MATH 130
Exam 4 Study Guide

David M. McClendon

Department of Mathematics
Ferris State University

Last updated May 2024

Contents

Chapter 1

Exam 4 Information

1.1 Exam 4 content

Exam 4 covers Chapter 4 in the 2024 version of my MATH 130 lecture notes.

This chapter is about transcendental functions, meaning:

Trig functions: $\sin$, $\cos$, $\tan$ (and the less important $\csc$, $\sec$, $\cot$)

Inverse trig functions: $\arcsin$, $\arctan$ (and the less important $\arccos$)

Exponential functions: $\exp$ a.k.a. e^x (and the less important b^x)

Logarithms: $\ln$ (and the less important $\log$ and $\log_b$)

1.2 Tasks for Exam 4

NOTE: This guide is not meant to be an exact representation of exam material. I always reserve the right to ask some questions that use the course material in a creative way.

1. Answer questions involving course vocabulary.
2. Classify statements as true or false.
3. Graph any transformation of a transcendental function, and use that graph to answer questions about the function (domain, range, maximum/minimum

values, increasing/decreasing, period, asymptotes, number of solutions to equations, etc.).

4. Given a graph of a transformed transcendental function, write a rule for the function.
5. Given the graph of an unknown function, sketch a graph of a transformation of that function.
6. Compute and simplify expressions involving transcendental functions (including the use of trig identities, exponent rules and log rules).
7. Rewrite expressions using log rules and/or change of base.
8. Solve basic equations involving transcendental functions.
9. Interpret $\arctan$ and $\arcsin$ expressions in the context of graphs, triangles and/or the unit circle; interpret logarithmic expressions in terms of graphs.
10. Write an equation of an exponential function passing through two points (when the x-coordinate of one of the points is 0).
11. Determine which of two functions is larger when the input is very large.
12. Diagram and/or reverse-diagram functions.

Chapter 2

Old MATH 130 Exam 4s

2.1 Spring 2024 Exam 4

1. In each part of this problem, you are given an expression.
   - If the expression does not exist, write "DNE" or something equivalent.
   - If the expression exists but cannot be reasonably simplified, just draw a box around it and move on.
   - If the expression exists and can be simplified, simplify it.

   a) (4.1) $\csc \frac{\pi}{6}$

   b) (4.1) $\sin^2 \frac{5\pi}{4}$

   c) (4.3) $\arctan 4$

   d) (4.4) $8 \arcsin \sqrt{2}$

   e) (4.3) $\arctan(-1) + \arctan \sqrt{3}$

   f) (4.1) $\cos 0 \tan \frac{\pi}{3}$

   g) (4.1) $\cot \pi$

   h) (4.1) $\sin 3$

2. Same directions as # 1:

   a) (4.7) $e^{4 \ln 2} - 5$

   b) (4.5) $\frac{2}{3}e^0$

   c) (4.6) $\ln(-6) + \ln 12$

   d) (4.6) $8e^{\ln 5}$

   e) (4.6) $\ln e + 3$

   f) (4.6) $\ln(3 - 2)$

   g) (4.7) $\frac{\ln 81}{\ln 3}$

   h) (4.7) $\ln 120 - \ln 4$

3. (4.5) Throughout this problem, let $f(x) = 7 - 4e^x$.

   a) What is the domain of f?

   b) Does f have a y-intercept? If so, what is its y-intercept?

c) Write the equation(s) of any horizontal asymptote(s) of f, if any.

d) Write the equation(s) of any vertical asymptote(s) of f, if any.

e) How many solutions does the equation $f(x) = 10$ have?

f) Which is larger, $f(200)$ or $f(300)$?

4. Sketch a graph of each function:

a) (4.1) $f(x) = 3\cos x + 2$

b) (4.6) $f(x) = \ln(x+3)$

c) (4.1) $f(x) = -\dfrac{1}{2}\sin 4\pi x$

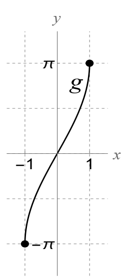
d) (4.5) $f(x) = -3^x$

e) (4.6) $f(x) = \log_4(-x)$

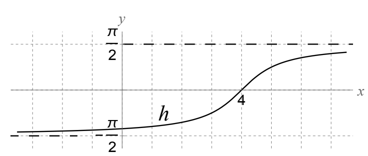
f) (4.1) $f(x) = \cos(x - \pi) + 4$

5. Write a rule for each function graphed here:

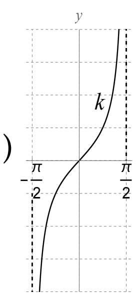
a) (4.4)



(b) (4.3)

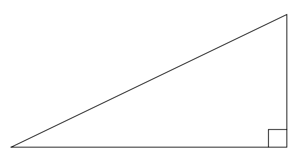


(c) (4.1)



6. a) (4.2) Simplify the expression $\dfrac{\cos(-x)}{\sec x} - \dfrac{\sin(-x)}{\csc x}$ using trig identities.

b) (4.4) Here is a picture of a right triangle:



Label this picture appropriately to explain what $\arcsin \dfrac{1}{3}$ means in the context of a right triangle.

7. a) (4.5) What is a reasonable decimal approximation of the number e?

b) (4.5) What is the point of the number e? In other words, why do we care about it so much?

c) (4.7) Expand the expression $\ln \dfrac{x^2}{19\sqrt[3]{y}}$ as much as possible.

d) (4.7) Rewrite $(x+2)^x$ as an exponential term with base e.

8. Solve each equation:

a) (4.4) $\sin x = \dfrac{2}{3}$

b) (4.4) $6\cos x = -3\sqrt{2}$

c) (4.7) $4e^x + 7 = 31$

d) (4.7) $\dfrac{2}{3}(\ln x + 4) = \dfrac{1}{2}(5\ln x - 3)$

Solutions

1. a) $\csc\frac{\pi}{6} = \frac{1}{\sin\frac{\pi}{6}} = \frac{1}{\frac{1}{2}} = \boxed{2}$.

   b) $\sin^2\frac{5\pi}{4} = \left(\sin\frac{5\pi}{4}\right)^2 = \left(-\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \boxed{\frac{1}{2}}$.

   c) $\boxed{\arctan 4}$ exists but cannot be simplified.

   d) $8\arcsin\sqrt{2}$ $\boxed{\text{DNE}}$ since $\sqrt{2} > 1$.

   e) $\arctan(-1) + \arctan\sqrt{3} = -\frac{\pi}{4} + \frac{\pi}{3} = -\frac{3\pi}{12} + \frac{4\pi}{12} = \boxed{\frac{\pi}{12}}$.

   f) $\cos 0 \tan\frac{\pi}{3} = 1\cdot\sqrt{3} = \boxed{\sqrt{3}}$.

   g) $\cot\pi = \frac{1}{\tan\pi} = \frac{1}{0}$ which $\boxed{\text{DNE}}$.

   h) $\boxed{\sin 3}$ exists but cannot be simplified.

2. a) $e^{4\ln 2} - 5 = 2^4 - 5 = 16 - 5 = \boxed{11}$.

   b) $\frac{2}{3}e^0 = \frac{2}{3}(1) = \boxed{\frac{2}{3}}$.

   c) $\ln(-6) + \ln 12$ $\boxed{\text{DNE}}$ since you can't take a log of a non-positive number.

   d) $8e^{\ln 5} = 8(5) = \boxed{40}$.

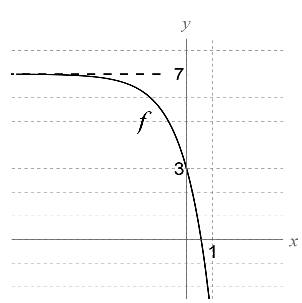
   e) $\ln e + 3 = 1 + 3 = \boxed{4}$.

   f) $\ln(3-2) = \ln 1 = \boxed{0}$.

   g) $\frac{\ln 81}{\ln 3} = \log_3 81 = \boxed{4}$.

   h) $\ln 120 - \ln 4 = \ln\frac{120}{4} = \boxed{\ln 30}$.

3. To answer these questions, first sketch a graph of f. Start with e^x, reflect it across the x-axis (which moves $(0,1)$ to $(0,-1)$), stretch it vertically by a factor of 4 (moving $(0,-1)$ to $(0,-4)$) and finally shift it up 7 units (which moves $(0,-4)$ up to $(0,-4+7) = (0,3)$ and moves the HA up from $y=0$ to $y=7$). This gives the following graph:



a) The domain of f is $\boxed{\mathbb{R}}$, the set of all real numbers, because the graph of f covers every value of x.

b) f has a y-intercept, since $f(0) = 7 - 4e^0 = 7 - 4(1) = 3$ so the y-intercept is $\boxed{(0, 3)}$. (You can also figure this out by tracking how the point $(0, 1)$ gets shifted.)

c) The HA of f is $\boxed{y = 7}$.

d) f, being an exponential function, has $\boxed{\text{no VA}}$.

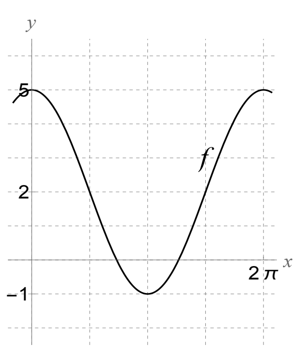
e) The equation $f(x) = 10$ has $\boxed{\text{no solution}}$ since the graph of f never hits the horizontal line $y = 10$.

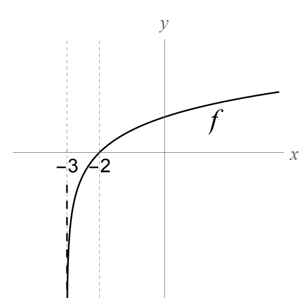
f) $\boxed{f(200)}$ is larger since the graph of f goes down from left to right.

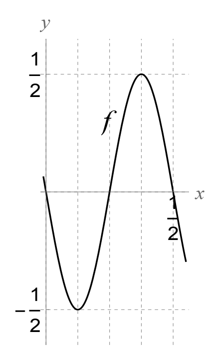
4. a) Start with $\cos x$, shift it up 2 units and stretch by a factor of 3 (so that it goes up as far as $2 + 3 = 5$ and down as far as $2 - 3 = -1$). The period isn't changed, so the graph looks like the one shown below at left.

b) This is a log graph, shifted left by 3 units so that its VA is $x = -3$ and that it passes through $(1-3, 0) = (-2, 0)$. This is the graph shown below in the center.

c) This function is a flipped sine graph that has been compressed vertically by a factor of $\frac{1}{2}$, meaning that it goes up as far as $\frac{1}{2}$ and down as far as $-\frac{1}{2}$. There is also a horizontal compression, making the period $T = \frac{2\pi}{B} = \frac{2\pi}{4\pi} = \frac{1}{2}$. This makes the graph the one shown below at right.



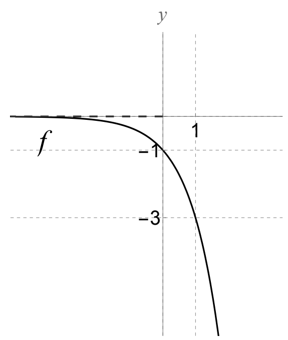


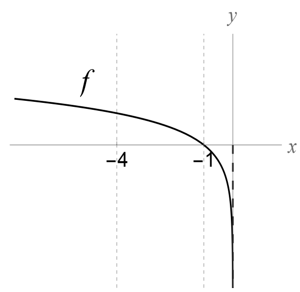


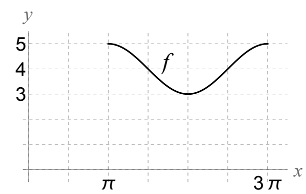
d) This is the exponential function 3^x, reflected across the y-axis so that its HA is still $y = 0$ but it now goes through $(0, -1)$ and $(1, -3)$; this graph is shown below at left.

e) Start with the graph of $\log_4 x$ and reflect it across the y-axis, so that it now passes through $(-1, 0)$ and $(-4, 1)$ (the VA is still $x = 0$). This graph is shown below in the center.

f) Take the graph of $\cos x$ and shift it right π units and up 4 units. This graph will now go up as far as $4+1=5$ and down as far as $4-1=3$, and since the period is still 2π, one period of the graph ends at $\pi + 2\pi = 3\pi$. This gives the graph shown below at right.







