MATH 130
Exam 3 Study Guide

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Contents

Chapter 1

Exam 3 Information

1.1 Exam 3 content

Exam 3 covers Chapter 3 in the 2024 version of my MATH 130 lecture notes.

NOTE: This guide is not meant to be an exact representation of exam material. I always reserve the right to ask some questions that use the course material in a creative way.

1.2 Tasks for Exam 3

1. Answer questions involving course vocabulary.
2. Classify statements as true or false.
3. Translate between the rule of the function and the graph for these types of functions:
   - any linear function;
   - any quadratic function;
   - any semicircle;
   - any transformation of any of these functions:

$$x^n \qquad \frac{1}{x} \qquad \sqrt[n]{x} \qquad |x| \qquad \frac{|x|}{x}$$

> **GROUND RULE:** you can assume that any power functions you see are transformations of either x^2 or x^3, and that any root functions you see are transformations of either $\sqrt{x}$ or $\sqrt[3]{x}$.

4. Given the graph of a function, sketch a graph of a transformation of that function.
5. Compute the slope of a line.
6. Interpret the meaning of slope in an applied problem.
7. Write the equation of a linear function with given properties.
8. Translate between the standard form and vertex form of a quadratic.
9. Compute the vertex of a parabola.
10. Solve linear and quadratic equations, and systems of 2 linear equations.
11. Solve equations with powers.
12. Rewrite radical and negative exponent expressions as $\square x^{\square}$.
13. Find intersection point(s) and x- and y-intercepts of lines and parabolas.
14. Identify whether or not a given function is a polynomial; if it is, identify its degree, leading coefficient and tail behavior.
15. Given the graph of a function, identify its horizontal and vertical asymptotes
16. For the types of functions studied in Chapter 3, identify their domain / range / symmetry / intercepts / VA / HA / inverse / maximum and minimum values / etc.
17. Simplify rational expressions and compound fractions; write rules for compositions of rational functions

Chapter 2

Old MATH 130 Exam 3s

2.1 Spring 2024 Exam 3

1. Sketch a graph of each function:

   a) (3.9) $f(x) = -|x-2|$

   b) (3.6) $f(x) = x^4 + 5$

   c) (3.5) $f(x) = -\frac{1}{2}(x-3)^2 - 1$

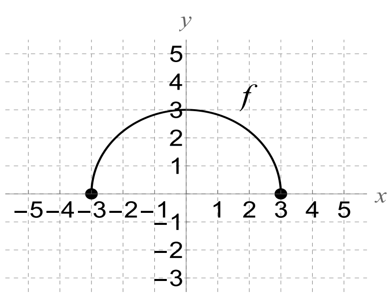
   d) (3.5) $f(x) = x^2 - 4x$

   e) (3.2) $f(x) = 2 + \frac{1}{4}(x-3)$

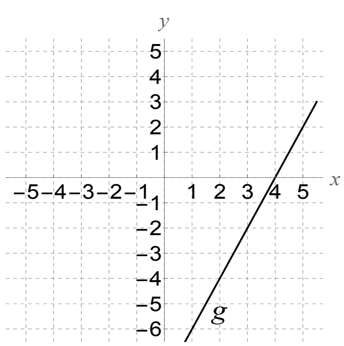
   f) (3.2) $f(x) = \sqrt{x+4}$

2. Write a rule for each function graphed here:

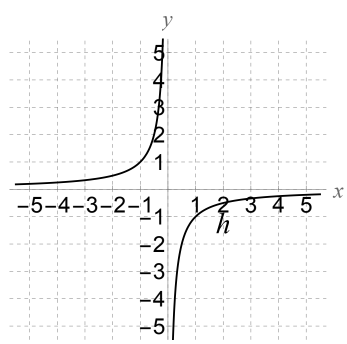
   a) (3.10)



   b) (3.2)



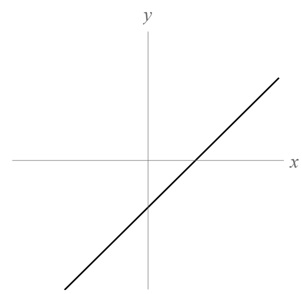
   c) (3.8)



3. a) (3.2) Write the equation of the line with slope 2 passing through $(-1, 5)$.

   b) (3.2) Compute the slope of the line $3x + 4y = 17$.

   c) (3.1) Estimate the slope of the line graphed here, assuming that the scales on the x- and y-axes are the same.



   d) (3.2) Write the equation of the line passing through $\left(-\frac{1}{4}, \frac{2}{5}\right)$ and $\left(\frac{1}{2}, \frac{9}{10}\right)$.

   e) (3.2) Write the equation of the line that makes an angle of $\frac{3\pi}{4}$ with the horizontal and has y-intercept $(0, 7)$.

4. (3.5) Throughout this problem, let $f(x) = 2x^2 + 20x + 30$.

   a) What is the vertex of f?

   b) Write the rule for f in vertex form.

   c) Does f have a maximum value, or a minimum value?

   d) At what x is $f(x)$ maximized/minimized?

   e) How many solutions does the equation $f(x) = -18$ have?

5. (3.8) Simplify $\dfrac{x^2 - 3x - 28}{x^2 - 2x - 35}$.

6. Simplify each expression; if possible, write it as $\square x^{\square}$, where the boxes are constants:

   a) (3.8) $\dfrac{7}{5x^3}$

   b) (3.7) $(2x^2)^3\sqrt{x}$

   c) (3.7) $\sqrt{x^2}$

   d) (3.7) $\left(\sqrt[4]{x}\right)^4$

7. Suppose $h(x) = \dfrac{3}{x-2}$ and $k(x) = \dfrac{5}{x+1}$.

   a) (3.8) Write the equation(s) of any vertical asymptote(s) of h. (If h has no VA, say so.)

   b) (3.8) Compute and simplify the rule for $h + k$.

c) (3.8) Compute and simplify the rule for $h \circ k$.

8. a) (3.3) Find the point where the lines $-11x - 2y = -1$ and $5x + 3y = 13$ intersect.

   b) (3.5) Find the x-intercept(s) of the function $\Gamma(x) = x^2 - 13x + 42$.

9. Solve for x in each equation:

   a) (3.5) $3x^2 - 4x - 2 = 0$

   b) (3.5) $\frac{x}{2}(x + 6) = x + 30$

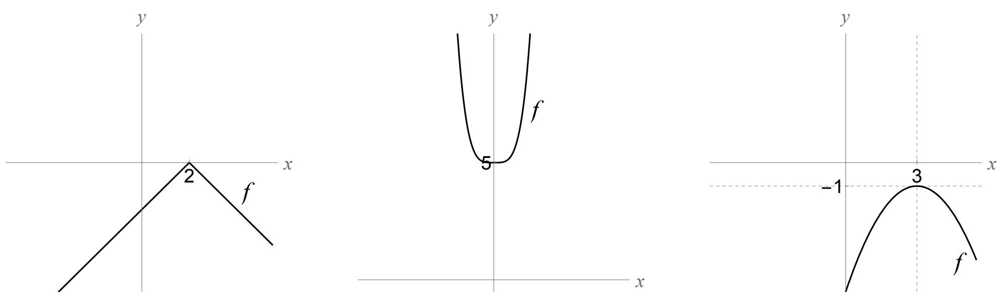
   c) (3.7) $3x^6 - 7 = 17$

Solutions

1. a) Take the graph of $|x|$, shift it right 2 units and reflect it across the x-axis to get the graph below at left.

   b) Take the graph of x^4 and shift it up 5 units to get the function graphed below in the middle.

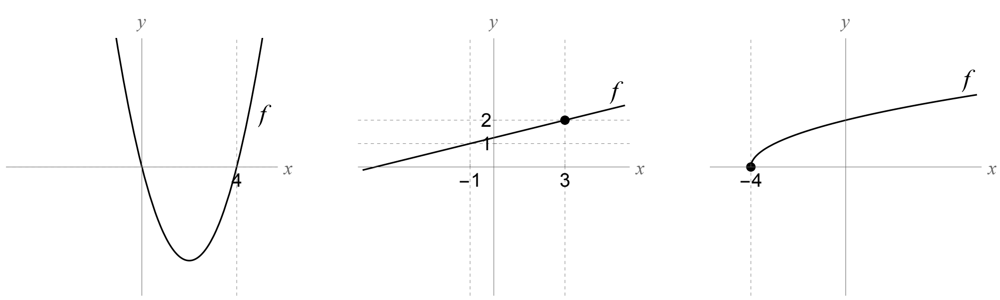
   c) This is a parabola with vertex $(3, -1)$ that opens downward, shown below at right.



   d) This is a parabola that opens upward with y-int $(0, 0)$ and x-ints $(0, 0)$ and $(4, 0)$, so it looks like the graph below at left.

   e) This is a line with slope $\frac{1}{4}$ passing through $(3, 2)$, as shown below in the middle.

   f) Take the graph of $\sqrt{x}$ and shift it left 4 units to get the graph shown below at right.



2. a) This is the top half of a circle of radius 3 centered at the origin, which has equation $\boxed{f(x) = \sqrt{9 - x^2}}$.

   b) This is a line of slope 2 passing through $(4, 0)$, so by the point-slope formula its equation is $g(x) = 0 + 2(x - 4)$, i.e. $\boxed{g(x) = 2x - 8}$.

   c) This is the graph of $\frac{1}{x}$, reflected across the x-axis. Therefore its rule is $\boxed{h(x) = -\frac{1}{x}}$.

3. a) By the point-slope formula, this is $\boxed{y = 5 + 2(x+1)}$.

   b) We solve for y to put the line in slope-intercept form. To do this, subtract $3x$ from both sides to get $4y = -3x + 17$; then divide by 4 to get $y = -\frac{3}{4}x + \frac{17}{4}$; the slope is the coefficient on the x term which is $\boxed{-\frac{3}{4}}$.

   c) This line goes up 1 unit for every 1 unit it goes to the right, so it has slope $\frac{1}{1} = \boxed{1}$.

   d) First, find the slope: $m = \frac{\triangle y}{\triangle x} = \frac{\frac{9}{10} - \frac{2}{5}}{\frac{1}{2} - \left(-\frac{1}{4}\right)} = \frac{\frac{9}{10} - \frac{4}{10}}{\frac{2}{4} + \frac{1}{4}} = \frac{\frac{5}{10}}{\frac{3}{4}} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$.

   So by the point-slope formula, an equation is $\boxed{y = \frac{9}{10} + \frac{2}{3}\left(x - \frac{1}{2}\right)}$.

   e) This line has slope $m = \tan\frac{3\pi}{4} = -1$ so by the slope-intercept formula, its equation is $\boxed{y = -x + 7}$.

4. a) First, the x-coordinate of the vertex is $h = -\frac{b}{2a} = -\frac{20}{2(2)} = -\frac{20}{4} = -5$. Next, the y-coordinate is $k = f(h) = f(-5) = 2(-5)^2 + 20(-5) + 30 = 2(25) - 100 + 30 = 50 - 70 = -20$, so the vertex is $(h,k) = \boxed{(-5,-20)}$.

   b) The vertex form of f is $f(x) = a(x-h)^2 + k$, i.e. $\boxed{f(x) = 2(x+5)^2 - 20}$.

   c) Since $a > 0$, the graph of f is a parabola that opens up, so f has a $\boxed{\text{minimum}}$ value.

   d) f is minimized at the x-coordinate of the vertex, which is $\boxed{x = -5}$.

   e) Since the parabola opens upward and -18 is above the y-coordinate of the vertex, the graph of f will have height -18 at two points, meaning $f(x) = -18$ has $\boxed{2}$ solutions.

