}}{2}}$ (Quadrant I, ref. angle $60°$).

f) $\cos\dfrac{3\pi}{4}=\boxed{-\dfrac{\sqrt{2}}{2}}$ (Quadrant II, ref. angle $45°$).

g) $\tan\dfrac{-5\pi}{6} = \boxed{\dfrac{1}{\sqrt{3}}}$ (Quadrant III, ref. angle $30°$).

h) $\cos\dfrac{3\pi}{2} = \boxed{0}$ (quadrantal; point on unit circle is $(0,-1)$).

A2. a) Subtract x^2 from both sides, add $2x$ to both sides and add 7 to both sides to get $\boxed{2x^2+6x+7=0}$.

b) Distribute on the left-hand side to get $4x^3-8xy = 5yx+2$. Then subtract $5yx+2$ from both sides to get $\boxed{4x^3-13xy-2=0}$.

A3. a) $2(x^5)^3 = \boxed{2x^{15}}$.

b) $(2x^5)^3 = 2^3(x^5)^3 = \boxed{8x^{15}}$.

c) $x^5(2x)^3 = x^5 8x^3 = \boxed{8x^8}$.

d) $x^5 2x^3 = \boxed{2x^8}$.

e) $\sqrt[5]{x^2} = \boxed{x^{2/5}}$.

f) $\dfrac{-2}{3x\sqrt[3]{x}} = -\dfrac{2}{3}\cdot\dfrac{1}{xx^{1/3}} = -\dfrac{2}{3}\cdot\dfrac{1}{x^{1+1/3}} = -\dfrac{2}{3}\cdot\dfrac{1}{x^{4/3}} = \boxed{-\dfrac{2}{3}x^{-4/3}}$.

A4. a) $(x-2)^2 = (x-2)(x-2) = x^2-2x-2x+4 = \boxed{x^2-4x+4}$.

b) $(2x-3)(x+4)-(x+2)x = 2x^2+8x-3x-12-x^2-2x = \boxed{x^2+3x-12}$

A5. a) $x^4-4x^3-32x^2 = x^2(x^2-4x-8) = \boxed{x^2(x-6)(x+2)}$.

b) $6x^2+18x-108 = 6(x^2+3x-18) = \boxed{6(x+6)(x-3)}$.

c) $10x^2-70 = 10(x^2-7) = \boxed{10(x-\sqrt{7})(x+\sqrt{7})}$.

d) $6x^2-x-15 = \boxed{(3x-5)(2x+3)}$.

A6. a) Subtract 8 from both sides to get $-3x > -15$. Divide both sides by 3 and flip the inequality sign to get $\boxed{x>5}$.

b) Divide both sides by 2 to get $\boxed{x \geq 3}$.

A7. $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-1-(-2))^2+(6-3)^2} = \sqrt{1^2+3^2} = \boxed{\sqrt{10}}$.

A8. The center is $\boxed{(0,5)}$; the radius is $\boxed{\sqrt{14}}$.

3.2 Practice Exam B

B1. Compute each quantity

a) $3 \cdot \dfrac{-4}{5}$

b) $\left(\dfrac{5}{3}\right)^{-2}$

c) $(-2)^4$

d) 2^{-4}

e) $10 \div 5 \cdot 2$

f) $\sqrt{12}\sqrt{3}$

g) $\dfrac{5^{13}}{(5^3)^4}$

h) $\sqrt[3]{64}$

i) $\cos(-\pi)$

j) $\tan\dfrac{\pi}{2}$

k) $\cos\dfrac{4\pi}{3}$

l) $\sin\dfrac{\pi}{4}$

B2. Rewrite each expression in the form $\square x^{\square}$, where the boxes are numbers:

a) $\dfrac{1}{x^6}$

b) $3\left(2x^3\right)^2$

c) $\left(2\sqrt[3]{x}\right)^4$

d) $4x\sqrt[3]{5x}$

B3. Perform the indicated operations and simplify completely:

a) $(\sqrt{x}+3)\left(x^{3/2}+4\right)$

b) $x^2(3x+4)-2x(4x-3)$

B4. Factor each expression completely:

a) x^8-4x^6

b) x^2+6x+9

c) $x^2-12x+27$

d) $28x^2-100x-48$

B5. Compute the distance between the points $(4,0)$ and (x,x^3).