5. a) This is $\arcsin$, stretched vertically by a factor of 2: $\boxed{g(x) = 2\arcsin x}$.

   b) This is $\arctan$, shifted right by 4 units: $\boxed{h(x) = \arctan(x-4)}$.

   c) This is an unshifted tangent graph: $\boxed{k(x) = \tan x}$.

6. a) First, use odd-even identities to get rid of the $(-)$ on the $\cos(-x)$ term and pull out the $(-)$ from the $\sin(-x)$ term. This gives

$$\frac{\cos(-x)}{\sec x} - \frac{\sin(-x)}{\csc x} = \frac{\cos x}{\sec x} + \frac{\sin x}{\csc x}.$$

   Now, write everything in terms of sines and cosines to get

$$\frac{\cos x}{\frac{1}{\cos x}} + \frac{\sin x}{\frac{1}{\sin x}}$$

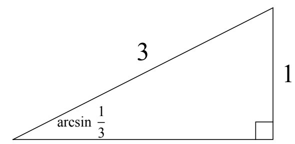
   and next, perform the division by flipping the second fraction over and multiplying. This gives

$$\cos x \cdot \frac{\cos x}{1} + \sin x \cdot \frac{\sin x}{1}.$$

   Last, perform the algebra and simplify using a Pythagorean identity

$$\cos x \cdot \frac{\cos x}{1} + \sin x \cdot \frac{\sin x}{1} = \cos^2 x + \sin^2 x = \boxed{1}.$$

   b) If $x = \arcsin\frac{1}{3}$, then $\sin x = \frac{1}{3} = \frac{\text{opposite}}{\text{hypotenuse}}$. So we label the opposite side 1 and the hypotenuse 3, and $\arcsin\frac{1}{3}$ is therefore the angle opposite the 1:



7. a) $e \approx \boxed{2.7}$ (or 2.71828 if you know a lot about e).

   b) What makes e special is that if you make the function $f(x) = e^x$, then the rate of change of f is the same as the output of f.

   c) $\ln \dfrac{x^2}{19\sqrt[3]{y}} = \ln x^2 - \ln 19y^{1/3} = 2\ln x - (\ln 19 + \ln y^{1/3}) = \boxed{2\ln x - \ln 19 - \frac{1}{3}\ln y}$.

   d) Use the change of base formula to get $(x+2)^x = \boxed{e^{x\ln(x+2)}}$.

8. a) To solve a sine equation, use arcsine: $x = \boxed{2\pi n + \arcsin\frac{2}{3}, 2\pi n + \pi - \arcsin\frac{2}{3}}$.

   b) First, divide both sides by 6 to isolate the $\cos x$ term. This gives $\cos x = -\frac{\sqrt{2}}{2}$. Now, solve using arccosine: $x = 2\pi n \pm \arccos -\frac{\sqrt{2}}{2} = \boxed{2\pi n \pm \frac{3\pi}{4}}$.

   c) Isolate the e^x term first. To do this, subtract 7 from both sides and then divide by 4. This yields $4e^x = 24$, i.e. $e^x = 6$. That means $x = \boxed{\ln 6}$.

   d) First, multiply through both sides by 6 to clear the fractions; then distribute and combine like terms:

$$\begin{aligned}
\frac{2}{3}(\ln x + 4) &= \frac{1}{2}(5\ln x - 3) \\
6\left[\frac{2}{3}(\ln x + 4)\right] &= 6\left[\frac{1}{2}(5\ln x - 3)\right] \\
4(\ln x + 4) &= 3(5\ln x - 3) \\
4\ln x + 16 &= 15\ln x - 9 \\
25 &= 11\ln x \\
\frac{25}{11} &= \ln x
\end{aligned}$$

   To get rid of the $\ln$, exponentiate. This means $x = \boxed{\exp\frac{25}{11}} = \boxed{e^{25/11}}$.

2.2 Relevant exam questions from Spring 2018

1. Sketch crude graphs of each of these functions:

   a) $f(x) = \ln x$
   b) $f(x) = \cos x$
   c) $f(x) = \arctan x$
   d) $f(x) = 4^x$

2. Evaluate each of the following expressions:

   a) $\log_9 3$
   b) $\log 10000$
   c) $\ln e^7$
   d) $e^{3\ln 2}$
   e) $\sin \frac{5\pi}{3}$
   f) $\tan \frac{\pi}{2}$
   g) $\tan \frac{5\pi}{4}$
   h) $\cos \frac{-\pi}{6}$
   i) $\arctan \sqrt{3}$
   j) $\arcsin \frac{-1}{2}$

3. Find all solutions of each of the following equations:

   a) $\sin x = \frac{\sqrt{3}}{2}$
   b) $5 \tan x = 5$

4. Classify each of the following statements as true or false:

   a) $\log_4 \frac{A}{B} = \log_4 A - \log_4 B$.
   b) The graph of a logarithmic function has a horizontal asymptote.
   c) $\tan(\arctan x) = x$.
   d) $\sin(A + B) = \sin A + \sin B$.
   e) The function $f(x) = 2^x$ is an even function.
   f) The statement $\log_y z = w$ is the same as the statement $z^w = y$.
   g) $\ln 1 = 0$.
   h) $\sin(-x) = -\sin x$.

5. Sketch the graph of each of the following functions:

   a) $f(x) = \arctan(x + 5)$
   b) $f(x) = e^{-x}$
   c) $f(x) = -\sin 2x$

6. Sketch the graph of each of the following functions:

a) $f(x) = -\log_3(x+6)$

b) $f(x) = \cos\dfrac{x}{3} + 2$

c) $f(x) = 3\sin\left(x + \dfrac{3\pi}{2}\right)$

7. Answer each question, writing your answers with correct notation. If any of these things fail to exist, say so.

   a) What is/are the horizontal asymptote(s) of the function $f(x) = \ln x + 5$?

   b) What is/are the vertical asymptote(s) of the function $f(x) = \ln(x+2) - 7$?

   c) What is the period of the function $f(x) = 3\sin 4x - 1$?

   d) Find the largest value of y obtained by this function:

   $$f(x) = 2\cos(x+\pi) - 5$$

   e) What is/are the y-intercept(s) of the function $f(x) = e^{-x} - 3$?

8. Rewrite each expression in terms of natural logarithms and/or natural exponentials:

   a) $\log_5 73$

   b) 4^x

9. Evaluate each expression:

   a) $\sin\dfrac{2\pi}{3}$

   b) $\tan\pi$

   c) $\cos\dfrac{-\pi}{4}$

   d) $e^{\ln 4}$

   e) $\log_5\dfrac{1}{5}$

   f) $\log_6 9 + 2\log_6 2$

   g) $4!$

   h) $\arcsin\dfrac{1}{2}$

   i) $\arctan -1$

10. Classify the following statements as true or false:

    a) To compute $\sin^2 x$, first take the sine of x, then square the answer.

    b) $\sin(x-y) = \sin x - \sin y$.

    c) $\sin(-x) = -\sin x$.

    d) $\tan(\arctan x) = x$.

    e) $\arctan(\tan x) = x$.

    f) e^x has a graph which is continuous, but not smooth.

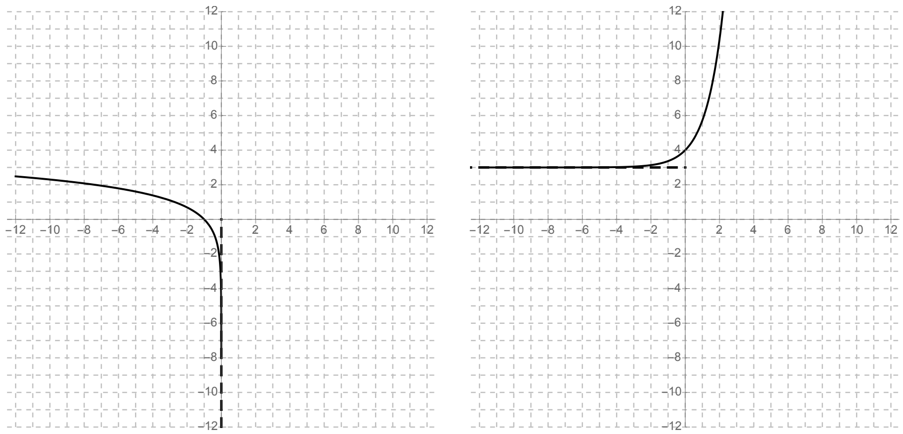
11. The function $\cos x$ is an example of an **even** function.

    a) What property does the word "even" refer to about the graph of $\cos x$?

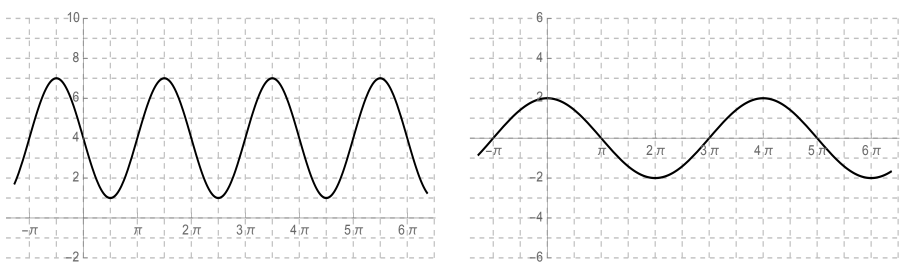
    b) What algebraic fact / trig identity does the fact that $\cos x$ is even refer to?

12. Write the rule for each function graphed here. Assume that any exponential and/or logarithmic functions are base e.

(a-b)



(c-d)

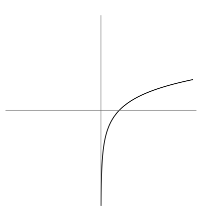


13. a) One of these two functions has a vertical asymptote. Circle the one that has a VA, and write the equation of its vertical asymptote.

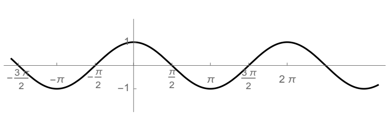
$$F(x) = \ln(x-2) \qquad\qquad G(x) = e^{x-2}$$

Solutions

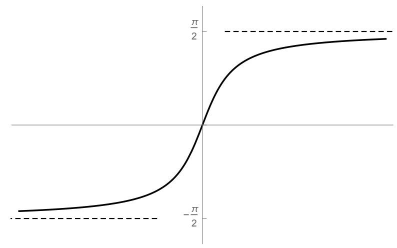
1. a)

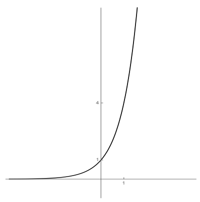


b)



c)



d) 

2. a) $\log_9 3 = \boxed{\dfrac{1}{2}}$ (because $9^{1/2} = \sqrt{9} = 3$).
   b) $\log 10000 = \boxed{4}$ (because $10^4 = 10000$).
   c) $\ln e^7 = \boxed{7}$ (by a Cancellation Law).
   d) $e^{3\ln 2} = 2^3 = \boxed{8}$ (by a log/exp rule).
   e) $\sin \dfrac{5\pi}{3} = \boxed{\dfrac{\sqrt{3}}{2}}$ (Quadrant II (+), ref. angle $\frac{\pi}{3} = 60°$)
   f) $\tan \dfrac{\pi}{2}$ $\boxed{\text{DNE}}$ (slope at $90°$ is undefined).
   g) $\tan \dfrac{5\pi}{4} = \boxed{1}$ (Quadrant III (−), ref. angle $\dfrac{\pi}{4} = 45°$ where the slope is 1)
   h) $\cos \dfrac{-\pi}{6} = \cos \dfrac{\pi}{6} = \boxed{\dfrac{\sqrt{3}}{2}}$.
   i) $\arctan \sqrt{3} = \boxed{\dfrac{\pi}{3}}$.
   j) $\arcsin \dfrac{-1}{2} = -\arcsin \dfrac{1}{2} = \boxed{-\dfrac{\pi}{6}}$.

3. a) One solution is $x = \arcsin \dfrac{\sqrt{3}}{2} = \dfrac{\pi}{3}$. A second solution is $\pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$. Therefore all solutions are $\boxed{\dfrac{\pi}{3} + 2\pi n}$ and $\boxed{\dfrac{2\pi}{3} + 2\pi n}$ where n is any integer.
   b) First, divide both sides by 5 to get $\tan x = 1$. Then one solution is $x = \arctan 1 = \dfrac{\pi}{4}$; all solutions are $\boxed{\dfrac{\pi}{4} + \pi n}$ where n is any integer.