5. $\frac{x^2 - 3x - 28}{x^2 - 2x - 35} = \frac{(x-7)(x+4)}{(x-7)(x+5)} = \boxed{\frac{x+4}{x+5}}$.

6. a) $\frac{7}{5x^3} = \boxed{\frac{7}{5}x^{-3}}$.

   b) (3.7) $(2x^2)^3\sqrt{x} = 2^3(x^2)^3x^{1/2} = 8x^6x^{1/2} = 8x^{6+1/2} = \boxed{8x^{13/2}}$.

   c) $\sqrt{x^2} = \boxed{|x|}$.

   d) $\left(\sqrt[4]{x}\right)^4 = \boxed{x}$.

7. a) Set the denominator of h equal to 0 to get $x - 2 = 0$, i.e. $x = 2$. This value of x does not make the numerator of h zero, so $\boxed{x = 2}$ is the VA of h.

b) Add the functions by finding a common denominator:

$$\begin{aligned}(h+k)(x) &= \frac{3}{x-2} + \frac{5}{x+1} \\ &= \frac{3(x+1)}{(x-2)(x+1)} + \frac{5(x-2)}{(x+1)(x-2)} \\ &= \frac{(3x+3)+(5x-10)}{(x-2)(x+1)} \\ &= \boxed{\frac{8x-7}{(x-2)(x+1)}}.\end{aligned}$$

c) Simplify the compound fraction:

$$\begin{aligned}h \circ k(x) = h(k(x)) = h\left(\frac{5}{x+1}\right) &= \frac{3}{\frac{5}{x+1} - 2} \\ &= \frac{3(x+1)}{\left(\frac{5}{x+1} - 2\right)(x+1)} \\ &= \frac{3(x+1)}{5-2(x+1)} \\ &= \frac{3(x+1)}{5-2x-2} = \boxed{\frac{3(x+1)}{3-2x}}.\end{aligned}$$

8. a) Solve the equations together as a system:

$$\begin{cases} -11x - 2y = -1 \xrightarrow{\times 3} \\ 5x + 3y = 13 \xrightarrow{\times 2} \end{cases} \quad \begin{array}{rrl} & -33x - 6y & = -3 \\ \oplus & 10x + 6y & = 26 \\ \hline & -23x & = 23 \\ & x & = -1 \end{array}$$

Since $x = -1$, we substitute into the first equation to get $-11(-1) - 2y = -1$, i.e. $11 - 2y = -1$, i.e. $-2y = -12$, i.e. $y = 6$. Thus the solution is $\boxed{(-1, 6)}$.

b) Set the function equal to 0 and solve for x:

$$\begin{aligned}0 &= \Gamma(x) \\ 0 &= x^2 - 13x + 42 \\ 0 &= (x-6)(x-7)\end{aligned}$$

Therefore $x = 6$ and $x = 7$, giving two x-intercepts $\boxed{(6,0),(7,0)}$.

9. a) Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$$
$$= \frac{4 \pm \sqrt{16 + 24}}{6}$$
$$= \boxed{\frac{4 \pm \sqrt{40}}{6}}.$$

b) First, distribute the terms on the left-hand side to get

$$\frac{1}{2}x^2 + 3x = x + 30.$$

Multiply everything through by 2 to clear the fraction to get

$$x^2 + 6x = 2x + 60.$$

At this point, move the terms to one side and factor:

$$x^2 + 4x - 60 = 0$$
$$(x + 10)(x - 6) = 0$$

Therefore $x = -10$ or $x = 6$, giving the solution set $\boxed{\{-10, 6\}}$.

c) Isolate the x^6-term and take $\pm$ sixth roots:

$$3x^6 - 7 = 17$$
$$3x^6 = 24$$
$$x^6 = 8$$
$$x = \boxed{\pm\sqrt[6]{8}}.$$

2.2 Relevant exam questions from Spring 2018

1. Perform the indicated operations and simplify:

   a) $\dfrac{\frac{2}{x+2} - 3}{2 - \frac{5}{x+2}}$

2. a) Find the slope of the line passing through the points $(3, -7)$ and $(-2, 8)$.

   b) (Write an equation of the line passing through the point $(-2, 5)$ whose slope is 11.

   c) Write an equation of the horizontal line passing through the point $(3, -1)$.

   d) Suppose two lines are parallel. If the first line has slope -3, what is the slope of the second line?

   e) Sketch the graph of the line $2x + 3y = 12$.

   f) Sketch the graph of the line $y = -2 + 3(x - 4)$.

3. Sketch crude graphs of each of these functions:

   a) $f(x) = |x|$

   b) $f(x) = -2(x-4)^2 - 1$

   c) $f(x) = x^4$

   d) $f(x) = \dfrac{1}{x}$

4. Find all horizontal and/or vertical asymptotes of the function

$$f(x) = \frac{2x^2 - x + 30}{x^2 + 2x - 15}.$$

5. Classify each of the following statements as true or false:

   a) The function $f(x) = x^2 - 7x + 4$ is one-to-one.

   b) If a polynomial has degree 8 and its leading coefficient is 3, then both of its tails point upward.

Solutions

1. a) $\dfrac{\frac{2}{x+2} - 3}{2 - \frac{5}{x+2}} = \dfrac{\left[\frac{2}{x+2} - 3\right](x+2)}{\left[2 - \frac{5}{x+2}\right](x+2)} = \dfrac{2 - 3(x+2)}{2(x+2) - 5} = \dfrac{2 - 3x - 6}{2x + 4 - 5} = \boxed{\dfrac{-3x - 4}{2x - 1}}.$

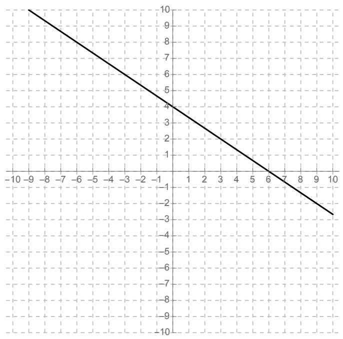
2. a) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - (-7)}{-2 - 3} = \dfrac{15}{-5} = \boxed{-3}.$

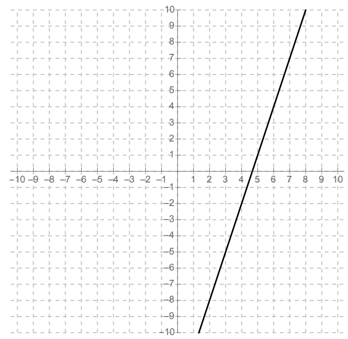
   b) From the point-slope formula, $\boxed{y = 5 + 11(x+2)}$.

c) $\boxed{y=-1}$.

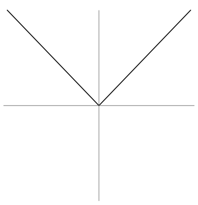
d) The lines have the same slope, so $\boxed{-3}$.

e) The line has intercepts $(0,4)$ and $(6,0)$; it is shown below at left.

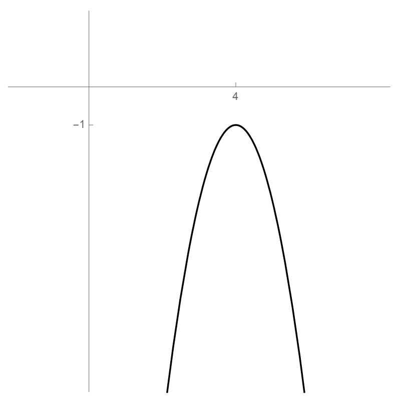


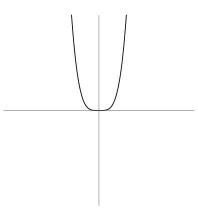


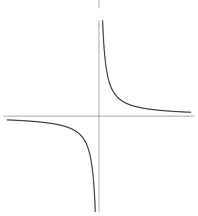
f) The line goes through $(4,-2)$ and has slope 3; it is shown above at right.

3. a) 

b) This is a parabola opening downward with vertex $(4,-1)$:



c) 

d) 

4. For the VAs, set the bottom equal to zero and solve for x:

$$x^2 + 2x - 15 = 0 \Rightarrow (x+5)(x-3) = 0 \Rightarrow x = -5, x = 3$$

Neither of these values of x make the numerator 0, so the VA are $\boxed{x = -5}$ and $\boxed{x = 3}$.

For the HAs, when x is large, $f(x) \approx \dfrac{2x^2}{x^2} = 2$ so the HA is $\boxed{y = 2}$.

5. a) $\boxed{\text{FALSE}}$ (the graph is a parabola which won't pass the Horizontal Line Test)

   b) $\boxed{\text{TRUE}}$ (even degree, positive leading coefficient)

Chapter 3

Additional Practice Exam 3s

3.1 Practice Exam A

A1. Sketch a graph of each function:

a) $f(x) = \sqrt{25 - x^2}$

b) $f(x) = 2(x-4) + 5$

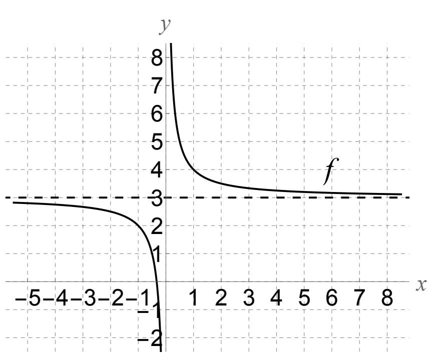
c) $f(x) = -\sqrt[5]{x+5} - 2$

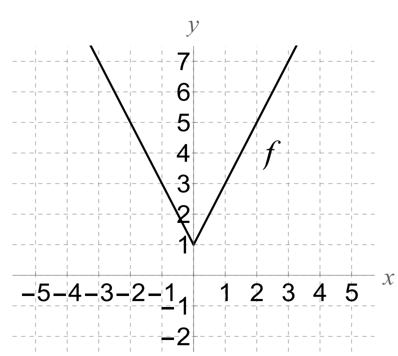
d) $f(x) = 4$

e) $f(x) = 7 - x^2$

f) $f(x) = \frac{1}{4}(x-1)^3 - 3$

A2. Write a rule for each function graphed here:

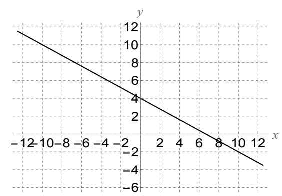
a) 

b) 

A3. Parts (a)-(e) of this question are unrelated to one another.

a) Compute the slope of the line passing through $(3, -5)$ and $(4, 11)$.

b) Estimate the slope of the line graphed here:



c) Write the parabola $f(x) = 2x^2 - 24x + 19$ in vertex form.

d) Find the maximum value of $g(x) = -|x - 5| + 6$.

e) What is the domain of $f(x) = 5\sqrt{49 - x^2}$?