B6. What is the center and radius of the circle $(x+4)^2+y^2=1$?

B7. Write the equation of the circle with center $(-5,3)$ that passes through the point $(-5,-1)$.

B8. Compute each quantity:

a) $\csc\dfrac{\pi}{6}$

b) $\cot\dfrac{-\pi}{2}$

c) $\sec\dfrac{2\pi}{3}$

d) $\cot\dfrac{3\pi}{4}$

Practice Exam B Solutions

B1. a) $3 \cdot \dfrac{-4}{5} = \dfrac{3}{1} \cdot \dfrac{-4}{5} = \boxed{\dfrac{-12}{5}}$.

b) $\left(\dfrac{5}{3}\right)^{-2} = \dfrac{1}{\left(\frac{5}{3}\right)^2} = \dfrac{1}{\frac{25}{9}} = \boxed{\dfrac{9}{25}}$.

c) $(-2)^4 = (-2)(-2)(-2)(-2) = \boxed{16}$.

d) $2^{-4} = \dfrac{1}{2^4} = \boxed{\dfrac{1}{16}}$.

e) $10 \div 5 \cdot 2 = 2 \cdot 2 = \boxed{4}$.

f) $\sqrt{12}\sqrt{3} = \sqrt{12 \cdot 3} = \sqrt{36} = \boxed{6}$.

g) $\dfrac{5^{13}}{(5^3)^4} = \dfrac{5^{13}}{5^{12}} = \boxed{5}$.

h) $\sqrt[3]{64} = \boxed{4}$.

i) $\cos(-\pi) = \boxed{-1}$ (quadrantal, point on unit circle is $(-1, 0)$).

j) $\tan \dfrac{\pi}{2}$ $\boxed{\text{DNE}}$ (quadrantal, point on unit circle is $(0, 1)$).

k) $\cos \dfrac{4\pi}{3} = \boxed{-\dfrac{1}{2}}$ (Quadrant III, ref. angle $60°$).

l) $\sin \dfrac{\pi}{4} = \boxed{\dfrac{\sqrt{2}}{2}}$ (Quadrant I, ref. angle $45°$).

B2. a) $\dfrac{1}{x^6} = \boxed{x^{-6}}$.

b) $3\left(2x^3\right)^2 = 3(4x^6) = \boxed{12x^6}$.

c) $\left(2\sqrt[3]{x}\right)^4 = 2^4(\sqrt[3]{x})^4 = \boxed{16x^{4/3}}$.

d) $4x\sqrt[3]{5x} = 4x\sqrt[3]{5}x^{1/3} = \boxed{4\sqrt[3]{5}\,x^{4/3}}$.

B3. a) $(\sqrt{x}+3)\left(x^{3/2}+4\right) = x^{1/2}x^{3/2} + 4\sqrt{x} + 3x^{3/2} + 12 = \boxed{x^2 + 4\sqrt{x} + 3x^{3/2} + 12}$.

b) $x^2(3x+4) - 2x(4x-3) = 3x^3 + 4x^2 - 8x^2 + 6x = \boxed{3x^3 - 4x^2 + 6x}$.

B4. a) $x^8 - 4x^6 = x^6(x^2-4) = \boxed{x^6(x-2)(x+2)}$.

b) $x^2 + 6x + 9 = \boxed{(x+3)(x+3)} = \boxed{(x+3)^2}$.

c) $x^2 - 12x + 27 = \boxed{(x-9)(x-3)}$.

d) $28x^2 - 100x - 48 = 4(7x^2 - 25x - 12) = \boxed{4(7x+3)(x-4)}$.

B5. $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(x^3-0)^2 + (x-4)^2} = \boxed{\sqrt{x^6 + (x-4)^2}}$.

B6. The center is $\boxed{(-4, 0)}$; the radius is $\boxed{1}$.

B7. The radius of the circle is $r = 3-(-1) = 4$, so the equation of the circle is $\boxed{(x+5)^2 + (y-3)^2 = 16}$.