4. a) $\boxed{\text{TRUE}}$ (log of a quotient is the difference of the logs)
   b) $\boxed{\text{FALSE}}$ (exponential functions have HA, logarithmic functions have VA)
   c) $\boxed{\text{TRUE}}$ (this is the “good” cancellation law for $\arctan$ and $\tan$)
   d) $\boxed{\text{FALSE}}$ (if $A = B = \dfrac{\pi}{2}$, the left-hand side is $\sin \pi = 0$ but the right-hand side is $1 + 1 = 2$)

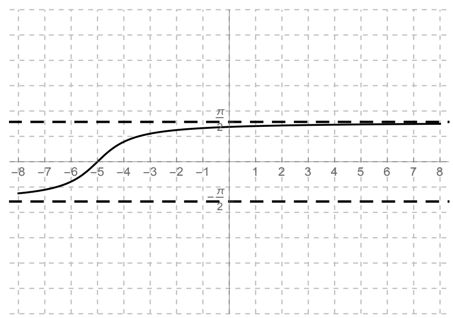
e) FALSE (the graph is not symmetric about the y-axis)

f) FALSE ($\log_y z = w$ is the same as $y^w = z$.

g) TRUE (since $e^0 = 1$).

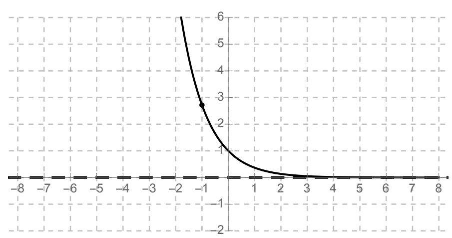
h) TRUE ($\sin$ is an odd function).

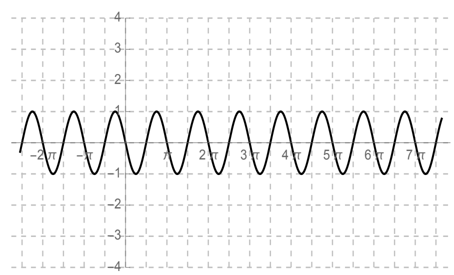
5. a) Take the graph of $\arctan x$, shift it left 5 units to get the graph below.



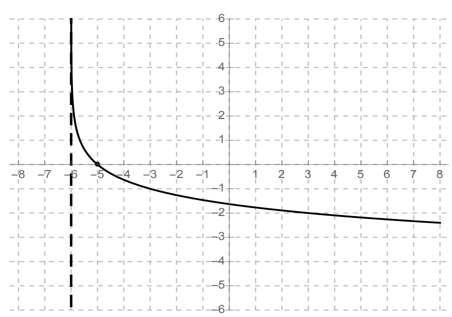
b) Take the graph of e^x and reflect it across the y-axis to get the bottom left below.

c) Take the graph of $\sin x$, smash it horizontally by a factor of 2, and then reflect across the x-axis to get the bottom right below.



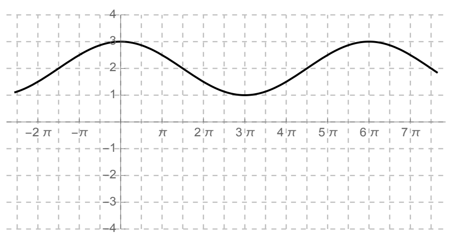


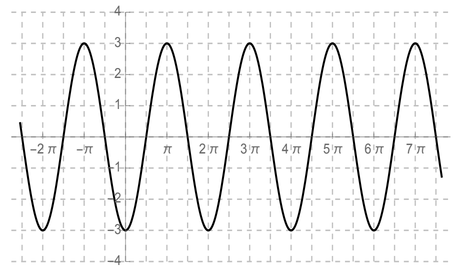
6. a) Take the graph of $\log_3 x$, shift it left 6 units, then reflect across the x-axis to get the graph below.



b) Take the graph of $\cos x$, stretch it horizontally by a factor of 3, then shift it up 2 units to get the bottom left graph below.

c) Take the graph of $\sin x$, shift it left $\frac{3\pi}{2}$ units, then stretch by a factor of 3 to get the bottom right graph below.





7. a) Logarithmic graphs do not have horizontal asymptotes, so there are $\boxed{\text{none}}$.

b) The VA of $\ln x$ is $x = 0$; since the graph is shifted two units left, the VA moves to $\boxed{x = -2}$.

c) The graph is smashed horizontally by a factor of 4, making the period $\frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$.

d) The graph is stretched by a factor of 2 and shifted down 5 units, so its range is $[-2-5, 2-5] = [-7, -3]$, making the answer to the question $\boxed{-3}$.

e) The y-intercept of e^x is $(0, 1)$; reflecting through the x-axis does not change this, but shifting down by 3 units moves the y-intercept to $\boxed{(0, -2)}$.

8. a) $\log_5 73 = \boxed{\frac{\ln 73}{\ln 5}}$.

b) $4^x = \boxed{e^{x \ln 4}}$.

9. a) $\sin \frac{2\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$ (Quadrant II; ref. angle $\frac{\pi}{3} = 60°$)

b) $\tan \pi = \boxed{0}$ (slope at $180°$ is zero)

c) $\cos \frac{-\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$ (Quadrant IV; ref. angle $\frac{\pi}{4} = 45°$)

d) $e^{\ln 4} = \boxed{4}$

e) $\log_5 \frac{1}{5} = \boxed{-1}$ (since $5^{-1} = \frac{1}{5}$)

f) $\log_6 9 + 2\log_6 2 = \log_6 9 + \log_6 2^2 = \log_6(9 \cdot 2^2) = \log_6 36 = \boxed{2}$.

g) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$.

h) $\arcsin \frac{1}{2} = \boxed{\frac{\pi}{6}}$.

i) $\arctan -1 = -\arctan 1 = \boxed{-\frac{\pi}{4}}$.

10. a) $\boxed{\text{TRUE}}$ (this is the definition of $\sin^2 x$)

    b) $\boxed{\text{FALSE}}$ (try $x = \pi, y = \frac{\pi}{2}$; then $\sin(x-y) = \sin\frac{\pi}{2} = 1$ but $\sin x - \sin y = 0 - 1 = -1$)

    c) $\boxed{\text{TRUE}}$ ($\sin$ is an odd function)

    d) $\boxed{\text{TRUE}}$ ($\tan$ inverts $\arctan$)

    e) $\boxed{\text{FALSE}}$ (only true if $\frac{-\pi}{2} < x < \frac{\pi}{2}$)

    f) $\boxed{\text{FALSE}}$ (since this graph has no sharp corners, it is smooth as well as continuous)

11. a) "Even" means the graph is symmetric about the y-axis.

    b) $\boxed{\cos(-x) = \cos x}$.

12. a) This is $\ln x$, reflected across the y-axis, i.e. $\boxed{f(x) = \ln(-x)}$.

    b) This is e^x, shifted up 3 units, i.e. $\boxed{f(x) = e^x + 3}$.

    c) This is $\sin x$, reflected across the x-axis, shifted up 4 units and stretched vertically by a factor of 3, i.e. $\boxed{f(x) = -3\sin x + 4}$.

    d) This is $\cos x$, stretched vertically by a factor of 2 and stretched horizontally by a factor of 2, i.e. $\boxed{f(x) = 2\cos\frac{x}{2}}$.

13. a) $\boxed{F(x) = \ln(x-2) \text{ has VA } x = 2}$ (its graph is the graph of $\ln x$, shifted right by 2 units).

Chapter 3

Additional Practice Exam 4s

3.1 Practice Exam A

A1. Sketch a graph of each function:

a) $f(x) = \sin 2x$

b) $f(x) = -\arctan x$

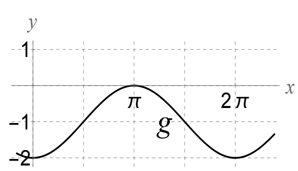
c) $f(x) = -\ln x + 2$

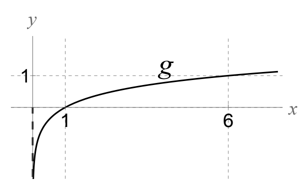
d) $f(x) = \cos(x-3)+4$

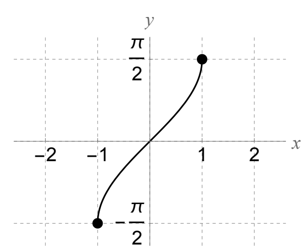
e) $f(x) = \tan x$

f) $f(x) = -3 - e^x$

A2. For each given graph, write a rule for the function that has that graph:

a) 

b) 

c) 

A3. Solve each equation:

a) $\cos x = -\dfrac{\sqrt{2}}{2}$

b) $\csc x = 0$

c) $\ln x = e^4$

d) $\arcsin x = 1$

e) $\tan x = 3$

f) $15 - 2e^x = 1$

g) $\ln x = 0$

A4. Simplify each quantity, if possible (if the quantity does not exist, say so). If the quantity exists but cannot be simplified, just draw a box around it and move to the next question.

a) $(2e^2)^3e^5$

b) $3\tan\pi$

c) $\sin\frac{2\pi}{3}$

d) $-3\cos\frac{3\pi}{2}$

e) $\frac{1}{\pi}\arcsin -\frac{\sqrt{3}}{2}$

f) $\arcsin 1 + 3\arctan 1$

g) $4\cos\frac{3\pi}{4}$

h) $3 - \ln\frac{1}{e}$

i) $\log -100$

j) $\frac{\ln 50 + \ln 2}{\ln 30 - \ln 3}$

k) $(\arcsin 0)\cos\frac{3\pi}{7}$

l) $\frac{1}{4}\sec^2\frac{\pi}{6}$

m) $7\arctan 2$

n) $e^{\ln\frac{2}{5}}$

o) $\tan\frac{\pi}{2}\arctan 1$

p) $7^{\log_7(-2)}$

q) $\cot\frac{\pi}{2}$

r) $e^{3\ln -1}$

s) $\log_4 64$

A5. Expand each expression using log rules:

a) $\log_3 9x^5$

b) $\ln\frac{5\sqrt{e}x^3}{\sqrt[4]{y}}$

A6. Simplify the following expression using trig identities:

$$\sin x + \cos(-x)\cot x$$

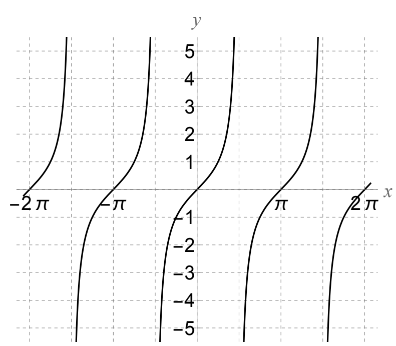
A7. a) Rewrite $\cos^2 x$ in terms of $\sin x$.

b) Rewrite $3 + 2\sec^2 x$ in terms of $\tan x$.

c) Rewrite $4 - \cot^2 w$ in terms of $\csc w$.

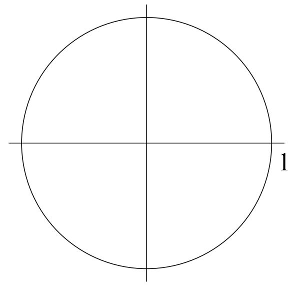
A8. Write the rule for a function of the form $f(x) = a\cdot b^x$ that passes through the points $(0, 12)$ and $(4, 3)$.

A9. a) Here is a graph of $\tan x$:



Explain, by drawing a picture on this graph, how you would find $\arctan -3$ using this graph.

b) Here is a picture of the unit circle:



Explain, by sketching a picture on this circle, how you would interpret $\arcsin .8$ in the context of the unit circle.

A10. Diagram each of these functions (remember that the only trig functions allowed in diagrams are sine, cosine and tangent, and the only exponential and logarithmic functions allowed are natural exponentials and logarithms):

a) $F(x) = 3\sec x$ b) $G(x) = 2^x$ c) $H(x) = \log_{15}(x-4)$

A11. Throughout this problem, let $k(x) = 3e^x + 2$.

a) What is the domain of k?

b) What is the range of k?

c) Write the equation(s) of any horizontal asymptote(s) of k, if any:

d) Write the equation(s) of any vertical asymptote(s) of k, if any:

e) Find the x-coordinate(s) of any points on the graph of k which have y-coordinate 11.

f) Evaluate $k(\ln 5)$.

g) As x goes from left to right, does the graph of k increase or decrease?