A4. Write an equation of each line with the indicated properties:

a) the line has slope $\frac{3}{8}$ and passes through $(-3, -5)$

b) the line passes through $(-5, 3)$ and makes an angle of $\frac{2}{3}$ radian with the horizontal

c) the line is horizontal and passes through $(4, -11)$

A5. Sketch a graph of each line:

a) $y = \frac{1}{4}(x - 3) - 2$ b) $-2x + 5y = 15$ c) $y = 3$

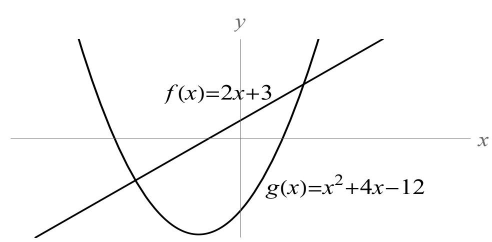
A6. Find all intersection points of the graphs of

$$f(x) = x^2 - 8x + 13$$

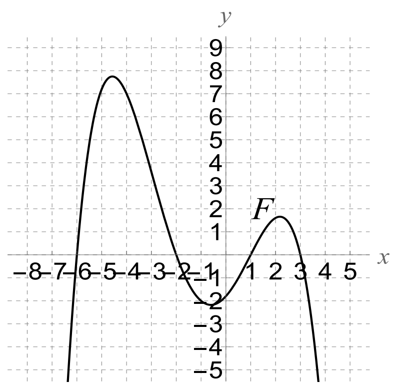
and

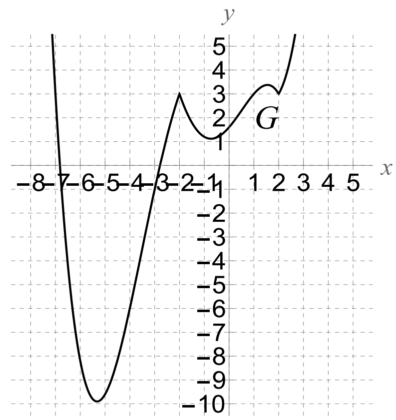
$$g(x) = -x^2 + 4x - 5.$$

A7. Find the intersection points of the graphs of functions f and g shown below:



A8. The graphs of two functions F and G are shown below:





a) Which function (F or G) is a polynomial?

b) For the function that is a polynomial, what do you know about its degree and leading coefficient?

c) For the function that is a polynomial, what is its constant term?

A9. Identify any horizontal and/or vertical asymptotes of the function $f(x) = \dfrac{x-3}{x^2-4}$.

A10. Simplify each expression, writing the answer as $\square x^{\square}$ if possible:

a) $(\sqrt{x})^2$

b) $\sqrt[4]{(2x)^4}$

c) $\dfrac{8x^2}{-16x^{4/3}}$

d) $\left(\dfrac{3}{\sqrt[4]{x}}\right)^{-2}$

e) $\sqrt[5]{x^{30}}$

f) $\dfrac{4}{(x^2)^3}$

A11. Simplify each expression, writing your answer in the form $\dfrac{\square}{\square}$:

a) $\dfrac{x^2+3x-54}{x^2-81}$

b) $\dfrac{1}{x} - \dfrac{1}{x-1}$

c) $\dfrac{\dfrac{3}{x^2-5x-14}}{\dfrac{6}{x^2+10x+16}}$

3.2 Practice Exam B

B1. Sketch a graph of each function:

a) $f(x) = 5 - \frac{1}{2}|x|$

b) $f(x) = 2(x+2)^2 + 5$

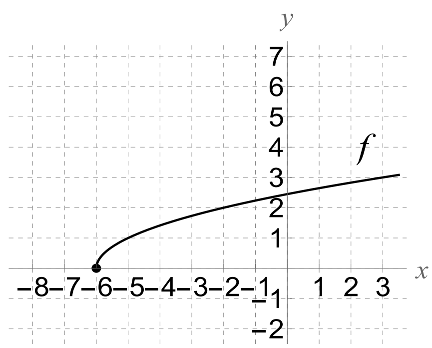
c) $f(x) = -2x + 7$

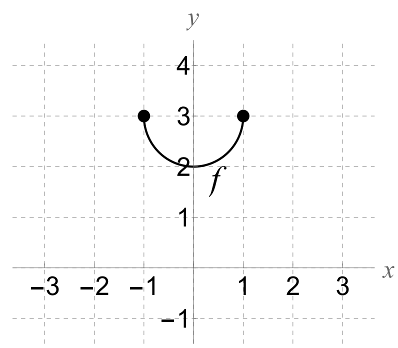
d) $f(x) = \frac{1}{x-5}$

e) $f(x) = -2(x-3) - 1$

f) $f(x) = x^2 - 4x - 5$

B2. Write a rule for each function graphed here:

a) 

b) 

B3. Parts (a)-(e) of this question are unrelated to one another.

a) Compute the slope of the line passing through $\left(-\frac{2}{3}, \frac{3}{8}\right)$ and $\left(\frac{5}{3}, \frac{5}{2}\right)$.

b) Compute the slope of the line with standard equation $3x - 5y = 7$.

c) If a linear function has slope -3, how much does its output change when its input is increased by 4?

d) Write the parabola $f(x) = 3(x-4)^2 - 5$ in standard form.

e) How many solutions does the equation $(x-5)^3 - 6 = 4$ have?

f) What is the range of $f(x) = 3(x-5)^4 - 7$?

B4. Write an equation of each line with the indicated properties:

a) the line passes through $(-4, -7)$ and $(3, -2)$

b) the line is vertical and passes through $(5, -2)$

c) the line passes through $(5, -1)$ and makes an angle of $\frac{\pi}{3}$ with the horizontal

B5. Find the x-coordinate(s) of all points on the graph of $f(x) = x^2 + 8x - 5$ which have y-coordinate 7.

B6. Solve each equation:

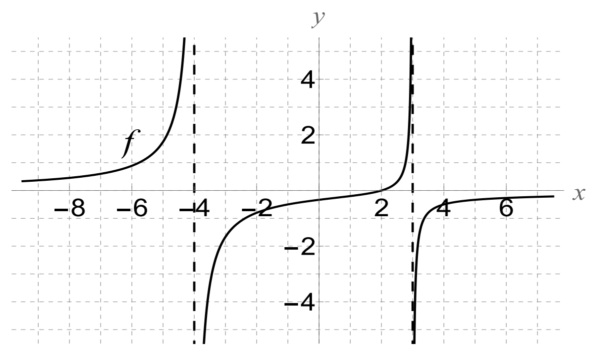
a) $x(x-2)+5=x(x-5)+7$

b) $x^2-3x=54$

c) $x^2-x-1=0$

d) $4x^7-9=31$

B7. Here is the graph of some unknown rational function f:



a) Identify any horizontal asymptote(s) of this function.

b) Identify any vertical asymptote(s) of this function.

c) What is the relationship between the degree of the numerator of f and the degree of the denominator of f? (Are they equal? If not, which is larger?)

B8. For each given function, determine whether it is even, odd, or neither:

a) $f(x)=|x|-3$

b) $f(x)=(x-4)^2$

c) $f(x)=3x^6$

d) $f(x)=\dfrac{1}{x}$

e) $f(x)=2$

f) $f(x)=\sqrt{x}$

B9. Simplify each expression, writing the answer as $\square x^{\square}$ if possible:

a) $\sqrt{3\sqrt{x}}$

b) $\sqrt[4]{x}\sqrt[3]{x^2}$

c) $\dfrac{12\sqrt{x}}{\dfrac{3x^2}{(x^2)^3}}$

d) $\dfrac{3\sqrt{x}}{4}\cdot\dfrac{8x^2}{5}\cdot\dfrac{35}{x^{4/3}}$

B10. In this problem, let $f(x)=\dfrac{x+7}{x-3}$ and $g(x)=\dfrac{x}{x+4}$. Compute and simplify the rule for each given function, writing your answer in the form $\dfrac{\square}{\square}$:

a) $f+2g$

b) g^2

c) fg

d) $f\circ g$

3.3 Practice Exam C

C1. Sketch a graph of each function:

a) $f(x) = -\sqrt{x}$

b) $f(x) = \dfrac{3}{x+4} - 2$

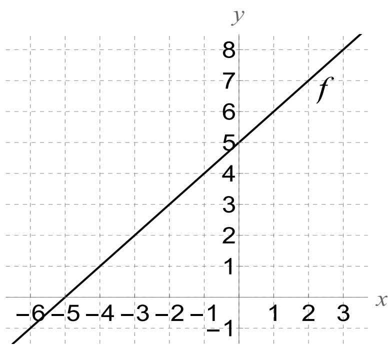
c) $f(x) = -\dfrac{1}{2}x$

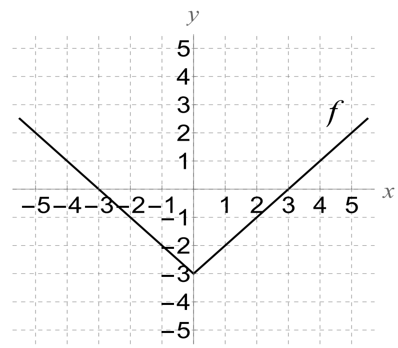
d) $f(x) = \sqrt{16-(x-3)^2}$

e) $f(x) = -x^2 + 4x + 32$

f) $f(x) = (x+3)^5$

C2. Write a rule for each function graphed here:

a) 

b) 

C3. Parts (a)-(e) of this question are unrelated to one another.

a) Compute the slope of the line passing through the points $(a+3b, 5c-4d)$ and $(4a-b, c+3d)$.

b) Compute the slope of a line which makes an angle of $\dfrac{3\pi}{4}$ with the horizontal.

c) Find the vertex of the parabola $f(x) = -\dfrac{1}{2}x^2 + \dfrac{3}{4}x + 1$.

d) Is the y-coordinate you found in part (c) the maximum value of f, or the minimum value of f?

e) What is the range of $f(x) = -\sqrt{16-(x+1)^2}$?

C4. Write an equation of each line with the indicated properties:

a) the line has slope $\sqrt{6}$ and passes through $(2\sqrt{7}, 3\sqrt{10})$

b) the line has slope 6 and passes through $(5, -2)$

c) the line is parallel to $2x - 5y = 8$ and passes through $(1, 3)$

C5. Solve the system of equations $\begin{cases} 2x - 3y &= 8 \\ 5x + y &= 3 \end{cases}$

C6. Find the x-coordinate(s) of all intersection points of the graphs of $f(x) = 2x^2 + 5x - 1$ and $g(x) = x^2 - 8x + 13$.

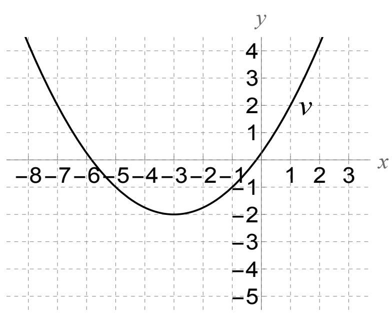
C7. Solve each equation:

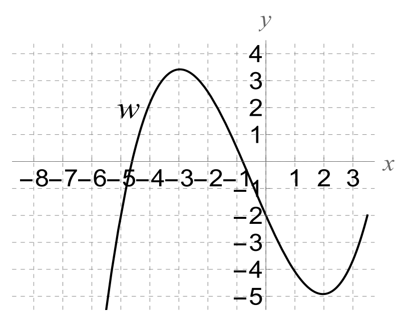
a) $3(x-5)+2=-7(x+4)-8$

b) $\frac{2}{3}x^2=\frac{5}{4}$

c) $x^2+2x=10$

C8. Here are the graphs of two functions v and w:





a) Which function (v or w) is quadratic?

b) For the function that is a quadratic, what is its vertex?

c) For the function that is a quadratic, is the coefficient on its x^2 term positive or negative?

d) For the function that is not quadratic, is its degree even or odd?

e) For the function that is not quadratic, give its turning points.

C9. Write down the rule for any rational function that has a vertical asymptote $x=3$ but has no horizontal asymptote.