B8. a) $\csc \dfrac{\pi}{6} = \boxed{2}$ (Quadrant I, ref. angle $30°$).

b) $\cot \dfrac{-\pi}{2} = \boxed{0}$ (quadrantal, point $(0, -1)$ on unit circle).

c) $\sec \dfrac{2\pi}{3} = \boxed{-\dfrac{1}{2}}$ (Quadrant II, ref. angle $60°$).

d) $\cot \dfrac{3\pi}{4} = \boxed{-1}$ (Quadrant II, ref. angle $45°$).

3.3 Practice Exam C

C1. Consider the equation $3x^2y + 5x - 3 + 4xy^2 = 2yx^2 - 7y + 4x - 3x^2y$

a) Which side of the equation contains like terms?

b) What is the constant term on the left-hand side?

c) What is the coefficient on the third term on the right-hand side?

C2. Compute each quantity

a) $\dfrac{6}{\frac{1}{2} + \frac{1}{4}}$

b) $\left(\dfrac{1}{\sqrt{5}}\right)^{-2}$

c) $9^{2/3}9^{4/3}$

d) $\sqrt[3]{-8}$

e) $\sqrt[6]{0}$

f) 0^{-3}

g) $\sin\dfrac{5\pi}{6}$

h) $\tan\dfrac{\pi}{4}$

i) $\cos\dfrac{-2\pi}{3}$

j) $\sin\dfrac{5\pi}{2}$

C3. Rewrite each expression in the form $\square x^{\square}$, where the boxes are numbers:

a) $\dfrac{1}{5x^2}$ b) $\dfrac{(4x^3)^2}{2x^4}$ c) $x\sqrt[4]{x}$ d) $\left(\dfrac{1}{\sqrt{x}}\right)^6$

C4. Write each expression using radical signs, so that it contains no fractional or negative exponents:

a) $t^{3/4}$

b) $19^{2/3}19^{1/2}$

c) $\dfrac{x^{-1}}{\sqrt{x}}$

d) $40^{-2/3}$

C5. Perform the indicated operations and simplify completely:

a) $(x-3)(3x+6) + (2x-5)4$ b) $8(x^2+2) - 4(x-5) + 3(x^2+5x)$

C6. Factor each expression completely:

a) $x^2 - 8x + 15$

b) $2x^3 + 4x^2 - 70x$

c) $4x^2 + 16x + 15$

d) $2x^2 - 24$

C7. Plot the points $(2, -4)$ and $(0, 6)$ on the same coordinate axes.

C8. Write the equation of the circle with center $(8, 0)$ with radius 5.

Practice Exam C Solutions

C1. a) The $\boxed{\text{right-hand side}}$ has like terms $2yx^2$ and $-3x^2y$.

b) -3

c) 4

C2. a) $\dfrac{6}{\frac{1}{2}+\frac{1}{4}} = \dfrac{6}{\frac{2}{4}+\frac{1}{4}} = \dfrac{6}{\frac{3}{4}} = 6 \cdot 43 = \boxed{8}$.

b) $\left(\dfrac{1}{\sqrt{5}}\right)^{-2} = \dfrac{1}{\left(\frac{1}{\sqrt{5}}\right)^2} = \dfrac{1}{\frac{1}{5}} = \boxed{5}$.

c) $9^{2/3}9^{4/3} = 9^{2/3+4/3} = 9^2 = \boxed{81}$.

d) $\sqrt[3]{-8} = \boxed{-2}$.

e) $\sqrt[6]{0} = \boxed{0}$.

f) $0^{-3} = \dfrac{1}{0^3} = \dfrac{1}{0}$ which $\boxed{\text{DNE}}$.

g) $\sin\dfrac{5\pi}{6} = \boxed{\dfrac{1}{2}}$ (Quadrant II, ref. angle $30°$).

h) $\tan\dfrac{\pi}{4} = \boxed{1}$ (Quadrant I, ref. angle $45°$).

i) $\cos\dfrac{-2\pi}{3} = \boxed{-\dfrac{1}{2}}$ (Quadrant III, ref. angle $60°$).

j) $\sin\dfrac{5\pi}{2} = \boxed{1}$ (quadrantal, point $(0,1)$ on the unit circle).