3.2 Practice Exam B

B1. Solve each equation:

a) $\sin x = \dfrac{\sqrt{3}}{2}$

b) $\ln x = -3$

c) $3 \arcsin x = \pi$

d) $e^x = 37$

e) $\dfrac{1}{\sqrt{3}} \cot x = 1$

f) $3 \cos x = 2$

B2. Sketch a graph of each function:

a) $f(x) = 3 \sin\left(x - \dfrac{\pi}{2}\right)$

b) $f(x) = 4 + \dfrac{1}{2} e^x$

c) $f(x) = -\ln(x+5)$

d) $f(x) = \tan \dfrac{x}{8}$

e) $f(x) = \left(\dfrac{1}{5}\right)^x$

f) $f(x) = 2 - 2\cos x$

g) $f(x) = \arcsin(-x)$

h) $f(x) = \log_6 x$

i) $f(x) = \dfrac{1}{\pi} \arctan x$

B3. Simplify each quantity, if possible (if the quantity does not exist, say so). If the quantity exists but cannot be simplified, just draw a box around it and move to the next question.

a) $\sin \dfrac{\pi}{2}$

b) $\tan -\dfrac{5\pi}{6}$

c) $\dfrac{-\ln 2}{\ln 4}$

d) $e^{\sin 0}$

e) $\sec \dfrac{\pi}{4}$

f) $3 \arcsin 0$

g) $2 \cos 0 + 3$

h) $\dfrac{e^8}{e^3}$

i) $\log_8 8^3$

j) $\exp\left(3 \ln \dfrac{1}{3}\right)$

k) $\log_3 9 + 1$

l) $8 \arctan \sqrt{3}$

m) $\ln(e^8 + 1)$

n) $8 \arcsin \dfrac{4}{3}$

o) $\ln 0$

p) $\ln(e^8) \sin \dfrac{3\pi}{4}$

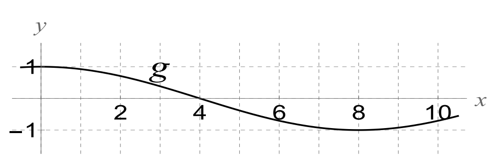
q) $\tan \dfrac{2\pi}{3}$

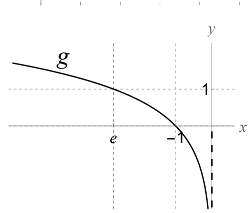
r) $6 \ln \sqrt[4]{e}$

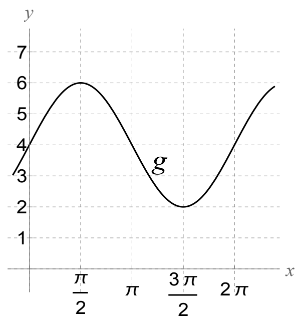
s) $6 \log_{64} 8$

t) $\dfrac{\ln 27}{\ln 3}$

B4. For each given graph, write a rule for the function that has that graph:

a) 

b) 

c) 

B5. Write each expression as a single logarithm:

a) $3\ln 2x - \ln y$

b) $\ln 5 + \frac{2}{7}\ln w$

B6. Suppose $a = \ln x$ and $b = \ln y$. Rewrite each expression in terms of a and/or b:

a) $3\ln 5x$

b) $\ln x^7\sqrt[3]{y}$

c) $\ln \frac{x}{y}$

d) $\log_y x$

e) x

B7. Simplify the following expression using trig identities:

$$\frac{1-\cos^2 x}{\sin x}$$

B8. In each part of this problem you are given two functions. Circle the function that is larger, when the input is very large:

a) $f(x) = 3^x$ $\qquad g(x) = 80^{100}\cdot 2^x$

b) $f(x) = 4^x 2^x$ $\qquad g(x) = 7^x$

c) $f(x) = 3^{-x}$ $\qquad g(x) = 2^{-x}$

d) $f(x) = x^{2024}$ $\qquad g(x) = \left(\frac{3}{2}\right)^x$

B9. Throughout this problem, let $g(x) = 2\sin x - 1$.

a) What is the minimum value of g?

b) What is the domain of g?

c) Write the equation(s) of any vertical asymptote(s) of f, if any:

d) Find the x-coordinates of any x-intercepts of g.

e) Compute $f\left(\frac{3\pi}{2}\right)$.

f) Suppose z is some number such that $g(z) = \frac{2}{3}$.

i. Do you know what $g(z) + 1$ is? If so, what is it?

ii. Do you know what $g(z + 1)$ is? If so, what is it?

iii. Do you know what $g(z + 2\pi)$ is? If so, what is it?

3.3 Practice Exam C

C1. Sketch a graph of each function:

a) $f(x) = 4\cos\frac{x}{8}$

b) $f(x) = -\frac{1}{2}\sin x + 1$

c) $f(x) = -2\tan x$

d) $f(x) = e^{x-2}$

e) $f(x) = \arctan 4x$

f) $f(x) = \frac{1}{4}\arcsin x$

C2. Simplify each quantity, if possible (if the quantity does not exist, say so). If the quantity exists but cannot be simplified, just draw a box around it and move to the next question.

a) $3\sin\frac{\pi}{8}$

b) $3^{\log_3 7}$

c) $\frac{2\ln 6 - \ln 4}{-4\ln 3}$

d) $\frac{\ln 3 + \ln 12}{\ln 6}$

e) $4\tan^2\frac{\pi}{3}$

f) $\cos\frac{\pi}{5}$

g) $\frac{\sqrt{e}}{e^2}$

h) $\ln 5$

i) $\arcsin\sin\frac{1}{5}$

j) $\log .01$

k) $4e^{-2\ln 6}$

l) $3\ln^2 e^5$

m) $\frac{1}{5}\arctan -\frac{1}{\sqrt{3}}$

n) $\sin -3\pi + \pi$

o) $\csc^4\frac{3\pi}{4}$

p) $2\tan\arctan -4$

q) $\log_5 5^8$

r) $\sin\frac{\pi}{4}\ln e$

s) $\cos^2\frac{\pi}{6}$

C3. Solve each equation:

a) $5(e^x + 2) = 27$

b) $4\tan x - 7 = -11$

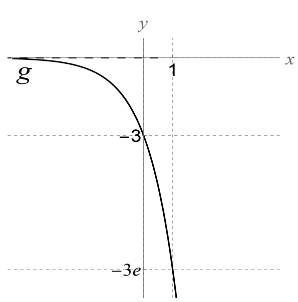
c) $2\ln x - 11 = 7$

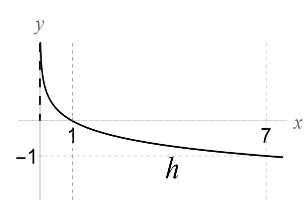
d) $4\sin x = -2$

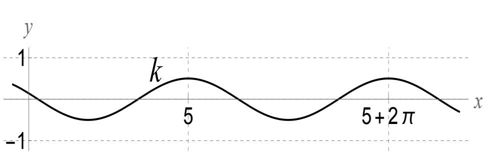
e) $\arctan x = \frac{2}{3}$

f) $7\sec x - 5 = 7$

C4. For each given graph, write a rule for the function that has that graph:

a) 

b) 

c) 

C5. Write each expression as a single logarithm:

a) $4 \ln x + 3 \ln y$

b) $\frac{3}{2} \log x + 2$

C6. Simplify the following expressions using trig identities, if possible. If an expression cannot be simplified, draw a box around it and move on:

a) $1 + \sec^2 x$

b) $\cot^2 4x + 1$

c) $3 - 3 \sin^2 x$

d) $\sin^2 x + \cos^2 2x$

C7. Write the rule for a function of the form $f(x) = a \cdot b^x$ that passes through the points $(0, -2)$ and $(2, -8)$.

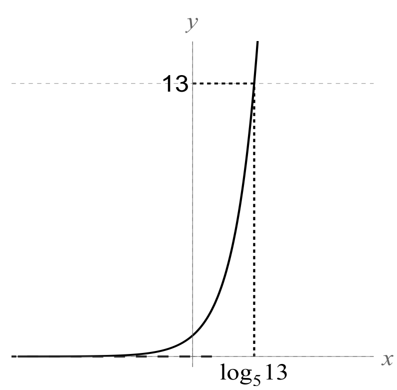
C8. Diagram each of these functions (remember that the only trig functions allowed in diagrams are sine, cosine and tangent, and the only exponential and logarithmic functions allowed are <u>natural</u> exponentials and logarithms):

a) $a(x) = x^{\sin x}$

b) $b(x) = 4x \ln x + 3$

c) $c(x) = \cot x$

C9. This picture shows how to interpret $\log_5 13$ in the context of graphs:



What is the rule for the function being graphed in this picture?

C10. In each part of this problem you are given two functions. Circle the function that is larger, when the input is very large:

a) $f(x) = 8\sqrt{x}$ $\quad g(x) = 8\sqrt[3]{x}$

b) $f(x) = \dfrac{1}{5}x^3$ $\quad g(x) = 100x^{2.75}$

c) $f(x) = x^{30}$ $\quad g(x) = 2^x$

d) $f(x) = 100 \cdot 2^x$ $\quad g(x) = (2.01)^x$

C11. Throughout this problem, let $f(x) = 5\cos\frac{\pi}{4}x + 2$.

a) What is the minimum value of f?

b) What is the maximum value of f?

c) Write the equation(s) of any horizontal asymptote(s) of f, if any:

d) How many solutions does the equation $f(x) = -1$ have?

e) How many solutions does the equation $f(x) = -4$ have?

f) What is the period of f?

3.4 Practice Exam D

D1. Simplify each quantity, if possible (if the quantity does not exist, say so). If the quantity exists but cannot be simplified, just draw a box around it and move to the next question.

a) $-2\sin\dfrac{-\pi}{4}$

b) $\ln\cos 0$

c) $8\cos\dfrac{\pi}{3}$

d) $\log_2\dfrac{1}{4}$

e) $\log_4 48$

f) $\log_7\dfrac{1}{7} + 5e^{\tan 0}$

g) $\left(e^7\right)^3$

h) $\tan\dfrac{\pi}{4}$

i) $\arcsin -1$

j) $\ln -3$

k) $4\arcsin\dfrac{1}{2}$

l) $4\ln e^7$

m) $-2\cot -\dfrac{\pi}{6}$

n) $\ln\exp 7$

o) $5\csc^2\dfrac{\pi}{3}$

p) $e^{4\ln 3} - 5$

q) $\log 1000$

r) $5 - \arctan 1$

s) $\log_5(15+10)$

D2. Solve each equation:

a) $4\sin x + 2 = 7$

b) $\dfrac{1}{2}\left(3\ln x + \dfrac{1}{4}\right) = \dfrac{1}{4}(7\ln x - 1)$

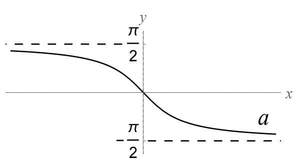
c) $\dfrac{1}{2}\cos x - \dfrac{1}{2} = 0$

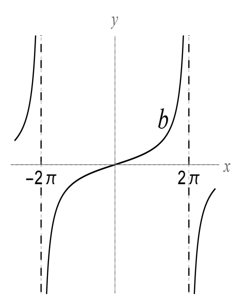
d) $8\arctan x + 5 = 23$

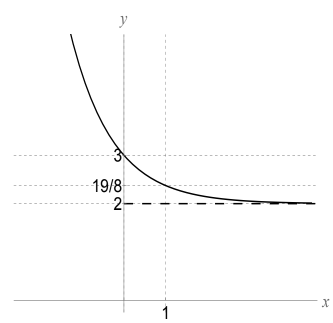
e) $2(\tan x - 1) = 5(\tan x + 2)$

f) $e^x = -2$

D3. For each given graph, write a rule for the function that has that graph:

a) 

b) 

c) 

D4. Sketch a graph of each function:

a) $f(x) = \ln(-x)$

b) $f(x) = -2^x$

c) $f(x) = -\cos 3\pi x$

d) $f(x) = e^{-x}$

e) $f(x) = \sin x - 6$

f) $f(x) = 3\ln(x-2)$

g) $f(x) = \arcsin x$

D5. Simplify the following expression using trig identities:

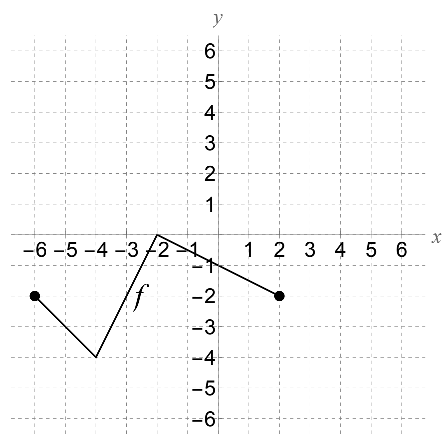
$$\frac{\cos x - \sec x}{\tan x}$$

D6. Expand each expression using log rules:

a) $\log_{1/2} 4\sqrt{y}$

b) $\ln e^6 x^{-4/3}(y-2)^2$

D7. Here is the graph of some unknown function f:



Use this graph to sketch a graph of each of these functions:

a) $f(-x)$

b) $-f(x)$

c) $f(2x)$

d) $\frac{1}{2}f\left(\frac{x}{3}\right)$

e) $-f(x+3)$

D8. Classify each statement as true or false:

a) The function $f(x) = e^{x-3} + 4$ has a horizontal asymptote.

b) The period of $f(x) = \cos \pi x$ is π.

c) $\arctan$ is an odd function.

d) The function $g(x) = -e^{-x}$ is increasing (from left to right).

e) The range of $f(x) = \ln x$ is $\mathbb{R}$, the set of all real numbers.

f) The equation $\cos x = \frac{5}{4}$ has exactly one solution.