C10. Simplify each expression, writing your answer in the form $\frac{\square}{\square}$:

a) $\frac{x^2+4x+3}{x^2-2x-15}$

b) $(x-3)^{-1}-4(x+2)^{-1}$

c) $\dfrac{\frac{1}{x}-\frac{3}{x+1}}{\frac{2}{x+1}-3}$

C11. Compute the inverse of each function:

a) $h(x)=(x+2)^3-8$

b) $H(x)=\sqrt{2x+1}$

3.4 Practice Exam D

D1. Sketch a graph of each function:

a) $f(x) = -\frac{1}{3}(x+7)$

b) $f(x) = x^6 - 4$

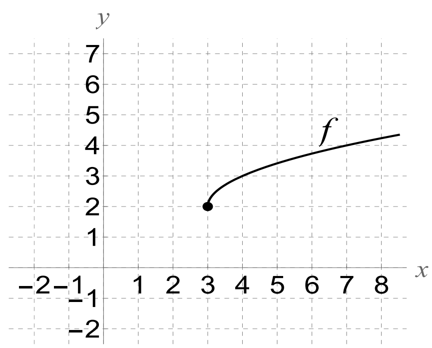
c) $f(x) = x(5-x)$

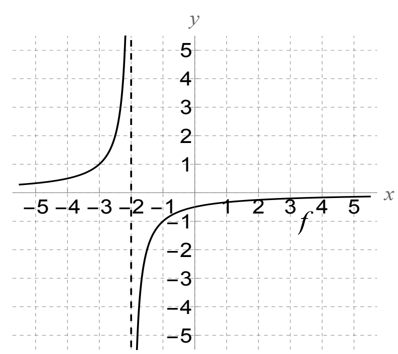
d) $f(x) = -\sqrt{9-x^2}$

e) $f(x) = -\frac{2}{5}x + 3$

f) $f(x) = \frac{|x|}{x}$

D2. Write a rule for each function graphed here:

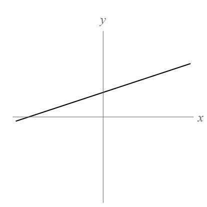
a) 

b) 

D3. Parts (a)-(e) of this question are unrelated to one another.

a) Compute the slope of the line passing through $(5, -3)$ and $(5, 4)$.

b) Estimate the slope of the line graphed here, assuming the scales on the x- and y-axes are the same:



c) If a linear function has slope 5, how much does its output change when its input is increased by 10?

d) How many solutions does the equation $2|x-5| + 3 = 4$ have?

e) At what value(s) of x is the function $f(x) = \sqrt{9-(x-2)^2}$ maximized?

D4. Write an equation of each line with the indicated properties:

a) the line has slope $\frac{1}{4}$ and has x-intercept $(8, 0)$

b) the line passes through $(-3, -2)$ and makes an angle of $\frac{\pi}{4}$ with the horizontal

c) the line has slope $\frac{13}{4}$ and passes through $\left(\frac{3}{7}, -\frac{20}{7}\right)$

D5. Find the point where the lines $5x + 4y = -8$ and $x - 2y = 1$ intersect.

D6. Find the intersection point of the two lines $\frac{3}{5}x + \frac{2}{3}y = \frac{22}{15}$ and $x + \frac{2}{5}y = \frac{2}{3}$.

D7. Solve each equation:

a) $2x^2 + 3x - 20 = 0$ b) $x^2 + 5x + 7 = 0$ c) $3x^5 = 27$

D8. Simplify each expression, writing the answer as $\Box x^{\Box}$ if possible:

a) $\sqrt[5]{-x^3}$ c) $\frac{8}{x^5}$ e) $\sqrt[4]{x}\sqrt[3]{2x}\sqrt{5x}$

b) $\sqrt{16x^2}$ d) $\frac{7}{x} \div \frac{21}{x^2}$ f) $\frac{3}{\sqrt[3]{7x^3}}$

D9. In this problem, let $F(x) = 2(x-1)^{-1}$ and $G(x) = 3(x+5)^{-1}$.

a) Compute and simplify the rule for $F - G$.

b) Compute and simplify the rule for $\frac{F}{G}$.

c) Determine all vertical asymptotes, if any, of F.

d) Determine all vertical asymptotes, if any, of the function H, where $H(x) = F(x-3)$.

e) Determine all vertical asymptotes, if any, of the function K, where $K(x) = 4F(x)$.

f) Determine all horizontal and/or vertical asymptotes of $F \circ G$.

D10. Let $f(x) = \sqrt{25 - (x-2)^2} + 4$.

a) What is the domain of f?

b) What is the minimum value of f?

c) At what value(s) of x is f maximized?

d) Identify all horizontal and/or vertical asymptotes of f.

e) What is the maximum value of g, where $g(x) = f(x-3)$?

f) What is the maximum value of h, where $h(x) = -f(x)$?

g) How many solutions does the equation $f(x) = 7$ have?

h) How many solutions does the equation $f(x) = 9$ have?

3.5 Practice Exam E

E1. Sketch a graph of each function:

a) $f(x) = -x^4$

b) $f(x) = 3x - 4$

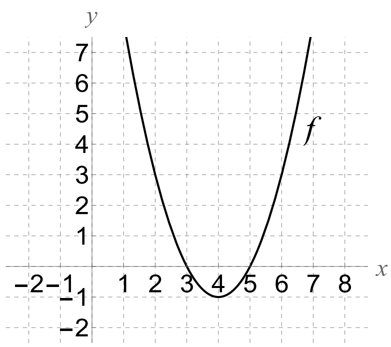
c) $f(x) = |x - 4| - 3$

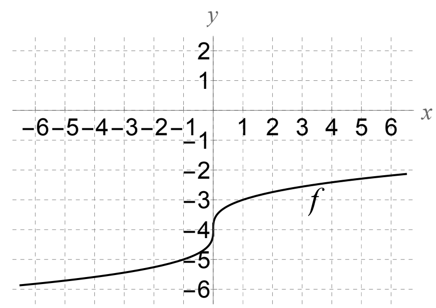
d) $f(x) = 2x^2 + 12x$

e) $f(x) = \dfrac{|x+4|}{x+4}$

f) $f(x) = \sqrt{36 - x^2} - 4$

E2. Write a rule for each function graphed here:

a) 

b) 

E3. Sketch a graph of each line:

a) $x = 2$

b) $2x - 7y = 14$

c) $3x + y = 9$

E4. Write an equation of each line with the indicated properties:

a) the line is perpendicular to $y = -3x + 4$ and passes through $(7, -3)$

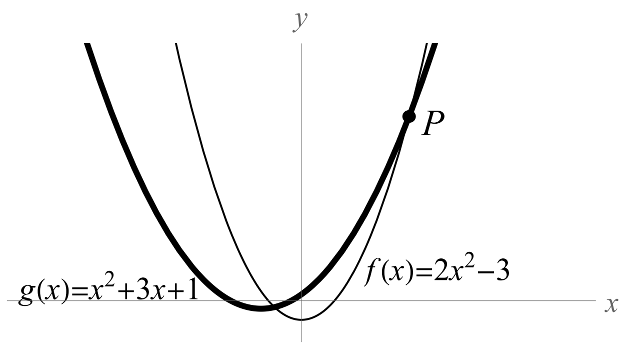
b) the line passes through $(3, 2)$ and is perpendicular to the line passing through $(0, 5)$ and $(4, -3)$

c) the line passes through $(8, 3)$ and has no x-intercept

d) the line has slope -2 and y-intercept $(0, 6)$

E5. Solve the system of equations $\begin{cases} y &= 2x - 7 \\ 4x + 3y &= 19 \end{cases}$

E6. Find the coordinates of the point P indicated in the picture below:



E7. Solve each equation:

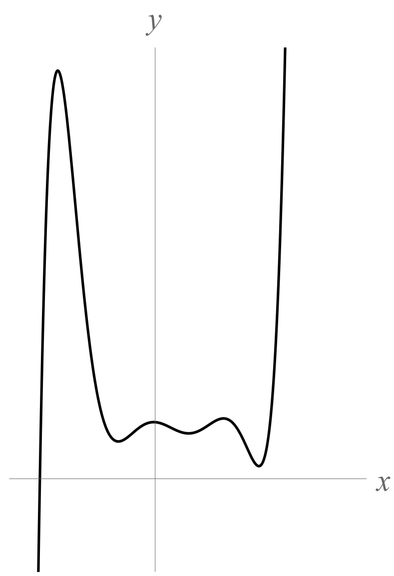
a) $\frac{3}{2}x - \frac{5}{4} = \frac{7}{4}x + 2$

b) $3x^2 + 12x - 180 = 0$

c) $8x^2 = 72$

d) $5x^2 + x - 3 = 0$

E8. Here is the graph of some unknown polynomial:



a) Is the degree of this polynomial even or odd?

b) What is the smallest possible degree of this polynomial?

c) Is the leading coefficient of this polynomial positive or negative?

d) How many x-intercepts does this polynomial have?

E9. Identify any horizontal and/or vertical asymptotes of the function $f(x) = \frac{2x^2 + x - 3}{x^2 + 7x + 12}$.

E10. Simplify each expression, writing the answer as $\square x^{\square}$ if possible:

a) $5 \div \dfrac{3}{x}$

b) $\dfrac{34\sqrt{x}}{2x^2}$

c) $20(2x^3)^{-2}(3\sqrt{x})^4$

d) $3x(2x\sqrt{x})^3$

E11. Simplify each expression, writing your answer in the form $\dfrac{\square}{\square}$:

a) $\dfrac{\dfrac{2}{x-5}+\dfrac{3}{2x+1}}{\dfrac{1}{2x+1}+\dfrac{4}{x-5}}$

b) $\dfrac{3}{x^2-7x-18}-\dfrac{1}{x-9}$

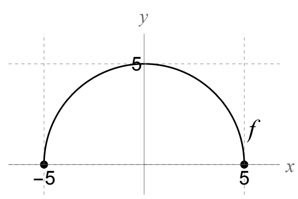
c) $\dfrac{x^2+9x+8}{x^2-7x}\cdot\dfrac{x^2-3x-28}{x^2-6x-7}$

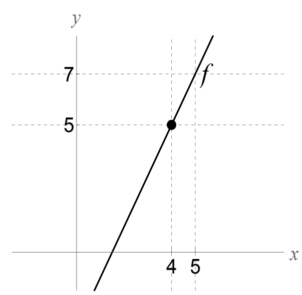
3.6 Solutions to Practice Exam A

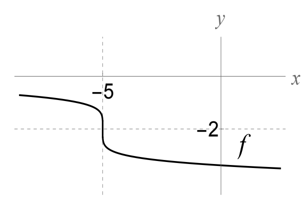
A1. a) This is the top half of a circle of radius 5, centered at the origin (shown below at left).

b) This is a line with slope 2 passing through $(4, 5)$, shown below in the middle:

c) Start with the graph of $\sqrt[5]{x}$; shift it left 5 units, reflect across the x-axis and then shift down 2 units to get the graoh shown below at right:



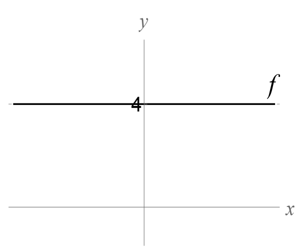


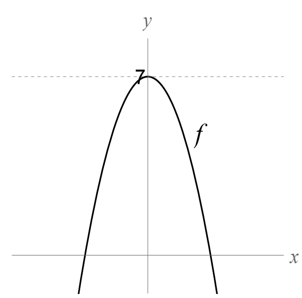


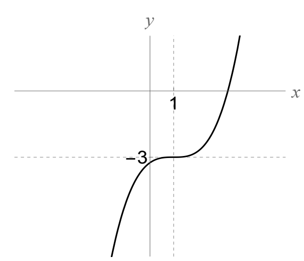
d) This is a horizontal line at height 4, shown below at left.

e) Start with the parabola x^2; reflect across the x-axis and shift up 7 units to get the graph shown below in the middle.

f) Start with the graph of x^3; shift it right 1 unit, compress it vertically so that it is $\frac{1}{4}$ as high (this step won't show up on the graph much), and then shift it down 3 units to get the graph shown below, at right:







A2. a) This is the graph of $\frac{1}{x}$, shifted up 3 units, so $\boxed{f(x) = \frac{1}{x} + 3}$.

b) This is the graph of $|x|$, stretched vertically by a factor of 2 (since it goes up/down 2 units for every 1 unit change in x) and shifted up 1 unit, so $\boxed{f(x) = 2|x| + 1}$.