C3. a) $\dfrac{1}{5x^2} = \boxed{\dfrac{1}{5}x^{-2}}$.

b) $\dfrac{(4x^3)^2}{2x^4} = \dfrac{16x^6}{2x^4} = \boxed{8x^2}$.

c) $x\sqrt[4]{x} = \boxed{x^{1/4}}$.

d) $\left(\dfrac{1}{\sqrt{x}}\right)^6 = (x^{-1/2})^6 = x^{(-1/2)\cdot 6} = \boxed{x^{-3}}$.

C4. a) $t^{3/4} = \boxed{\sqrt[4]{t^3}} = \boxed{\left(\sqrt[4]{t}\right)^3}$.

b) $19^{2/3}19^{1/2} = 19^{2/3+1/2} = 19^{4/6+3/6} = 19^{7/6} = \boxed{\sqrt[6]{19^7}} = \boxed{\left(\sqrt[6]{19}\right)^7}$.

c) $\dfrac{x^{-1}}{\sqrt{x}} = x^{-1-1/2} = x^{-3/2} = \dfrac{1}{x^{3/2}} = \boxed{\dfrac{1}{\sqrt[2]{x^3}}}$ (there are other ways to write this).

d) $40^{-2/3} = \boxed{\dfrac{1}{\sqrt[3]{40^2}}}$ (there are other ways to write this).

C5. a) $(x-3)(3x+6)+(2x-5)4 = 3x^2+6x-9x-18+8x-20 = \boxed{3x^2+5x-38}$.

b) $8(x^2+2)-4(x-5)+3(x^2+5x) = 8x^2+16-4x+20+3x^2+15x = \boxed{11x^2+11x+36}$.

C6. a) $x^2-8x+15 = \boxed{(x-3)(x-5)}$.

b) $2x^3 + 4x^2 - 70x = 2x(x^2 + 2x - 35) = \boxed{2x(x+7)(x-5)}$.

c) $4x^2 + 16x + 15 = \boxed{(2x+3)(2x+5)}$.

d) $2x^2 - 24 = 2(x^2 - 12) = \boxed{2(x - \sqrt{12})(x + \sqrt{12})}$.

C7. $(2, -4)$ is 2 units left and 4 down from the origin; $(0, 6)$ is 6 units up from the origin.

C8. $(x - 8)^2 + y^2 = 25$.

3.4 Practice Exam D

D1. Compute each quantity

a) $-4 \div \dfrac{-3}{-2}$

b) $10(5 - 3 \cdot 2)$

c) $4^{5/2}$

d) 4^{-3}

e) $\left(\dfrac{4}{9}\right)^{-1/2}$

f) $2\sqrt[3]{125}$

g) $\sin\left(-\dfrac{5\pi}{6}\right)$

h) $\cos\dfrac{-3\pi}{4}$

i) $\tan 4\pi$

j) $\sin\dfrac{3\pi}{2}$

D2. Rewrite each expression in the form $\square x^{\square}$, where the boxes are numbers:

a) $x\sqrt[3]{x}\sqrt[3]{x}$

b) $\dfrac{3}{x^4}$

c) $\dfrac{4}{x} \cdot \dfrac{3}{2x} \cdot \dfrac{12}{5x}$

d) $-(x^3)^4$

D3. Classify each statement as true or false (remember that in mathematics, for a statement to be true it must always be true):

a) $(xy)^3 = x^3y^3$

b) $(x + y)^3 = x^3 + y^3$

c) $\sqrt{x - y} = \sqrt{x} - \sqrt{y}$

d) $\dfrac{x^8}{x^7} = x$

e) $|x + y| = |x| + |y|$

f) $7 + 5\sqrt{x} = 12\sqrt{x}$

g) $\sqrt{5x} = \sqrt{5}\sqrt{x}$

D4. Perform the indicated operations and simplify completely:

a) $\sqrt{2x}\left(\sqrt{3x} + 4\sqrt{8}\right)$

b) $(3x - 2)(3y + 10)$

D5. Factor each expression completely:

a) $8x^5 - 88x^4 + 80x^3$

b) $4x^2 - 121$

c) $10x^2 - 3x - 1$

d) $x^2 + 9x + 18$

D6. Compute the distance between the points $(3, -4)$ and $(0, 8)$.