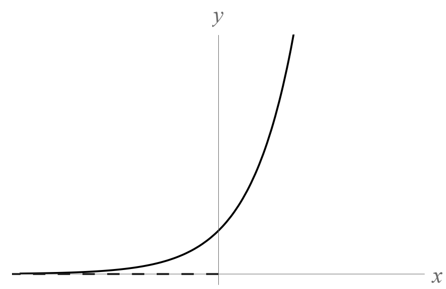
g) For any number x, $1 + \sec^2 x = \tan^2 x$.

h) For any numbers a and b, $\ln(ab) = \ln a + \ln b$.

i) The equation $4^x = 19$ has exactly one solution.

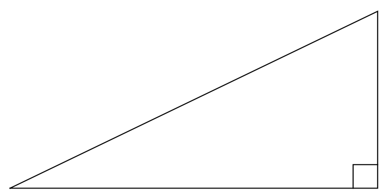
j) The maximum value obtained by $f(x) = 4\cos 3(x-2)$ is 4.

D9. a) Here is a graph of e^x:



Explain, by drawing a picture on this graph, how you would find $\ln 6$ using this graph.

b) Here is a picture of a right triangle:



Explain, by sketching a picture on this circle, how you would interpret $\arctan \frac{2}{5}$ in the context of this right triangle.

D10. Throughout this problem, let $h(x) = \ln(x-5)$.

a) Write the equation(s) of any horizontal asymptote(s) of h, if any:

b) Write the equation(s) of any vertical asymptote(s) of h, if any:

c) How many solutions does the equation $h(x) = -2$ have?

d) How many x-intercepts does h have?

e) How many y-intercepts does h have?

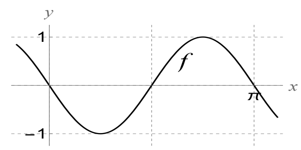
f) What is the domain of h?

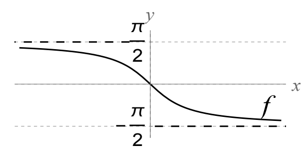
g) Which is greater, $h(10)$ or $h(20)$?

3.5 Solutions to Practice Exam A

A1. Sketch a graph of each function:

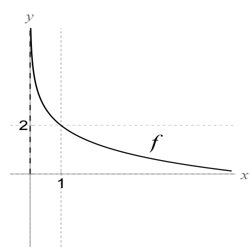
a) This is a sinusoidal graph with period $T = \dfrac{2\pi}{B} = \dfrac{2\pi}{2} = \pi$; see below at left.

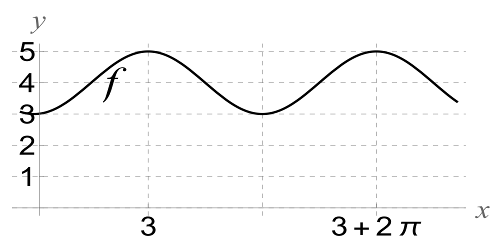




b) This is $\arctan$, reflected across the x-axis; see above at right.

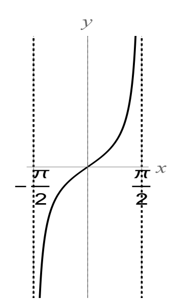
c) This is $\ln$, reflected across the x-axis and then shifted up 2 units; see below at left.

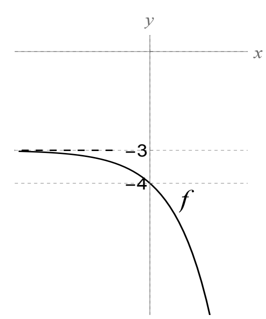




d) This is a cosine graph, shifted right 3 units and up 4 units; see above at right.

e) The graph of $\tan x$ is shown below at left.





f) This is the exponential function, reflected across the x-axis and then shifted down 3 units; see above at right.

A2. a) This is a flipped cosine graph which is shifted down one unit, so $\boxed{g(x) = -\cos x - 1}$.

b) This is a log graph passing through $(1, 0)$ and $(6, 1)$, which is $\boxed{g(x) = \log_6 x}$.

c) This is $\boxed{g(x) = \arcsin x}$.

A3. a) $x = 2\pi n \pm \arccos -\dfrac{\sqrt{2}}{2} = \boxed{2\pi n \pm -\dfrac{\pi}{4}} = \boxed{2\pi n \pm \dfrac{\pi}{4}}$.

b) Take reciprocals of both sides of $\csc x = 0$ to get $\sin x = \frac{1}{0}$ which DNE. Thus there is $\boxed{\text{no solution}}$.

c) $\ln x = e^4$ means $x = \boxed{e^{e^4}} = \boxed{\exp(e^4)}$.

d) $\arcsin x = 1$ means $x = \boxed{\sin 1}$.

e) $\tan x = 3$ means $x = \boxed{n\pi + \arctan 3}$.

f) First, isolate the exponential term: subtract 15 from both sides to get $-2e^x = -14$; divide through by 2 to get $e^x = 7$; then $x = \boxed{\ln 7}$.

g) $\ln x = 0$ means $x = e^0 = \boxed{1}$.

A4. a) $(2e^2)^3e^5 = 2^3(e^2)^3e^5 = 8e^6e^5 = \boxed{8e^{11}}$.

b) $3\tan\pi = 3(0) = \boxed{0}$.

c) $\sin\frac{2\pi}{3} = \boxed{-\frac{\sqrt{3}}{2}}$.

d) $-3\cos\frac{3\pi}{2} = -3(0) = \boxed{0}$.

e) $\frac{1}{\pi}\arcsin -\frac{\sqrt{3}}{2} = \frac{1}{\pi}\left(-\frac{\pi}{3}\right) = \boxed{-\frac{1}{3}}$.

f) $\arcsin 1 + 3\arctan 1 = \frac{\pi}{2} + 3\left(\frac{\pi}{4}\right) = \boxed{\frac{5\pi}{4}}$.

g) $4\cos\frac{3\pi}{4} = 4\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-2\sqrt{2}}$.

h) $3 - \ln\frac{1}{e} = 3 - \ln e^{-1} = 3 - (-1) = \boxed{4}$.

i) $\log -100\ \boxed{\text{DNE}}$.

j) $\frac{\ln 50 + \ln 2}{\ln 30 - \ln 3} = \frac{\ln 100}{\ln 10} = \log_{10} 100 = \boxed{2}$.

k) $(\arcsin 0)\cos\frac{3\pi}{7} = 0\cos\frac{3\pi}{7} = \boxed{0}$.

l) $\frac{1}{4}\sec^2\frac{\pi}{6} = \frac{1}{4}\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{1}{4}\left(\frac{4}{3}\right) = \boxed{\frac{1}{3}}$.

m) $\boxed{7\arctan 2}$ can't be simplified

n) $e^{\ln\frac{2}{5}} = \boxed{\frac{2}{5}}$.

o) $\tan\frac{\pi}{2}\arctan 1 = (\text{DNE})\frac{\pi}{4} = \boxed{\text{DNE}}$.

p) $7^{\log_7(-2)}\boxed{\text{DNE}}$.

q) $\cot\frac{\pi}{2} = \frac{1}{\tan\frac{\pi}{2}} = \frac{1}{\text{DNE}} = \boxed{0}$.

r) $e^{3\ln -1} = (-1)^3 = \boxed{-1}$.

s) $\log_4 64 = \boxed{3}$.

A5. a) $\log_3 9x^5 = \log_3 9 + \log_3 x^5 = \boxed{2 + 5\log_3 x}$.

b)

$$\begin{aligned}\ln\frac{5\sqrt{e}x^3}{\sqrt[4]{y}} &= \ln 5\sqrt{e}x^3 - \ln\sqrt[4]{y}\\ &= \ln 5 + \ln\sqrt{e} + \ln x^3] - \ln y^{1/4}\\ &= \ln 5 + \ln e^{1/2} + 3\ln x - \frac{1}{4}\ln y\\ &= \boxed{\ln 5 + \frac{1}{2} + 3\ln x - \frac{1}{4}\ln y}.\end{aligned}$$

A6.

$$\begin{aligned}\sin x + \cos(-x)\cot x &= \sin x + \cos x\cot x\\ &= \sin x + \cos x\frac{\cos x}{\sin x}\\ &= \sin x\frac{\sin x}{\sin x} + \frac{\cos^2 x}{\sin x}\\ &= \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}\\ &= \frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \boxed{\csc x}.\end{aligned}$$

A7. a) $\cos^2 x = \boxed{1 - \sin^2 x}$.

b) Rewrite $3 + 2\sec^2 x = 3 + 2(1 + \tan^2 x) = 3 + 2 + 2\tan^2 x = \boxed{5 + 2\tan^2 x}$.

c) Rewrite $4 - \cot^2 w = 4 - (\csc^2 w - 1) = \boxed{5 - \csc^2 w}$.

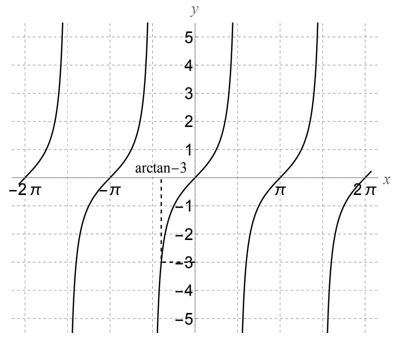
A8. Plug in the points $(0, 12)$ and $(4, 3)$ to $f(x) = a \cdot b^x$ to get

$$\begin{cases} 12 &= a \cdot b^0 = a \cdot 1 = a\\ 4 &= a \cdot b^3\end{cases}$$

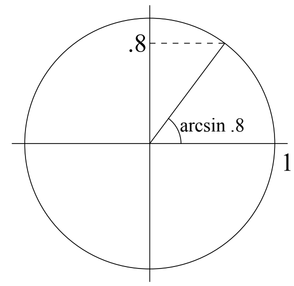
From the first equation, $a = 12$. Substitute into the other equation to get $4 = 12b^3$, i.e. $b^3 = \frac{4}{12} = \frac{1}{3}$, i.e. $b = \sqrt[3]{\frac{1}{3}}$. Therefore the function is $\boxed{f(x) = 12 \cdot \left(\sqrt[3]{\frac{1}{3}}\right)^x}$.