A3. a) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{11 - (-5)}{4 - 3} = \dfrac{16}{1} = \boxed{16}$.

b) The line goes down 3 units for every 5 units it goes to the right, so $m = \dfrac{\triangle y}{\triangle x} = \dfrac{-3}{5} = \boxed{-\dfrac{3}{5}}$.

c) We have $h = -\frac{b}{2a} = -\frac{-24}{2(2)} = 6$ and $k = f(h) = f(6) = 2(6^2) - 24(6) + 19 = 72 - 144 + 19 = -53$, so the vertex form of the parabola is $f(x) = a(x-h)^2 + k = \boxed{2(x-6)^2 - 53}$.

d) The graph of g is the graph of $|x|$, shifted right 5 units, flipped upside down and then shifted up 6 units. This graph is an upside-down V with the maximum value (the peak of the $\wedge$) at $(5, 6)$, so the maximum value of f is $\boxed{6}$.

e) f is a semicircle of radius 7 stretched upward by a factor of 5. This semicircle goes as far left as the point $(-7, 0)$ and as far right as the point $(7, 0)$, so the domain of f is $\boxed{[-7, 7]}$.

A4. a) By the point-slope formula, the equation is $\boxed{y = -5 + \frac{3}{8}(x+3)}$.

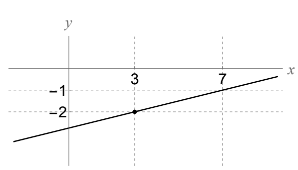
b) The line has slope $m = \tan\frac{2}{3}$, so by the point-slope formula, its equation is $\boxed{y = 3 + \tan\frac{2}{3}(x+5)}$.

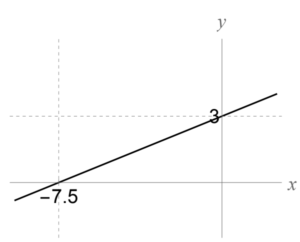
c) Since the line is horizontal, it has slope 0 so by the point-slope formula, its equation is $y = -11 + 0(x-4)$ which simplifies to $\boxed{y = -11}$.

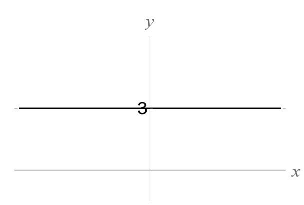
A5. a) $y = \frac{1}{4}(x-3) - 2$ goes through $(3, -2)$ with slope $\frac{1}{4}$, as shown below at left.

b) Find the x- and y-intercepts by setting the opposite variable equal to 0. If you do this, you will see that $-2x + 5y = 15$ has x-int $\left(-\frac{15}{2}, 0\right) = (-7.5, 0)$ and y-int $(0, 3)$, so its graph is shown in the center below.

c) $y = 3$ is a horizontal line of height 3, as shown below at right.







A6. Set the two functions equal and solve for x:

$$\begin{aligned} f(x) &= g(x) \\ x^2 - 8x + 13 &= -x^2 + 4x - 5 \\ 2x^2 - 12x + 18 &= 0 \\ 2(x^2 - 6x + 9) &= 0 \\ 2(x-3)(x-3) &= 0 \\ x &= 3 \end{aligned}$$

Therefore there is one intersection point, when $x = 3$. Last, find the y-coordinate: $y = f(3) = 3^2 - 8(3) + 13 = -2$ so the intersection point is $\boxed{(3, -2)}$.

A7. Set the two functions equal and solve for x:

$$\begin{aligned} f(x) &= g(x) \\ 2x + 3 &= x^2 + 4x - 12 \\ 0 &= x^2 + 2x - 15 \\ 0 &= (x+5)(x-3) \end{aligned}$$

Therefore there are two intersection points, when $x = -5$ and when $x = 3$. Last, find y-coordinates: when $x = -5$, $y = f(-5) = 2(-5) + 3 = -7$ giving the intersection point $\boxed{(-5, -7)}$. When $x = 3$, $y = f(3) = 2(3) + 3 = 9$ so the other intersection point is $\boxed{(3, 9)}$.

A8. a) $\boxed{F}$ is a polynomial (the graph of G is not smooth because of the sharp corners).

b) Since both of the tails of F point down, we know $\boxed{\text{the LC of } F \text{ is negative}}$ and $\boxed{\text{the degree of } F \text{ is even}}$. Last, we know that since F has three turning points, $\boxed{\text{the degree of } F \text{ is at least } 4}$.

c) The constant term of F is its y-intercept $F(0)$, which is $\boxed{-2}$.

A9. Since the degree of the numerator is less than the degree of the denominator, f has $\boxed{\text{HA } y = 0}$. To find the VA, set the denominator equal to zero:

$$x^2 - 4 = 0 \quad \Rightarrow \quad (x-2)(x+2) = 0 \quad \Rightarrow \quad x = 2, x = -2.$$

Neither $x = 2$ nor $x = -2$ make the numerator of f zero, so they are both VA, i.e. f has $\boxed{\text{VA } x = 2, x = -2}$.

A10. a) $(\sqrt{x})^2 = (x^{1/2})^2 = x^{1/2 \cdot 2} = x^1 = \boxed{x}$.

b) $\sqrt[4]{(2x)^4} = |2x| = \boxed{2|x|}$.

c) $\dfrac{8x^2}{-16x^{4/3}} = -\dfrac{1}{2}x^{2-4/3} = \boxed{-\dfrac{1}{2}x^{2/3}}$.

d) $\left(\dfrac{3}{\sqrt[4]{x}}\right)^{-2} = (3x^{-1/4})^{-2} = 3^{-2}x^{-1/4 \cdot -2} = \dfrac{1}{3^2}x^{1/2} = \boxed{\dfrac{1}{9}x^{1/2}}$.

e) $\sqrt[5]{x^{30}} = \left(x^{30}\right)^{1/5} = x^{30 \cdot 1/5} = \boxed{x^6}$.

f) $\dfrac{4}{(x^2)^3} = \dfrac{4}{x^6} = \boxed{4x^{-6}}$.

A11. a) $\dfrac{x^2 + 3x - 54}{x^2 - 81} = \dfrac{(x+9)(x-6)}{(x-9)(x+9)} = \boxed{\dfrac{x-6}{x-9}}$.

b) $\dfrac{1}{x} - \dfrac{1}{x-1} = \dfrac{x-1}{x(x-1)} = \dfrac{x}{x(x-1)} = \dfrac{x-1-x}{x(x-1)} = \boxed{\dfrac{-1}{x(x-1)}}.$

c)

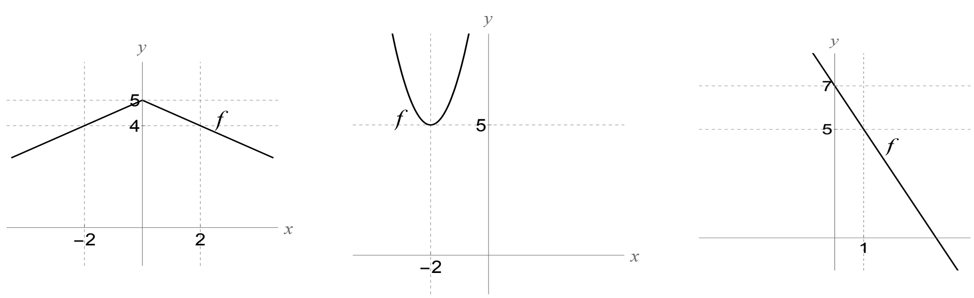
$$\frac{\frac{3}{x^2-5x-14}}{\frac{6}{x^2+10x+16}} = \frac{3}{x^2-5x-14} \cdot \frac{x^2+10x+16}{6}$$
$$= \frac{3}{(x-7)(x+2)} = \frac{(x+2)(x+8)}{6} = \boxed{\frac{(x+8)}{2(x-7)}}.$$

3.7 Solutions to Practice Exam B

B1. a) Start with the graph of $|x|$, compress it so that it is $\frac{1}{2}$ as tall, reflect it across the x-axis and then shift up 5 to get the graph shown below at left.

b) Start with the parabola $y = x^2$; shift it left 2 units, stretch vertically by a factor of 2 (this stretch won't really show up on the graph) and shift up 5 units to get the graph below, in the center.

c) This is a line with slope -2 and y-intercept $(0, 7)$, as shown below at right.

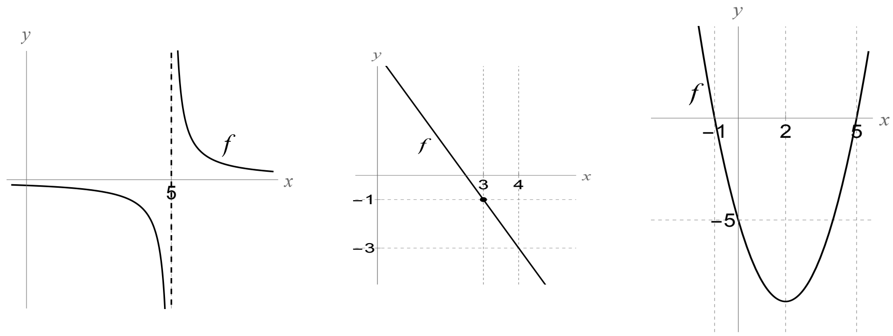


d) Start with the graph of $\frac{1}{x}$ and shift it 5 units right to get the graph shown below at left:

e) This is a line with slope -2 passing through $(3, -1)$, shown below in the center.

f) To graph this parabola, find its x- and y-intercepts. For the y-intercept, $y = f(0) = 0^2 - 4(0) - 5 = -5$ so the y-int is $(0, -5)$. For the $x-$ints, set $0 = f(x)$ to get $0 = x^2 - 4x - 5 = (x-5)(x+1)$ so the x-ints are $(5, 0)$ and $(-1, 0)$. Since

$a > 0$, the parabola opens upward so we get the graph shown below at right.



B2. a) This is the graph of $\sqrt{x}$, shifted left by 6 units, so $\boxed{f(x) = \sqrt{x+6}}$.

b) This is a semicircle of radius 1, reflected across the x-axis to get the bottom half of the circle, then shifted up 3 units, so $\boxed{f(x) = 3 - \sqrt{1-x^2}}$.

B3. a)

$$m = \frac{\triangle y}{\triangle x} = \frac{\frac{5}{2} - \frac{3}{8}}{\frac{5}{3} - \left(-\frac{2}{3}\right)} = \frac{\frac{20}{8} - \frac{3}{8}}{\frac{7}{3}} = \frac{17}{8} \div \frac{7}{3} = \frac{17}{8} \cdot \frac{3}{7} = \boxed{\frac{51}{24}}.$$

b) Solve for y to get $-5y = -3x + 7$, i.e. $y = \frac{3}{5}x - \frac{7}{5}$. So the slope is $\boxed{\frac{3}{5}}$.

c) The change in the output is the slope times the change in the input, which here is $4(-3) = -12$. Thus the output $\boxed{\text{decreases by } 12}$.

d) FOIL the rule for f to get $f(x) = 3(x-4)^2 - 5 = 3(x^2 - 8x + 16) - 5 = \boxed{3x^2 - 24x + 43}$.

e) The graph of the left-hand side $(x-5)^3 - 6$ is the graph of x^3 shifted right 5 units and down 6 units; this graph has height 4 at exactly one value of x, so the equation has $\boxed{1}$ solution.

f) The graph of f is the graph of x^4 shifted right 5 units, stretched by a factor of 3 and shifted down 7 units. This graph will cover every y-value from -7 upward, so the range of f is $\boxed{[-7, \infty)}$.