D7. Sketch the graph of the circle $(x - 2)^2 + (y + 3)^2 = 9$.

D8. Compute each quantity:

a) $\csc\dfrac{\pi}{4}$

b) $\sec\dfrac{\pi}{2}$

c) $\cot\dfrac{5\pi}{6}$

Practice Exam D Solutions

D1. a) $-4 \div \dfrac{-3}{-2} = -4 \cdot \dfrac{-2}{-3} = \boxed{-\dfrac{8}{3}}$.

b) $10(5 - 3 \cdot 2) = 10(5 - 6) = 10(-1) = \boxed{-10}$.

c) $4^{5/2} = \left(\sqrt{4}\right)^5 = 2^5 = 2(2)2(2)2 = \boxed{32}$.

d) $4^{-3} = \dfrac{1}{4^3} = \boxed{\dfrac{1}{64}}$.

e) $\left(\dfrac{4}{9}\right)^{-1/2} = \dfrac{1}{\sqrt{\frac{4}{9}}} = \dfrac{1}{\frac{2}{3}} = \boxed{\dfrac{3}{2}}$.

f) $2\sqrt[3]{125} = 2(5) = \boxed{10}$.

g) $\sin\left(-\dfrac{5\pi}{6}\right) = \boxed{-\dfrac{1}{2}}$ (Quadrant III, ref. angle $30°$).

h) $\cos\dfrac{-3\pi}{4} = \boxed{-\dfrac{\sqrt{2}}{2}}$ (Quadrant III, ref. angle $45°$).

i) $\tan 4\pi = \boxed{0}$ (quadrantal, point $(1, 0)$ on the unit circle).

j) $\sin\dfrac{3\pi}{2} = \boxed{-1}$ (quadrantal, point $(0, -1)$ on the unit circle).

D2. a) $x\sqrt[3]{x}\sqrt[3]{x} = xx^{1/3}x^{1/3} = x^{1+1/3+1/3} = \boxed{x^{5/3}}$.

b) $\dfrac{3}{x^4} = \boxed{3x^{-4}}$.

c) $\dfrac{4}{x} \cdot \dfrac{3}{2x} \cdot \dfrac{12}{5x} = \dfrac{72}{5} \cdot \dfrac{1}{x^3} = \boxed{\dfrac{72}{5}x^{-3}}$.

d) $-(x^3)^4 = \boxed{-x^{12}}$.

D3. a) true b) false c) false d) true e) false f) false g) true

D4. a) $\sqrt{2x}\left(\sqrt{3x} + 4\sqrt{8}\right) = \sqrt{6}\,x + 4\sqrt{16}\sqrt{x} = \boxed{\sqrt{6}\,x + 16\sqrt{x}}$.

b) $(3x - 2)(3y + 10) = \boxed{9xy + 30x - 6y - 20}$.

D5. a) $8x^5 - 88x^4 + 80x^3 = 8x^3(x^2 - 11x + 10) = \boxed{8x^3(x - 10)(x - 1)}$.

b) $4x^2 - 121 = (2x)^2 - 11^2 = \boxed{(2x - 11)(2x + 11)}$.

c) $10x^2 - 3x - 1 = \boxed{(5x + 1)(2x - 1)}$.

d) $x^2 + 9x + 18 = \boxed{(x + 3)(x + 6)}$.

D6. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(8 - (-4))^2 + (0 - 3)^2} = \sqrt{12^2 + 3^2} = \boxed{\sqrt{153}}$.

D7. The center is $(2, -3)$ and the radius is $\sqrt{9} = 3$, so the circle should go through $(5, -3)$, $(2, 0)$, $(-1, -3)$ and $(2, -6)$.

D8. a) $\csc \dfrac{\pi}{4} = \boxed{\dfrac{2}{\sqrt{2}}} = \boxed{\sqrt{2}}$ (Quadrant I, ref. angle $45°$).

b) $\sec \dfrac{\pi}{2} = \dfrac{1}{0}$ $\boxed{\text{DNE}}$ (quadrantal, point $(0, 1)$ on the unit circle).

c) $\cot \dfrac{5\pi}{6} = \boxed{-\sqrt{3}}$ (Quadrant II, ref. angle $30°$).