A9.

a)

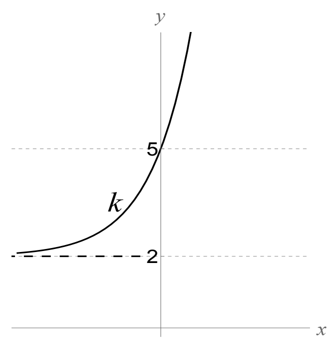


b)



A10. a) $\boxed{x \overset{\cos}{\longrightarrow} \overset{1/\cdot}{\longrightarrow} \overset{\times 3}{\longrightarrow} F(x)}$ b) $\boxed{x \overset{\times \ln 2}{\longrightarrow} \overset{\exp}{\longrightarrow} G(x)}$ c) $\boxed{x \overset{-4}{\longrightarrow} \overset{\ln}{\longrightarrow} \overset{\div \ln 15}{\longrightarrow} H(x)}$

A11. To solve this problem, first graph k (start with e^x, stretch vertically by a factor of 3 and then shift up 2 units). This gives you a graph like this:



Use the graph to address parts (a), (b), (c), (d) and (g) below:

a) The domain of k is $\boxed{\mathbb{R}}$, the set of all real numbers (this is the set of x-values covered by the graph).

b) The range of k is $\boxed{(2, \infty)}$ (the set of y-values covered by the graph).

c) $\boxed{y = 2}$ is the HA of k.

d) There is $\boxed{\text{no VA}}$ (exponential functions do not have VAs).

e) Set $k(x) = 11$ and solve for x:

$$\begin{aligned} 3e^x + 2 &= 11 \\ 3e^x &= 9 \\ e^x &= 3 \\ x &= \boxed{\ln 3}. \end{aligned}$$

f) $k(\ln 5) = 3e^{\ln 5} + 2 = 3(5) + 2 = \boxed{17}$.

g) As x goes from left to right, the graph of k $\boxed{\text{increases}}$.

3.6 Solutions to Practice Exam B

B1. a) $x = 2\pi n + \arcsin\frac{\sqrt{3}}{2}, 2\pi n + \pi - \arcsin\frac{\sqrt{3}}{2} = \boxed{2\pi n + \frac{\pi}{3}, 2\pi n + \pi - \frac{\pi}{3}} = \boxed{2\pi n + \frac{\pi}{3}, 2\pi n + \frac{2\pi}{3}}$.

b) $x = \boxed{e^{-3}}$

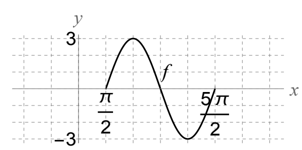
c) Divide through by 3 to get $\arcsin x = \frac{\pi}{3}$, so $x = \sin\frac{\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$.

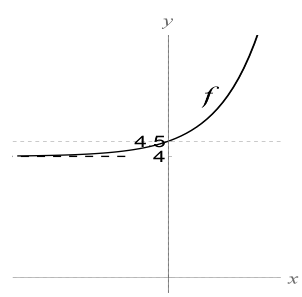
d) $x = \boxed{\ln 37}$

e) Multiply by $\sqrt{3}$ to get $\cot x = \sqrt{3}$; take reciprocals to get $\tan x = \frac{1}{\sqrt{3}}$. Therefore $x = \pi n + \arctan\left(\frac{1}{\sqrt{3}}\right) = \boxed{\pi n + \frac{\pi}{6}}$.

f) Divide by 3 to get $\cos x = \frac{2}{3}$. Thus $x = \boxed{2\pi n \pm \arccos\frac{2}{3}}$.

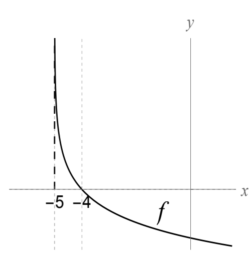
B2. a) This is a sine graph shifted right by $\frac{\pi}{2}$ and stretched vertically by a factor of 3; see below at left.

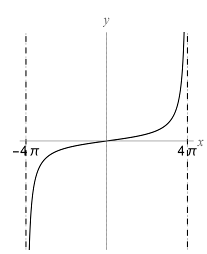




b) This is an exponential function compressed vertically by a factor of $\frac{1}{2}$ and shifted up 4; see above at right.

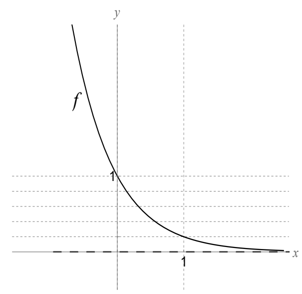
c) This is ln, shifted left 5 units and then reflected across the y-axis; see below at left.

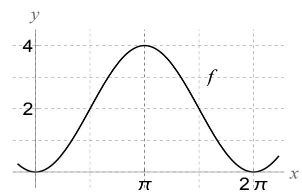




d) This is a tangent graph, stretched horizontally so that its period is $T = \frac{\pi}{B} = \frac{\pi}{\frac{1}{8}} = 8\pi$; thus its VA are $x = \pm 4\pi$. This graph is shown above at right.

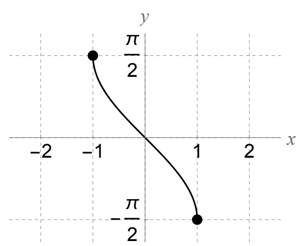
e) This an exponential decay graph going through $(0,1)$ and $(1, \frac{1}{5})$; see below at left.

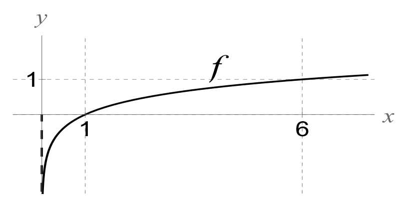




f) This is a cosine graph, flipped across the x-axis, stretched by a factor of 2 and shifted up 2 units; see above at right.

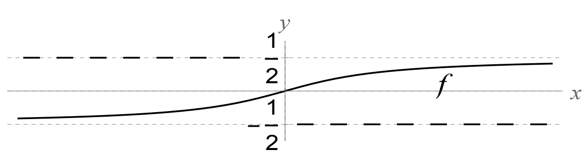
g) This is the graph of $\arcsin$, reflected across the y-axis; see below at left.





h) This is a logarithmic graph with VA $x = 0$, passing through $(1,0)$ and $(6,1)$; see above at right.

i) This is the graph of $\arctan$, stretched vertically so that its VA are $y = \pm\frac{1}{\pi} \cdot \frac{\pi}{2} = \pm\frac{1}{2}$, as shown below:



B3. a) $\sin \frac{\pi}{2} = \boxed{1}$.

b) $\tan -\frac{5\pi}{6} = \boxed{-\frac{1}{\sqrt{3}}}$.

c) $\frac{-\ln 2}{\ln 4} = -\log_4 2 = \boxed{-\frac{1}{2}}$.

d) $e^{\sin 0} = e^0 = \boxed{1}$.

e) $\sec \frac{\pi}{4} = \frac{1}{\cos \frac{\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$.

f) $3 \arcsin 0 = 3(0) = \boxed{0}$.

g) $2\cos 0 + 3 = 2(1) + 3 = \boxed{5}$.

h) $\frac{e^8}{e^3} = e^{8-3} = \boxed{e^5}$.

i) $\log_8 8^3 = \boxed{3}$ by a cancellation law.

j) $\exp\left(3 \ln \frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 = \boxed{\frac{1}{27}}$.

k) $\log_3 9 + 1 = 2 + 1 = \boxed{3}$.

l) $8\arctan\sqrt{3} = 8\left(\frac{\pi}{3}\right) = \boxed{\frac{8\pi}{3}}$.

m) $\boxed{\ln(e^8+1)}$ cannot be simplified.

n) $8\arcsin\frac{4}{3}$ $\boxed{\text{DNE}}$ since $\frac{4}{3} > 1$.

o) $\ln 0$ $\boxed{\text{DNE}}$.

p) $\ln(e^8)\sin\frac{3\pi}{4} = 8\left(\frac{\sqrt{2}}{2}\right) = \boxed{4\sqrt{2}}$.

q) $\tan\frac{2\pi}{3} = \boxed{-\sqrt{3}}$.

r) $6\ln\sqrt[4]{e} = 6\ln e^{1/4} = 6\left(\frac{1}{4}\right) = \boxed{\frac{3}{2}}$.

s) $6\log_{64} 8 = 6\left(\frac{1}{2}\right) = \boxed{3}$.

t) $\frac{\ln 27}{\ln 3} = \log_3 27 = \boxed{3}$.

B4. a) This is a cosine graph, horizontally stretched so that its period is 32 (you can tell this since $\frac{1}{4}$ of the period is 8, so the period is $8 \cdot 4$). Therefore $\frac{2\pi}{B} = 32$ so $32B = 2\pi$ so $B = \frac{\pi}{16}$. This makes the function $\boxed{g(x) = \cos\frac{\pi}{16}x}$.

b) This is a natural log graph, reflected across the y-axis. Thus $\boxed{g(x) = \ln(-x)}$.

c) This is a sine graph shifted up 4 units and stretched by a factor of 2, so $\boxed{g(x) = 2\sin x + 4}$.

B5. a) $3\ln 2x - \ln y = \ln(2x)^3 - \ln y = \ln 8x^3 - \ln y = \boxed{\ln\frac{8x^3}{y}}$.

b) $\ln 5 + \frac{2}{7}\ln w = \ln 5 + \ln w^{2/7} = \boxed{\ln 5w^{2/7}}$.

B6. a) $3\ln 5x = 3(\ln 5 + \ln x) = \boxed{3(\ln 5 + a)}$.

b) $\ln x^7\sqrt[3]{y} = \ln x^7 + \ln y^{1/3} = 7\ln x + \frac{1}{3}\ln y = \boxed{7a + \frac{1}{3}b}$.

c) $\ln\frac{x}{y} = \ln x - \ln y = \boxed{a - b}$.

d) $\log_y x = \frac{\ln x}{\ln y} = \boxed{\frac{a}{b}}$.

e) $x = \boxed{e^a}$.

B7. $\frac{1-\cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \boxed{\sin x}$.

B8. a) $\boxed{f(x) = 3^x}$ $\qquad g(x) = 80^{100} \cdot 2^x$

Reason: 3^x has a larger base than $c2^x$, for any constant c.

b) $\boxed{f(x) = 4^x 2^x}$ $\qquad g(x) = 7^x$

Reason: $4^x 2^x = 8^x$ has a larger base than 7^x.

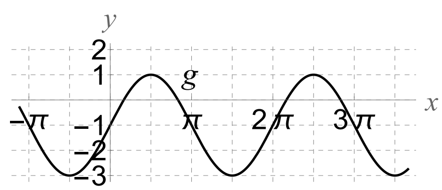
c) $f(x) = 3^{-x}$ $\qquad \boxed{g(x) = 2^{-x}}$

Reason: $2^{-x} = \left(\frac{1}{2}\right)^x$ has a larger base than $3^{-x} = \left(\frac{1}{3}\right)^x$.

d) $f(x) = x^{2024}$ $\quad$ $\boxed{g(x) = \left(\frac{3}{2}\right)^x}$

Reason: exponential functions with base greater than 1 grow faster than polynomials.

B9. We sketch a graph of g by stretching vertically by a factor of 2 and shifting down 1 unit. This gives the following graph:



which is useful in parts (a), (b) and (c).

a) The minimum value of g is $\boxed{-3}$.

b) The domain of g is $\boxed{\mathbb{R}}$, since the graph of g covers all values of x.

c) g has $\boxed{\text{no HA}}$ since sinusoidal graphs do not have HA.

d) To find the x-intercepts of g, set $g(x) = 0$ and solve for x:

$$\begin{aligned} 0 &= 2\sin x - 1 \\ -1 &= 2\sin x \\ -\frac{1}{2} &= \sin x \\ x &= 2\pi n + \arcsin -\frac{1}{2}, 2\pi n + \pi - \arcsin -\frac{1}{2} \\ x &= 2\pi n - \frac{\pi}{6}, 2\pi n + \pi + \frac{\pi}{6} \\ x &= \boxed{2\pi n - \frac{\pi}{6}, 2\pi n + \frac{7\pi}{6}}. \end{aligned}$$

e) $f\left(\frac{3\pi}{2}\right) = 2\sin\frac{3\pi}{2} - 1 = 2(-1) - 1 = \boxed{-3}$.

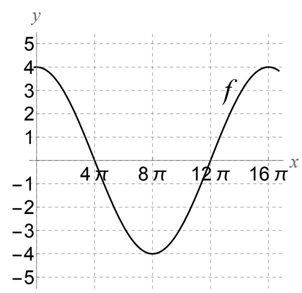
f) i. $g(z) + 1 = \frac{2}{3} + 1 = \boxed{\frac{5}{3}}$.

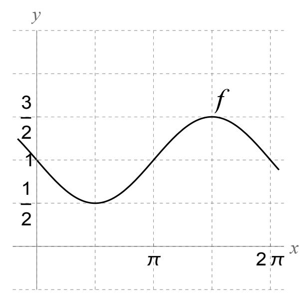
ii. We $\boxed{\text{do not know}}$ what $g(z+1)$ is.

iii. By periodicity, $g(z + 2\pi) = g(z) = \boxed{\frac{2}{3}}$.