B4. a) The line has slope $m = \frac{\triangle y}{\triangle x} = \frac{-2-(-7)}{3-(-4)} = \frac{5}{7}$, so by the point-slope formula its equation is $\boxed{y = -2 + \frac{5}{7}(x-3)}$.

b) Since the line is vertical, it has equation $x = \textit{constant}$, which here is $\boxed{x = 5}$.

c) The slope is $m = \tan\frac{\pi}{3} = \sqrt{3}$, so by the point-slope formula the equation is $\boxed{y = -1 + \sqrt{3}(x-5)}$.

B5. Solve the equation $f(x) = 7$:

$$\begin{aligned} x^2 + 8x - 5 &= 7 \\ x^2 + 8x - 12 &= 0 \end{aligned}$$

By the quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4(1)(-12)}}{2(1)} = \boxed{\frac{-8 \pm \sqrt{112}}{2}}.$$

B6. a)

$$\begin{aligned} x(x-2) + 5 &= x(x-5) + 7 \\ x^2 - 2x + 5 &= x^2 - 5x + 7 \\ 3x &= 2 \\ x &= \boxed{\frac{2}{3}} \end{aligned}$$

b) $x^2 - 3x = 54$

$$\begin{aligned} x^2 - 3x &= 54 \\ x^2 - 3x - 54 &= 0 \\ (x-9)(x+6) &= 0 \\ x - 9 &= 0 \quad \text{or } x + 6 = 0 \\ \boxed{x = 9} &\quad \boxed{x = -6} \end{aligned}$$

c) Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)} = \boxed{\frac{1 \pm \sqrt{5}}{2}}.$$

d) Isolate the x^7 term and then take seventh roots:

$$\begin{aligned} 4x^7 - 9 &= 31 \\ 4x^7 &= 40 \\ x^7 &= 10 \\ x &= \boxed{\sqrt[7]{10}}. \end{aligned}$$

B7. a) f has HA $\boxed{y = 0}$.

b) f has VA $\boxed{x = -4, x = 3}$.

c) Since the HA is $y = 0$, the degree of the denominator of f is larger than the degree of the numerator of f.

B8. To solve these, you can graph the function. If the graph is symmetric across the y-axis, the function is even; if it has $180°$ rotational symmetry around the origin, it is odd. (Alternatively, functions with only even powers of x are even; those with only odd powers of x are odd.)

a) $f(x) = |x| - 3$ is $\boxed{\text{even}}$.

b) $f(x) = (x-4)^2 = x^2 - 8x + 16$ is $\boxed{\text{neither}}$ even nor odd.

c) $f(x) = 3x^6$ is $\boxed{\text{even}}$.

d) $f(x) = \dfrac{1}{x} = x^{-1}$ is $\boxed{\text{odd}}$.

e) $f(x) = 2 = 2x^0$ is $\boxed{\text{even}}$.

f) $f(x) = \sqrt{x}$ is $\boxed{\text{neither}}$ even nor odd.

B9. a) $\sqrt{3\sqrt{x}} = \sqrt{3}\sqrt{\sqrt{x}} = \sqrt{3}(x^{1/2})^{1/2} = \boxed{\sqrt{3}x^{1/4}}$.

b) $\sqrt[4]{x}\sqrt[3]{x^2} = x^{1/4}x^{2/3} = x^{1/4+2/3} = \boxed{x^{11/12}}$.

c) $\dfrac{12\sqrt{x}}{\dfrac{3x^2}{(x^2)^3}} = \dfrac{12x^{1/2}}{3x^2/x^6} = \dfrac{12x^{1/2}}{3x^{-4}} = 4x^{1/2-(-4)} = \boxed{4x^{9/2}}$.

d) $\dfrac{3\sqrt{x}}{4} \cdot \dfrac{8x^2}{5} \cdot \dfrac{35}{x^{4/3}} = 3(2)(7)x^{1/2+2-4/3} = \boxed{42x^{7/6}}$.

B10. a) $(f+2g)(x) = f(x)+2g(x) = \dfrac{x+7}{x-3} + \dfrac{2x}{x+4} = \dfrac{(x+7)(x+4)}{(x-3)(x+4)} + \dfrac{2x(x-3)}{(x-3)(x+4)} = \dfrac{x^2+11x+44+2x^2-6x}{(x-3)(x+4)} = \boxed{\dfrac{3x^2+5x+44}{(x-3)(x+4)}}$.

b) $g^2(x) = \left(\dfrac{x}{x+4}\right)^2 = \boxed{\dfrac{x^2}{(x+4)2}}$.

c) $fg(x) = f(x)g(x) = \boxed{\dfrac{(x+7)x}{(x-3)(x+4)}}$.

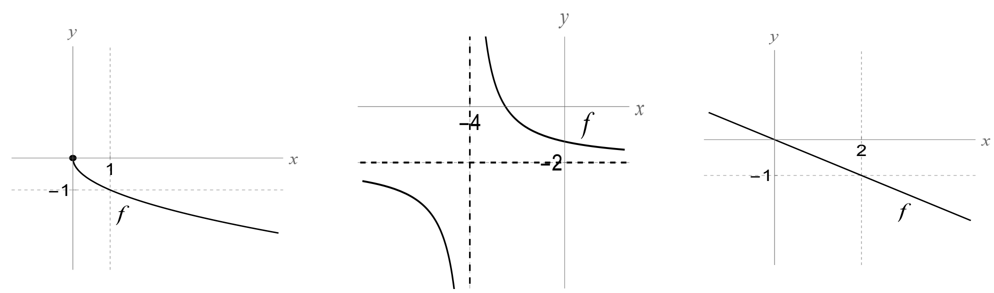
d) $f \circ g(x) = f(g(x)) = f\left(\dfrac{x}{x+4}\right) = \dfrac{\frac{x}{x+4}+7}{\frac{x}{x+4}-3} = \dfrac{x+7(x+4)}{x-3(x+4)} = \dfrac{8x+28}{-2x-12} = \dfrac{4(2x+7)}{-2(x+6)} = \boxed{\dfrac{-2(2x+7)}{x+6}}$.

3.8 Solutions to Practice Exam C

C1. a) Reflect the graph of $\sqrt{x}$ across the y-axis to get the graph shown below at left.

b) Start with the graph of $\dfrac{1}{x}$, shift it left 4 units, stretch it vertically by a factor of 3 (which won't show up on the graph much), and then shift down 2 units to get the graph shown below in the center.

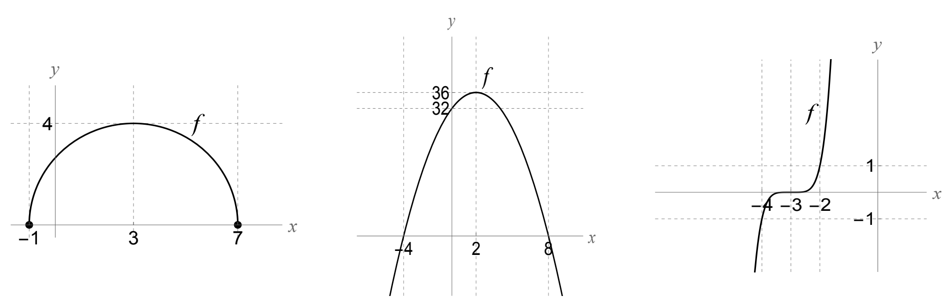
c) This is a line passing through the origin with slope $-\frac{1}{2}$, as shown below at right.



d) Start with the semicircle $\sqrt{16 - x^2}$ of radius 4, and shift it right 3 units to get the graph shown below at left.

e) Graph this parabola by finding intercepts. For the x-ints, set $0 = f(x)$ to get $0 = -x^2 + 4x + 32$, i.e. $0 = -(x-8)(x+4)$ so the x-ints are $(8,0)$ and $(-4,0)$. For the y-int, $y = f(0) = 32$ so the y-int is $(0, 32)$. Since $a < 0$, the parabola opens downward, so we get the graph shown below in the center.

f) Start with the graph of x^5 and shift it left 3 units to get the graph shown below at right.



C2. a) This is a line with slope 1 and y-int $(0,5)$, so the equation is $\boxed{f(x) = x + 5}$.

b) This is $|x|$ shifted down 3 units, so $\boxed{f(x) = |x| - 3}$.

C3. a) $m = \dfrac{\triangle y}{\triangle x} = \dfrac{(c+3d)-(5c-4d)}{(4a-b)-(a+3b)} = \boxed{\dfrac{-4c+7d}{3a-4b}}$.

b) $m = \tan \dfrac{3\pi}{4} = \boxed{-1}$.

c) The x-coordinate is $h = -\dfrac{b}{2a} = -\dfrac{\frac{3}{4}}{2\left(-\frac{1}{2}\right)} = -\dfrac{\frac{3}{4}}{-1} = \dfrac{3}{4}$, and the y-coordinate is

$$\begin{aligned} k = f(h) &= -\frac{1}{2}\left(\frac{3}{4}\right)^2 + \frac{3}{4}\left(\frac{3}{4}\right) + 1 \\ &= -\frac{1}{2}\left(\frac{9}{16}\right) + \frac{9}{16} + 1 \\ &= -\frac{9}{32} + \frac{18}{32} + \frac{32}{32} = \frac{41}{32}, \end{aligned}$$

so the vertex is $(h, k) = \boxed{\left(\frac{3}{4}, \frac{41}{32}\right)}$.

d) Since $a < 0$, the parabola opens downward so the vertex is the $\boxed{\text{maximum}}$ of the parabola.

e) The graph of this function is a semicircle of radius 4, shifted left 1 and then reflected across the x-axis. So the graph of f goes up as far as $y = 0$ and down as far as $y = -4$, so the range is $\boxed{[-4, 0]}$.

C4. a) By the point-slope formula, the equation is $\boxed{y = 3\sqrt{10} + \sqrt{6}\left(x - 2\sqrt{7}\right)}$.

b) By the point-slope formula, the equation is $\boxed{y = -2 + 6(x - 5)}$.

c) The given line can be rewritten as $y = \frac{2}{5}x - \frac{8}{5}$, so it has slope $\frac{2}{5}$. Since our line is parallel, it has the same slope, so by the point-slope formula our equation is $\boxed{y = 3 + \frac{2}{5}(x - 1)}$.

C5. Multiply the second equation by 3, then add them to eliminate y:

$$\begin{cases} 2x - 3y &= 8 \xrightarrow{\times 1} 2x - 3y &= 8 \\ 5x + y &= 3 \xrightarrow{\times 3} 15x + 3y &= 9 \end{cases}$$

Adding the equations, we get $17x = 17$, so $x = 1$., Substituting into the second equation, we get $5(1) + y = 3$, so $y = -2$. Thus the solution is $\boxed{(1, -2)}$.

C6. Set the functions equal and solve for x:

$$\begin{aligned} f(x) &= g(x) \\ 2x^2 + 5x - 1 &= x^2 - 8x + 13 \\ x^2 + 13x - 14 &= 0 \\ (x + 14)(x - 1) &= 0 \\ x + 14 = 0 \quad &\text{or } x - 1 = 0 \\ \boxed{x = -14} \quad &\boxed{x = 1} \end{aligned}$$

C7. a) Combine like terms on each side:

$$\begin{aligned}3(x-5)+2&=-7(x+4)-8\\3x-15+2&=-7x-28-8\\3x-13&=-7x-36\\10x&=-23\\x&=\boxed{-\frac{23}{10}}\end{aligned}$$

b) Isolate the x^2 term and take square roots:

$$\begin{aligned}\frac{2}{3}x^2&=\frac{5}{4}\\x^2&=\frac{5}{4}\cdot 32=\frac{15}{8}\\x&=\boxed{\pm\sqrt{\frac{15}{8}}}.\end{aligned}$$

c) Set one side equal to zero and use the quadratic formula: $x^2+2x-10=0$ so

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{2^2-4(1)(-10)}}{2(1)}=\boxed{\frac{-2\pm\sqrt{44}}{2}}.$$

C8. a) $\boxed{v}$ is quadratic since its graph is a parabola.

b) The vertex of v is its turning point, which is $\boxed{(-3,-2)}$.

c) Since the parabola opens upward, its leading coefficient is $\boxed{\text{positive}}$.

d) w has $\boxed{\text{odd}}$ degree since its tails point in opposite directions.

e) The turning points of w are $\boxed{(-3,3.5)}$ and $\boxed{(2,-5)}$.

C9. To get VA $x=3$, we need an $x-3$ in the denominator and not in the numerator. To have no HA, we need the degree of the numerator to be greater than the degree of the denominator, so something like $\boxed{f(x)=\frac{x^2}{x-3}}$ works (other answers are possible).

C10. a) $\frac{x^2+4x+3}{x^2-2x-15}=\frac{(x+3)(x+1)}{(x+3)(x-5)}=\boxed{\frac{x+1}{x-5}}$.

b) $(x-3)^{-1}-4(x+2)^{-1}=\frac{1}{x-3}-\frac{4}{x+2}=\frac{x+2}{(x-3)(x+2)}-\frac{4(x-3)}{(x-3)(x+2)}=\frac{x+2-(4x-12)}{(x-3)(x+2)}=\boxed{\frac{-3x+10}{(x-3)(x+2)}}$.

c) $$\frac{\frac{1}{x}-\frac{3}{x+1}}{\frac{2}{x+1}-3} = \frac{\frac{x(x+1)}{x}-\frac{3x(x+1)}{x}}{\frac{2x(x+1)}{x+1}-3x(x+1)} = \frac{x+1-3(x+1)}{2x-3x^2-3x} = \boxed{\frac{-2x-2}{-3x^2-x}}.$$

C11. a) h diagrams as $x \xrightarrow{+2} \xrightarrow{\wedge 3} \xrightarrow{-8} h(x)$; inverting each arrow gives $x \xleftarrow{-2} \xleftarrow{\sqrt[3]{\ }} \xleftarrow{+8} h(x)$ so reverse-diagramming these arrows left-to-right we get $\boxed{h(x) = \sqrt[3]{x+8}-2}$.

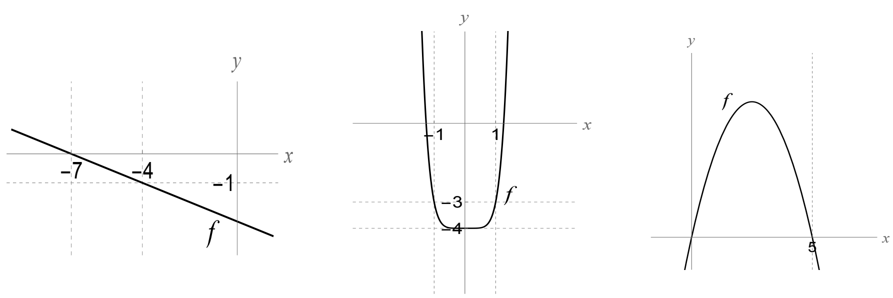
b) H diagrams as $x \xrightarrow{\times 2} \xrightarrow{+1} \xrightarrow{\sqrt{\ }} H(x)$; inverting each arrow gives $x \xleftarrow{\div 2} \xleftarrow{-1} \xleftarrow{\wedge 2} H(x)$ so reverse-diagramming these arrows right-to-left we get $\boxed{H^{-1}(x) = \frac{x^2-1}{2}}$.

3.9 Solutions to Practice Exam D

D1. a) This is a line with slope $-\frac{1}{3}$ passing through $(-7,0)$; this line is shown below at left.

b) Shift the graph of x^6 down 4 units to get the graph shown below in the middle.

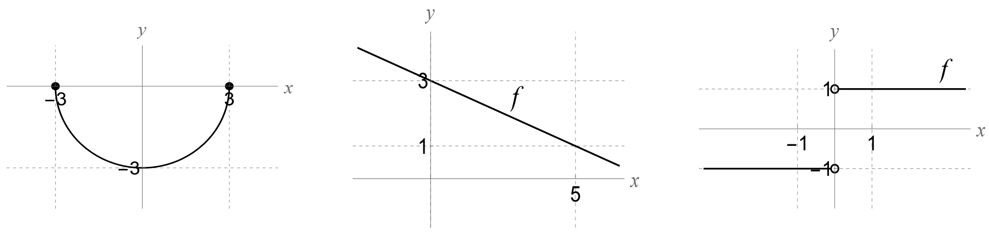
c) Multiply out the rule for f to get $f(x) = -x^2 + 5x$. Therefore f is a parabola with y-int $(0,0)$ and x-ints $(0,0)$ and $(5,0)$; since $a < 0$ the parabola opens downward, giving the picture shown below at right.



d) Take the semicircle of radius 3 centered at the origin and reflect it across the x-axis (because of the $(-)$ sign) to get the bottom half of the circle. This is graphed below at left.

e) This is a line with slope $-\frac{2}{5}$ and y-intercept $(0,3)$, graphed below in the center.

f) $f(x) = \dfrac{|x|}{x}$ is the signum function, graphed below at right.



D2. a) This is the graph of $\sqrt{x}$ shifted right 3 and up 2 units, so $\boxed{f(x) = \sqrt{x-3}+2}$.

b) this is the graph of $\dfrac{1}{x}$ shifted left 2 units, so $\boxed{f(x) = \dfrac{1}{x+2}}$.

D3. a) $m = \dfrac{\triangle y}{\triangle x} = \dfrac{4-(-3)}{5-5} = \dfrac{7}{0}$ which is $\boxed{\text{undefined}}$ (which makes sense since this line is vertical).

b) The slope is definitely positive and less than 1; I'd estimate at as $\boxed{\dfrac{1}{3}}$.

c) The change in the output is the slope times the change in the input, which in this problem is $5(10) = \boxed{50}$.

d) The graph of this function is a V, shifted right 5 units and up 3 units. The horizontal line $y = 4$ will hit this V twice, so the equation has $\boxed{2}$ solutions.

e) The graph of this function is a semicircle of radius 3 shifted right 2 units; the highest point on this semicircle is right in the middle, when $\boxed{x = 2}$.

D4. a) By the point-slope formula, this is $\boxed{y = 0 + \dfrac{1}{4}(x-8)}$.

b) The line has slope $m = \tan\dfrac{\pi}{4} = 1$, so by the point-slope formula the equation is $\boxed{y = -2 + 1(x+3)}$.

c) By the point-slope formula, this is $\boxed{y = -\dfrac{20}{7} + \dfrac{13}{4}\left(x - \dfrac{3}{7}\right)}$.

D5. From the second equation, $x = 2y - 1$. Substituting into the first equation, we get $5(2y-1) + 4y = -8$, i.e. $10y - 5 + 4y = -8$, i.e. $14y = -3$, i.e. $y = -\dfrac{3}{14}$. Using the equation $x = 2y - 1$, we have $x = 2\left(-\dfrac{3}{14}\right) - 1 = -\dfrac{3}{7} - 1 = -\dfrac{10}{7}$, so the intersection point is $\boxed{\left(-\dfrac{10}{7}, \dfrac{3}{14}\right)}$.

D6. We solve the two equations together as a system. First, clear the denominators by multiplying everything through by 15:

$$\begin{cases} \frac{3}{5}x + \frac{2}{3}y &= \frac{22}{15} \\ x + \frac{2}{5}y &= \frac{2}{3} \end{cases} \begin{array}{c} \xrightarrow{\times 15} \\ \xrightarrow{\times 15} \end{array} \begin{cases} 9x + 10y &= 22 \\ 15x + 6y &= 10 \end{cases}$$

We solve this system with addition/elimination:

$$\begin{cases} 9x + 10y &= 22 \\ 15x + 6y &= 10 \end{cases} \begin{array}{c} \xrightarrow{\times -5} \\ \xrightarrow{\times 3} \end{array} \begin{cases} -45x - 50y &= -110 \\ 45x + 18y &= 30 \end{cases}$$

Add the equations to get $-32y = 80$, i.e. $y = \frac{80}{-32} = -\frac{5}{2}$. Back-substitute in the equation $x + \frac{2}{5}y = \frac{2}{3}$ to get $x - 1 = \frac{2}{3}$, i.e. $x = \frac{5}{3}$. This makes the solution $\boxed{\left(\frac{5}{3}, -\frac{5}{2}\right)}$.

D7. a) Solve by factoring (or use the quadratic formula):

$$\begin{aligned} 2x^2 + 3x - 20 &= 0 \\ (2x+5)(x-4) &= 0 \\ 2x + 5 = 0 &\quad \text{or } x - 4 = 0 \\ \boxed{x = -\frac{5}{2}} &\quad \boxed{x = 4}. \end{aligned}$$

b) Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 4(1)(7)}}{2(1)} = \frac{-5 \pm \sqrt{-3}}{2},$$

meaning this equation has $\boxed{\text{no solution}}$.

c) Divide by 3 to get $x^5 = 9$, then take fifth roots of each side to get $x = \boxed{\sqrt[5]{9}}$.

D8. a) $\sqrt[5]{-x^3} = \boxed{-x^{3/5}}$.

b) $\sqrt{16x^2} = 4\sqrt{x^2} = \boxed{4|x|}$.

c) $\frac{8}{x^5} = \boxed{8x^{-5}}$.

d) $\frac{7}{x} \div \frac{21}{x^2} = \frac{7}{x} \cdot \frac{x^2}{21} = \boxed{\frac{1}{3}x}$

e) $\sqrt[4]{x}\sqrt[3]{2x}\sqrt{5x} = x^{3/4}\sqrt[3]{2}x^{2/3}\sqrt{5}x^{1/2} = \sqrt[3]{2}\sqrt{5}x^{3/4+2/3+1/2} = \boxed{\sqrt[3]{2}\sqrt{5}x^{25/12}}$.

f) $\frac{3}{\sqrt[3]{7x^3}} = \frac{3}{\sqrt[3]{7}} \cdot \frac{1}{x^{3/3}} = \boxed{\frac{3}{\sqrt[3]{7}}x^{-1}}$.

D9. a) $(F-G)(x) = 2(x-1)^{-1} - 3(x+5)^{-1} = \dfrac{2}{x-1} - \dfrac{3}{x+5} = \dfrac{2(x+5)}{(x-1)(x+5)} - \dfrac{3(x-1)}{(x-1)(x+5)} = \dfrac{2x+10-(3x-3)}{(x-1)(x+5)} = \boxed{\dfrac{-x+13}{(x-1)(x+5)}}$.

b) $\dfrac{F}{G}(x) = \dfrac{2}{x-1} \div \dfrac{3}{x+5} = \dfrac{2}{x-1} \cdot \dfrac{x+5}{3} = \boxed{\dfrac{2(x+5)}{3(x-1)}}$.

c) $F(x) = \dfrac{2}{x-1}$; the denominator is zero when $x = 1$ so $\boxed{x = 1}$ is the VA of F.

d) The graph of H is the graph of F shifted right 3 units. This shifts the VA of F 3 units right, from $x = 1$ to $\boxed{x = 4}$.