3.7 Solutions to Practice Exam C

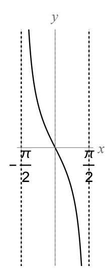
C1. a) This is a cosine graph stretched horizontally so that its period is $T = \dfrac{2\pi}{B} = \dfrac{2\pi}{\frac{1}{8}} = 2\pi \cdot 8 = 16\pi$ and stretched vertically by a factor of 4; see below at left.

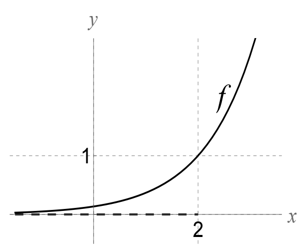




b) This is a sine function compressed vertically by a factor of $\frac{1}{2}$, reflected across the x-axis, and shifted up 1; see above at right.

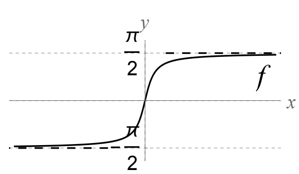
c) This is a tangent graph, reflected across the x-axis and stretched by a factor of 2 vertically (you won't be able to see this stretching much on the graph); see below at left.

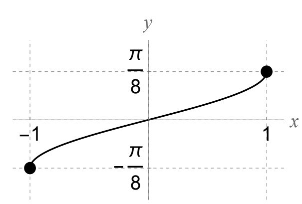




d) This is an exponential graph, shifted right 2 units so that the point $(0, 1)$ gets moved to $(2, 1)$. This graph is shown above at right.

e) This is the graph of $\arctan$, compressed horizontally by a factor of 4. You won't see this stretching much on the graph, which is shown below at left.





f) This is the graph of $\arcsin$, compressed vertically by a factor of 4. This graph is shown above at right.

C2. a) $\boxed{3\sin\frac{\pi}{8}}$ cannot be reasonably simplified

b) $3^{\log_3 7} = \boxed{7}$ by a cancellation law.

c) $\dfrac{2\ln 6 - \ln 4}{-4\ln 3} = \dfrac{\ln 6^2 - \ln 4}{-4\ln 3} = \dfrac{\ln \frac{36}{4}}{-4\ln 3} = \dfrac{\ln 9}{-4\ln 3} = -\dfrac{1}{4}\log_3 9 = -\dfrac{1}{4}(2) = \boxed{-\dfrac{1}{2}}$.

d) $\dfrac{\ln 3 + \ln 12}{\ln 6} = \dfrac{\ln 36}{\ln 6} = \log_6 36 = \boxed{2}$.

e) $4\tan^2\dfrac{\pi}{3} = 4(\sqrt{3})^2 = 4(3) = \boxed{12}$.

f) $\boxed{\cos\dfrac{\pi}{5}}$ cannot be reasonably simplified.

g) $\dfrac{\sqrt{e}}{e^2} = e^{1/2-2} = \boxed{e^{-3/2}}$.

h) $\boxed{\ln 5}$ cannot be reasonably simplified.

i) $\arcsin\sin\dfrac{1}{5} = \boxed{\dfrac{1}{5}}$.

j) $\log .01 = \log 10^{-2} = \boxed{-2}$.

k) $4e^{-2\ln 6} = 4(6^{-2}) = 4\left(\dfrac{1}{36}\right) = \boxed{\dfrac{1}{9}}$.

l) $3\ln^2 e^5 = 3(\ln e^5)^2 = 3(5^2) = \boxed{75}$.

m) $\dfrac{1}{5}\arctan -\dfrac{1}{\sqrt{3}} = \dfrac{1}{5}\left(-\dfrac{\pi}{6}\right) = \boxed{-\dfrac{\pi}{30}}$.

n) $\sin -3\pi + \pi = 0 + \pi = \boxed{\pi}$.

o) $\csc^4\dfrac{3\pi}{4} = \left(\dfrac{1}{\sin\frac{3\pi}{4}}\right)^4\left(\dfrac{1}{\frac{\sqrt{2}}{2}}\right)^4 = \left(\sqrt{2}\right)^4 = \boxed{4}$.

p) $2\tan\arctan -4 = 2(-4) = \boxed{-8}$.

q) $\log_5 5^8 = \boxed{8}$ by a cancellation law.

r) $\sin\dfrac{\pi}{4}\ln e = \left(\dfrac{\sqrt{2}}{2}\right)(1) = \boxed{\dfrac{\sqrt{2}}{2}}$.

s) $\cos^2\dfrac{\pi}{6} = \left(\dfrac{\sqrt{3}}{2}\right)^2 = \boxed{\dfrac{3}{4}}$.

C3. a) Divide through by 5 to get $e^x + 2 = \dfrac{27}{5}$; then subtract 2 to get $e^x = \dfrac{17}{5}$. Thus $x = \boxed{\ln\dfrac{17}{5}}$.

b) Add 7 to both sides to get $4\tan x = -4$; divide both sides by 4 to get $\tan x = -1$; therefore $x = \pi n + \arctan -1 = \boxed{\pi n + \dfrac{\pi}{4}}$.

c) Add 11 to both sides to get $2\ln x = 18$; divide through by 2 to get $\ln x = 9$; therefore $x = \boxed{e^9}$.

d) Divide through by 4 to get $\sin x = -\frac{1}{2}$; therefore $x = 2\pi n + \arcsin -\frac{1}{2}, 2\pi n + \pi - \arcsin -\frac{1}{2} = \boxed{2\pi n - \frac{\pi}{6}, 2\pi n + \pi + \frac{\pi}{6}} = \boxed{2\pi n - \frac{\pi}{6}, 2\pi n + \frac{7\pi}{6}}$.

e) $x = \boxed{\tan \frac{2}{3}}$.

f) Add 5 to both sides to get $7\sec x = 12$; then $\sec x = \frac{12}{7}$ so $\cos x = \frac{7}{12}$. Thus $x = \boxed{2\pi n \pm \arccos \frac{7}{12}}$ which cannot be simplified further.

C4. a) This is an exponential function, reflected across the x-axis and stretched by a factor of 3, so it is $\boxed{g(x) = -3e^x}$.

b) This is a logarithmic function with base 7, reflected across the x-axis to give $\boxed{h(x) = -\log_7 x}$.

c) This is a cosine graph, shifted right by 5 and compressed vertically by a factor of $\frac{1}{2}$ to give $\boxed{k(x) = \frac{1}{2}\cos(x - 5)}$.

C5. a) $4\ln x + 3\ln y = \ln x^4 + \ln y^3 = \boxed{\ln x^4 y^3}$.

b) $\frac{3}{2}\log x + 2 = \log x^{3/2} + \log 10^2 = \boxed{\log 100x^{3/2}}$.

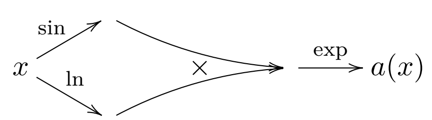
C6. a) $\boxed{1 + \sec^2 x}$ cannot be simplified.

b) $\cot^2 4x + 1 = \boxed{\csc^2 4x}$.

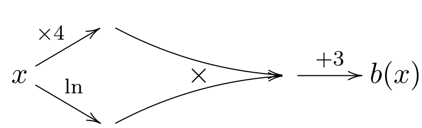
c) $3 - 3\sin^2 x = 3(1 - \sin^2 x) = \boxed{3\cos^2 x}$.

d) $\boxed{\sin^2 x + \cos^2 2x}$ cannot be simplified.

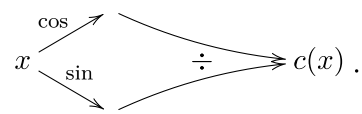
C7. Plug in the given points to get $-2 = a \cdot b^0 = a \cdot 1 = a$ and $-8 = a \cdot b^2$. From the first equation, $a = -2$. Substitute into the second equation to get $-8 = -2b^2$, i.e. $b^2 = 4$ and since b has to be positive, $b = \sqrt{4} = 2$. Thus the function is $\boxed{f(x) = -2 \cdot 2^x}$.

C8. a) $a(x) = x^{\sin x} = e^{\sin x \ln x}$ so this diagrams as 

.

b) 

c) $c(x) = \cot x = \dfrac{\cos x}{\sin x}$ so this diagrams as



Note: tan is not an elementary function, so it isn't allowed in a diagram.

C9. The function being graphed is $\boxed{f(x) = 5^x}$.

C10. a) $\boxed{f(x) = 8\sqrt{x}}$ $\quad g(x) = 8\sqrt[3]{x}$

Reason: f has a larger exponent ($\frac{1}{2} > \frac{1}{3}$).

b) $\boxed{f(x) = \frac{1}{5}x^3}$ $\quad g(x) = 100x^{2.75}$

Reason: f has a larger exponent ($3 > 2.75$).

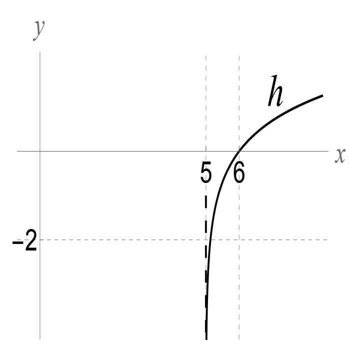
c) $f(x) = x^{30}$ $\quad \boxed{g(x) = 2^x}$

Reason: exponential functions with base > 1 are larger than polynomials when x is large.

d) $f(x) = 100 \cdot 2^x$ $\quad \boxed{g(x) = (2.01)^x}$

Reason: g has a greater base than f.

C11. Start by graphing f (it is a cosine graph, shifted up 2 and stretched by a factor of 5, and stretched horizontally so that its period is $T = \dfrac{2\pi}{B} = \dfrac{2\pi}{\frac{\pi}{4}} = 2\pi \cdot \dfrac{4}{\pi} = 8$:



a) The minimum value of f is $2 - 5 = \boxed{-3}$.

b) The maximum value of f is $2 + 5 = \boxed{7}$.

c) f is sinusoidal, so it has $\boxed{\text{no HA}}$.

d) $f(x) = -1$ has $\boxed{\text{infinitely many}}$ solutions, since the graph will pass through $y = -1$ infinitely many times (due to periodicity).

e) The equation $f(x) = -4$ has $\boxed{\text{no solution}}$, since the graph never crosses the horizontal line $y = -4$.

f) As computed earlier, the period of f is $\boxed{8}$.

3.8 Solutions to Practice Exam D

D1. a) $-2\sin\frac{-\pi}{4} = -2\left(\frac{\sqrt{2}}{2}\right) = \boxed{-\sqrt{2}}$.

b) $\ln\cos 0 = \ln 1 = \boxed{0}$.

c) $8\cos\frac{\pi}{3} = 8\left(\frac{1}{2}\right) = \boxed{4}$.

d) $\log_2\frac{1}{4} = \boxed{-2}$ (since $2^{-2} = \frac{1}{4}$).

e) $\boxed{\log_4 48}$ cannot be simplified.

f) $\log_7\frac{1}{7} + 5e^{\tan 0} = -1 + 5e^0 = -1 + 5 = \boxed{4}$.

g) $\left(e^7\right)^3 = e^{7\cdot 3} = \boxed{e^{21}}$.

h) $\tan\frac{\pi}{4} = \boxed{1}$.

i) $\arcsin -1 = -\arcsin 1 = \boxed{-\frac{\pi}{2}}$.

j) $\ln -3$ $\boxed{\text{DNE}}$.

k) $4\arcsin\frac{1}{2} = 4\left(\frac{\pi}{6}\right) = \boxed{\frac{2\pi}{3}}$.

l) $4\ln e^7 = 4(7) = \boxed{28}$.

m) $-2\cot -\frac{\pi}{6} = -2\left(\frac{1}{\tan\frac{\pi}{6}}\right) = -2\left(\frac{1}{\frac{1}{\sqrt{3}}}\right) = \boxed{-2\sqrt{3}}$.

n) $\ln\exp 7 = \boxed{7}$ by a cancellation law.

o) $5\csc^2\frac{\pi}{3} = 5\left(\frac{1}{\sin\frac{\pi}{3}}\right)^2 = 5\left(\frac{1}{\frac{\sqrt{3}}{2}}\right)^2 = 5\left(\frac{2}{\sqrt{3}}\right)^2 = 5\left(\frac{4}{3}\right) = \boxed{\frac{20}{3}}$.

p) $e^{4\ln 3} - 5 = 3^4 - 5 = 81 - 5 = \boxed{76}$.

q) $\log 1000 = \boxed{3}$ (since $10^3 = 1000$).

r) $5 - \arctan 1 = \boxed{5 - \frac{\pi}{4}}$.

s) $\log_5(15 + 10) = \log_5 25 = \boxed{2}$.

D2. a) Subtract 2 from both sides to get $4\sin x = 5$; then divide both sides by 4 to get $\sin x = \frac{5}{4}$. Since $\frac{5}{4} > 1$, this equation has $\boxed{\text{no solution}}$.

b) First, distribute on both sides to get $\frac{3}{2}\ln x + \frac{1}{8} = \frac{7}{4}\ln x - \frac{1}{4}$. Then, clear the fractions by multiplying everything through by 8 to get $12\ln x + 1 = 14\ln x - 2$. Combine like terms to get $3 = 2\ln x$; so $\frac{3}{2} = \ln x$ so $x = \boxed{e^{3/2}}$.

c) Add $\frac{1}{2}$ to both sides to get $\frac{1}{2}\cos x = \frac{1}{2}$; multiply both sides by 2 to get $\cos x = 1$ so $x = 2\pi n \pm \arccos 1 = \boxed{2\pi n \pm 0} = \boxed{2\pi n}$.

d) Subtract 5 from both sides to get $8\arctan x = 16$; divide both sides by 8 to get $\arctan x = 2$, so $x = \boxed{\tan 2}$.

e) Distribute on both sides to get $2\tan x - 2 = 5\tan x + 10$. Combine like terms on each side to get $-3\tan x = 12$; divide through by -3 to get $\tan x = -4$. Thus $x = \boxed{\pi n + \arctan -4} = \boxed{\pi n - \arctan 4}$.

f) Since $e^x = -2$, it must be that $x = \ln(-2)$, which $\boxed{\text{DNE}}$. This equation has no solution.

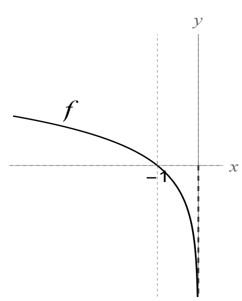
D3. a) This is arctan, reflected across the x-axis, so $\boxed{a(x) = -\arctan x}$.

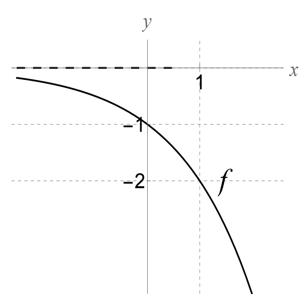
b) This is a tangent graph, stretched horizontally so that its period is $2\pi - (-2\pi) = 4\pi$, so $T = \frac{\pi}{B} = 4\pi$,so $B = \frac{1}{4}$. This makes the rule $\boxed{b(x) = \tan \frac{1}{4}x}$.

c) This is an exponential function with HA $y = 2$, so it is shifted up 2 units. Thus the point $(1, \frac{19}{8})$ on the graph must have been shifted up from $(1, \frac{19}{8} - 2) = (1, \frac{3}{8})$, so the base of the exponential function is $\frac{3}{8}$. This makes the rule of the function $\boxed{c(x) = \left(\frac{3}{8}\right)^x + 2}$.

D4. a) This is the graph of $\ln x$, reflected across the y-axis so that its VA is still $x = 0$ but that it now passes through $(-1, 0)$. This graph is shown below at left.

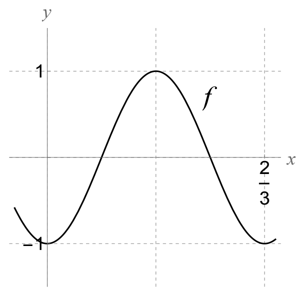
b) This is the graph of 2^x, reflected across the x-axis so that its HA is still $y = 0$ and that it passes through $(0, -1)$ and $(1, -2)$. This is the graph shown below at right.

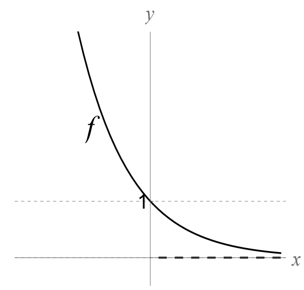




c) This is a cosine graph compressed horizontally so that its period is $T = \frac{2\pi}{B} = \frac{2\pi}{3\pi} = \frac{2}{3}$, and reflected across the x-axis. This makes the graph shown below at left.

d) Reflect the graph of the exponential function across the y-axis, as shown below at right.

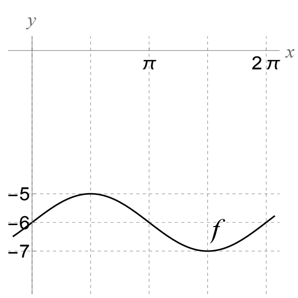


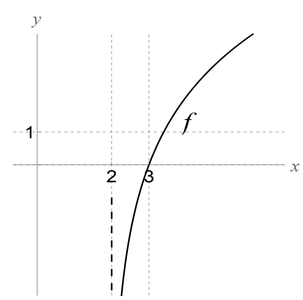


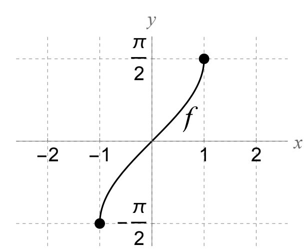
e) This is the graph of $\sin x$, shifted down 6 units; the graph is below at left.

f) This is a log graph, shifted right 2 units (so that its VA is $x = 2$ and the graph passes through $(3, 0)$) and stretched by a factor of 3 vertically (you won't see the stretch much). This produces the graph shown below in the center.

g) The graph of $\arcsin x$ is shown below at right.







D5.

$$\frac{\cos x - \sec x}{\tan x} = \frac{\cos x - \frac{1}{\cos x}}{\frac{\sin x}{\cos x}}.$$

This is a compound fraction, so you multiply through the top and bottom of the big fraction by the small denominator $\cos x$ to get

$$\frac{\left(\cos x - \frac{1}{\cos x}\right)\cos x}{\left(\frac{\sin x}{\cos x}\right)\cos x} = \frac{\cos^2 x - 1}{\sin x} = \frac{\sin^2 x}{\sin x} = \boxed{-\sin x}.$$

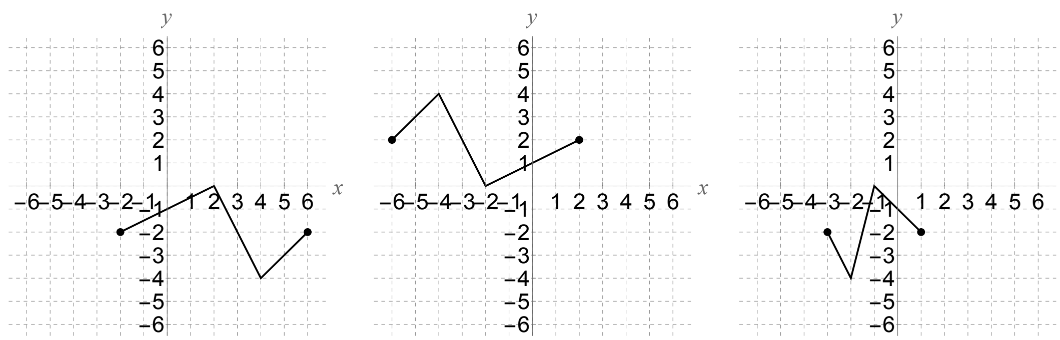
D6. a) $\log_{1/2} 4\sqrt{y} = \log_{1/2} 4 + \log_{1/2}\sqrt{y} = -2 + \log_{1/2} y^{1/2} = \boxed{-2 + \frac{1}{2}\log_{1/2} y}.$

b) $\ln e^6 x^{-4/3}(y-2)^2 = \ln e^6 + \ln x^{-4/3} + \ln(y-2)^2 = \boxed{6 - \frac{4}{3}\ln x + 2\ln(y-2)}.$

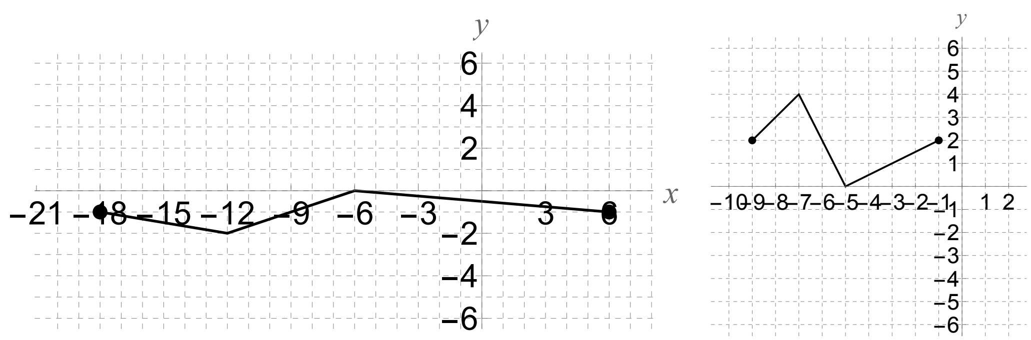
D7. a) Reflect the given graph across the y-axis to get the graph shown below at left.

b) Reflect the given graph across the x-axis to get the graph shown below in the center.

c) Compress the graph horizontally by a factor of 2, freezing the y-intercept $(0, -1)$, to get the graph shown below at right.



d) First stretch the graph horizontally by a factor of 3, then compress it vertically by a factor of $\frac{1}{2}$ to get the graph shown below at left.



e) Shift the graph left 3 units, then reflect it across the x-axis to get the graph shown below at right: $-f(x+3)$

D8. a) TRUE (exponential graphs have HAs).

b) FALSE (the period is $T = \frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$).

c) TRUE ($\arctan(-x) = -\arctan x$).

d) TRUE (to graph this, reflect e^x across both axes).

e) TRUE (if you graph $\ln$, it covers every y-value eventually).

f) FALSE ($\cos x = \frac{5}{4}$ has no solution since $\frac{5}{4} > 1$).

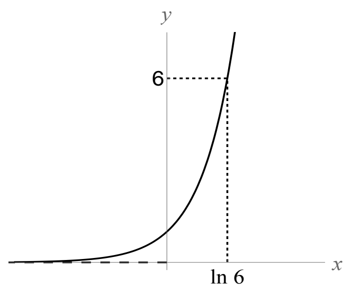
g) FALSE ($1 + \tan^2 x = \sec^2 x$, not the other way around).

h) TRUE (logarithms convert $\times$ to $+$).

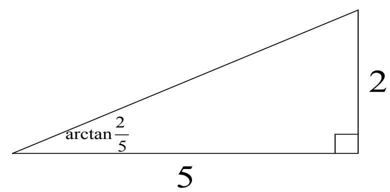
i) $\boxed{\text{TRUE}}$ (the graph of 4^x hits $y = 19$ in one point).

j) $\boxed{\text{TRUE}}$ (this function is stretched vertically by a factor of 4 and not shifted up or down).

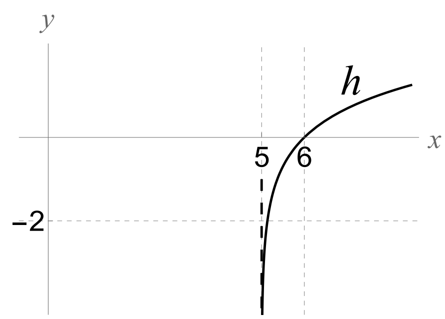
D9. a)



b)



D10. First, graph this function (it is the graph of $\ln x$, shifted right 5 units):



Use this graph to answer the questions:

a) h, being a log graph, has $\boxed{\text{no HA}}$.

b) $\boxed{x = 5}$ is a VA of h.

c) $h(x) = -2$ has $\boxed{\text{one}}$ solution since the graph of h hits the horizontal line $y = -2$ once.

d) h has $\boxed{1}$ x-int (at $(6, 0)$).

e) h has $\boxed{\text{no } y\text{-int}}$ since the graph never crosses the y-axis.

f) The domain of h is $\boxed{(5, \infty)}$, the set of x-values covered by the graph.

g) $\boxed{h(20)}$ is greater since the graph of h increases from left to right.