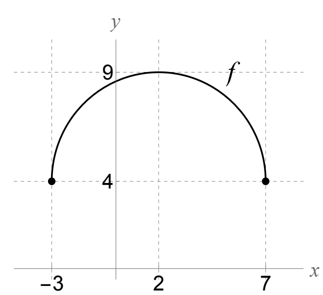
e) The graph of K is the graph of F, stretched by a factor of 4. This does not change the VA, so the VA of K is the same as that of F, namely $\boxed{x = 1}$.

f) The rule of $F \circ G$ is

$$F \circ G(x) = F\left(\frac{3}{x+5}\right) = \frac{2}{\frac{3}{x+5} - 1} = \frac{2(x+5)}{\frac{3(x+5)}{x+5} - (x+5)} = \frac{2x+10}{8-x}.$$

The denominator is zero when $x = 8$ and when $x = 8$, the numerator isn't zero. So $F \circ G$ has $\boxed{\text{VA } x = 8}$. Since the degree of the numerator and denominator of $F \circ G$ are equal, the HA is $y = \dfrac{\text{LC(top)}}{\text{LC(bottom)}} = \dfrac{2}{-1}$, i.e. $F \circ G$ has $\boxed{\text{HA } y = -2}$.

D10. These questions all use the fact that the graph of f is the top half of the circle of radius 5, shifted right 2 units and up 4 units. This graph looks like this:



From this graph, we can read off all the answers.

a) The domain of f is the set of x-values covered by the graph, which is $\boxed{[-3, 7]}$.

b) The minimum value of f is $\boxed{4}$.

c) f is minimized when $\boxed{x = -3 \text{ and } x = 7}$.

d) f has $\boxed{\text{no HA nor VA}}$.

e) The graph of g is the graph of f shifted right 3 units. This doesn't change the maximum value, so g has maximum value $\boxed{4}$.

f) What is the maximum value of h, where $h(x) = -f(x)$? The graph of h is the graph of f reflected across the x-axis. This graph will go as far down as $y = -9$ and as far up as $y = -4$, so its maximum value is $\boxed{-4}$.

g) $f(x) = 7$ has $\boxed{2}$ solutions since the graph has two points with y-coordinate 7.

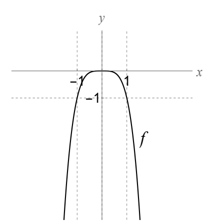
h) $f(x) = 9$ has $\boxed{1}$ solution since the graph has y-coordinate 9 in only one place (at the center of the circle, when $x = 2$).

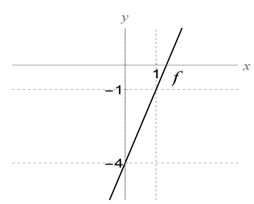
3.10 Solutions to Practice Exam E

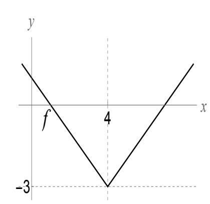
E1. a) Reflect the graph of x^4 across the x-axis to get the graph sketched below at left.

b) This is the line with y-intercept -4 and slope 3, shown below in the middle.

c) Start with the graph of $|x|$; shift it right 4 units and down 3 units to get the graph shown below at right.



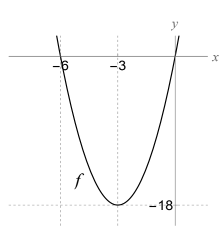


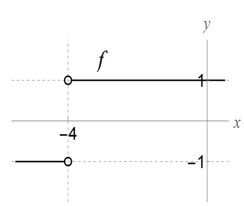


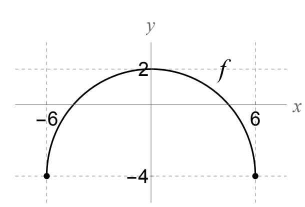
d) This is a parabola that opens upward; its vertex has x-coordinate $h = -\dfrac{b}{2a} = -\dfrac{12}{2(2)} = -3$ and has y-coordinate $k = f(h) = f(-3) = 2(9) + 12(-3) = -18$. Since the vertex is $(h, k) = (-3, -18)$ and it opens upward with y-int $(0, 0)$, it looks like the graph shown below in the middle.

e) Take the graph of the signum function and shift it left 4 units to get the graph shown below, in the middle.

f) Take the semicircle of radius 6 centered at the origin and shift it down 6 units to get the graph shown below, at right.







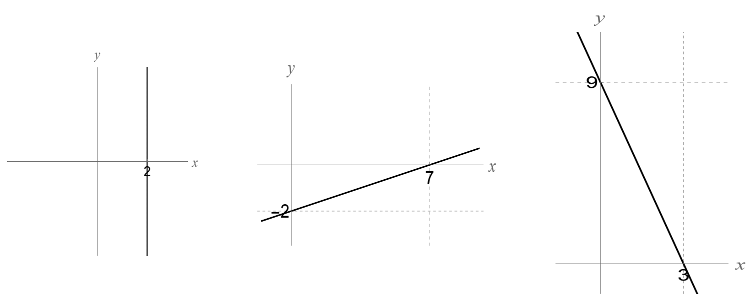
E2. a) This is the parabola x^2 shifted right 4 units and down 1 unit, so its rule is $\boxed{f(x) = (x-4)^2 - 1}$.

b) This is the graph of $\sqrt[3]{x}$ shifted down by 4 units, so its rule is $\boxed{f(x) = \sqrt[3]{x} - 4}$.

E3. a) $x = 2$ is the vertical line graphed below at left.

b) To graph this line, find its intercepts. For the x-intercept, set $y = 0$ to get $2x = 14$, i.e. $x = 7$. Thus its x-intercept is $(7, 0)$. For the y-int, set $x = 0$ to get $-7y = 14$, i.e. $y = -2$ so the y-int is $(0, -2)$. Graph the intercepts and connect them to get the line shown below in the middle.

c) By similar methods as in part (b), $3x + y = 9$ has x-intercept $(3, 0)$ and y-intercept $(0, 9)$, so it has the graph shown below at right.



E4. a) The given line has slope -3, so the line we want has slope $\frac{-1}{-3} = \frac{1}{3}$. By the point-slope formula, our line is $\boxed{y = -3 + \frac{1}{3}(x-7)}$.

b) The line passing through $(0, 5)$ and $(4, -3)$ has slope $m = \frac{\triangle y}{\triangle x} = \frac{-3-5}{4-0} = \frac{-8}{4} = -2$, so the line we want has slope $\frac{-1}{2}$. By the point-slope formula, the equation is $\boxed{y = 2 - \frac{1}{2}(x-3)}$.

c) Since the line has no x-intercept, it must be horizontal so it has slope 0. By the point-slope formula, its equation is $y = 3 + 0(x - 8)$, i.e. $\boxed{y = 3}$.

d) By the slope-intercept formula, this line has equation $\boxed{y = -2x + 6}$.

E5. Substitute the first equation into the second to get

$$\begin{aligned} 4x + 3(2x-7) &= 19 \\ 4x + 6x - 21 &= 19 \\ 10x &= 40 \\ x &= 4 \end{aligned}$$

Substitute into the first equation to get $y = 2(4) - 7 = 1$, so the solution is $\boxed{(4, 1)}$.

E6. Set the functions equal and solve for x:

$$\begin{aligned} f(x) &= g(x) \\ x^2+3x+1 &= 2x^2-3 \\ 0 &= x^2-3x-4 \\ 0 &= (x-4)(x+1) \end{aligned}$$

Therefore $x=4$ and $x=-1$. Since P has positive x-coordinate, we want $x=4$. For the y-coordinate, $y=f(4)=2(4^2)-3=29$ so the coordinates of P are $\boxed{(4,29)}$.

E7. a) Multiply through by 4 to clear the denominators, then combine like terms:

$$\begin{aligned} \frac{3}{2}x-\frac{5}{4} &= \frac{7}{4}x+2 \\ 6x-5 &= 7x+8 \\ \boxed{-13} &= x \end{aligned}$$

b) Factor the left-hand side as $3(x^2+4x-60)=3(x+10)(x-6)$, so $x+10=0$ or $x-6=0$. Thus $x=-10$ or $x=6$, so the solution set is $\boxed{\{-10,6\}}$.

c) Divide by 8 to get $x^2=9$; then take $\pm\sqrt{\ }$ of both sides to get $x=\pm\sqrt{9}=\boxed{\pm 3}$.

d) Use the quadratic formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1-4(5)(-3)}}{2(5)}=\boxed{\frac{-1\pm\sqrt{61}}{10}}.$$

E8. a) Since the tails point in opposite directions, the degree is $\boxed{\text{odd}}$.

b) This graph has 6 turning points, so the degree is at least $\boxed{7}$.

c) Since the right-hand tail points upward, the LC is $\boxed{\text{positive}}$.

d) This polynomial has $\boxed{1}$ x-intercept since it crosses the x-axis once.

E9. Since the degrees of the numerator and denominator are equal, the HA is $y=\dfrac{\text{LC(top)}}{\text{LC(bot)}}=\dfrac{2}{1}$, i.e. $\boxed{y=2}$.

For the VA, set the denominator equal to 0 to get $0=x^2+7x+12=(x+3)(x+4)$, which gives $x=-3$ and $x=-4$. Testing these in the numerator, we see $2(3^2)+3-3=18\neq 0$ and $2(16)-4-3\neq 0$, so f has two VA $\boxed{x=-3, x=-4}$.

E10. a) $5\div\dfrac{3}{x}=5\cdot\dfrac{x}{3}=\boxed{\dfrac{5}{3}x}$.

b) $\dfrac{34\sqrt{x}}{2x^2}=17x^{1/2-2}=\boxed{17x^{-3/2}}$.

c) $20(2x^3)^{-2}(3\sqrt{x})^4 = 20 \cdot \dfrac{1}{2^2(x^3)^2} \cdot 3^4 x^{1/2 \cdot 4} = 20 \cdot \dfrac{1}{4x^6} \cdot 81x^2 = 5(81)x^{2-6} = \boxed{405x^{-4}}$.

d) $3x(2x\sqrt{x})^3 = 3x(2x^{3/2})^3 = 3x(2^3 x^{3/2 \cdot 3}) = 3x(8x^{9/2}) = 24x^{1+9/2} = \boxed{24x^{11/2}}$.

E11. a) $\dfrac{\dfrac{2}{x-5} + \dfrac{3}{2x+1}}{\dfrac{1}{2x+1} + \dfrac{4}{x-5}} = \dfrac{\dfrac{2(x-5)(2x+1)}{x-5} + \dfrac{3(x-5)(2x+1)}{2x+1}}{\dfrac{(x-5)(2x+1)}{2x+1} + \dfrac{4(x-5)(2x+1)}{x-5}} = \dfrac{2(2x+1)+3(x-5)}{x-5+4(2x+1)} = \dfrac{4x+2+3x-15}{x-5+8x+4} = \boxed{\dfrac{7x-13}{9x-1}}$.

b) $\dfrac{3}{x^2-7x-18} - \dfrac{1}{x-9} = \dfrac{3}{(x-9)(x+2)} - \dfrac{1}{x-9} = \dfrac{3}{(x-9)(x+2)} - \dfrac{x+2}{(x-9)(x+2)} = \dfrac{3-(x+2)}{(x-9)(x+2)} = \boxed{\dfrac{-x+1}{(x-9)(x+2)}}$.

c) $\dfrac{x^2+9x+8}{x^2-7x} \cdot \dfrac{x^2-3x-28}{x^2-6x-7} = \dfrac{(x+8)(x+1)}{x(x-7)} \cdot \dfrac{(x-7)(x+4)}{(x-7)(x+1)} = \boxed{\dfrac{(x+8)(x+4)}{x(x-7)}}$.