1. (Rewrite each expression in the form $\square^{\square}$, where your answer contains no radicals nor fractions in the base of any exponent:

   (a) $\frac{1}{p^7}$ (b) $\sqrt[3]{p^4}$ (c) $x^2\sqrt{x}$ (d) $\frac{1}{(x^3)^{-2/3}}$

2. Perform the indicated operations and simplify:

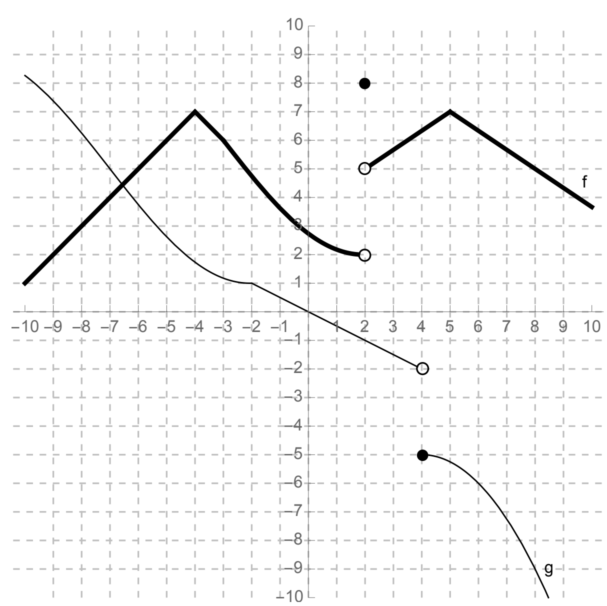
   (a) $(2x-1)^2 - x(x+3)$ (b) $\frac{\frac{2}{x+2}-3}{2-\frac{5}{x+2}}$

3. (a) Find the slope of the line passing through the points $(3,-7)$ and $(-2,8)$.
   (b) (Write an equation of the line passing through the point $(-2,5)$ whose slope is 11.
   (c) Write an equation of the horizontal line passing through the point $(3,-1)$.
   (d) Suppose two lines are parallel. If the first line has slope -3, what is the slope of the second line?
   (e) Sketch the graph of the line $2x+3y=12$.
   (f) Sketch the graph of the line $y=-2+3(x-4)$.

4. Throughout this problem, assume $f(x)=2x^2-x$ and $g(x)=\frac{x+3}{2}$. Compute and simplify each of the following expressions:

   (a) $(f\circ g)(x)$ (b) $4f(2x)$ (c) $(f-g)(3)$ (d) $g^{-1}(x)$

5. Throughout this problem, suppose f and g are functions whose graphs are given below (f is the thick curve, g is the thin curve):



Suppose also that h is a one-to-one function with the following table of values:

x	-3	-2	-1	0	1	2	3	4
$h(x)$	5	-3	0	4	-2	1	7	2

Use this information to answer the following questions:

(a) What is $f(-3)$?

(b) What is $f(2)$?

(c) What is $g^{-1}(-1)$?

(d) For what value(s) of x, if any, does $g(x) = -3$?

(e) For what value(s) of x, if any, does $f(x) = g(x)$?

(f) What is the average change in h from $x = 1$ to $x = 4$?

(g) What is the instantaneous rate of change in g at $x = 6$?

(h) Find $(f \circ g)(6)$.

(i) Find $(fh)(3 - 6)$.

(j) Find $(h^{-1} \circ g \circ h)(1)$.

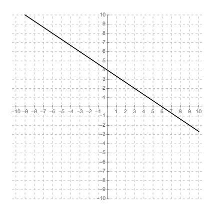
6. (a) A rectangle is four inches longer than it is wide. A 2-inch by 2-inch square is cut out of one corner of the rectangle. If the remaining L-shaped region has area 41 square inches, what was the width of the original rectangle?

(b) The energy emitted by a radioactive particle decreases at a constant rate. If the particle emits 300 J initially ("J" is the abbreviation for joules, which is the unit in which you measure energy) and emits 240 J five seconds later, how much energy will the particle emit seven seconds after that?

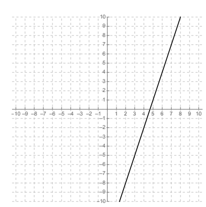
1. (a) $\frac{1}{p^7} = p^{-7}$.
   (b) $\sqrt[3]{p^4} = p^{4/3}$.
   (c) $x^2\sqrt{x} = x^2x^{1/2} = x^{2+1/2} = x^{5/2}$.
   (d) $\frac{1}{(x^3)^{-2/3}} = \frac{1}{x^{3(-2/3)}} = \frac{1}{x^{-2}} = x^{-(-2)} = x^2$.

2. (a) $(2x-1)^2 - x(x+3) = (2x-1)(2x-1) - x(x+3) = 4x^2 - 2x - 2x + 1 - (x^2+3x) = 4x^2 - 4x + 1 - x^2 - 3x = 3x^2 - 7x + 1$.
   (b) $\frac{\frac{2}{x+2}-3}{2-\frac{5}{x+2}} = \frac{\left[\frac{2}{x+2}-3\right](x+2)}{\left[2-\frac{5}{x+2}\right](x+2)} = \frac{2-3(x+2)}{2(x+2)-5} = \frac{2-3x-6}{2x+4-5} = \frac{-3x-4}{2x-1}$.

3. (a) $m = \frac{y_2-y_1}{x_2-x_1} = \frac{8-(-7)}{-2-3} = \frac{15}{-5} = -3$.
   (b) From the point-slope formula, $y = 5 + 11(x+2)$.
   (c) $y = -1$.
   (d) The lines have the same slope, so 3.
   (e) The line has intercepts $(0,4)$ and $(6,0)$:



   (f) The line goes through $(4,-2)$ and has slope 3:



4. (a) $(f\circ g)(x) = f(g(x)) = f\left(\frac{x+3}{2}\right) = 2\left(\frac{x+3}{2}\right)^2 - \left(\frac{x+3}{2}\right) = 2\left(\frac{x^2+6x+9}{4}\right) - \frac{x+3}{2} = \frac{x^2+6x+9}{2} - \frac{x+3}{2} = \frac{x^2+5x+6}{2}$.
   (b) $4f(2x) = 4\left[2(2x)^2 - (2x)\right] = 4\left[2(4x^2) - 2x\right] = 4[8x^2 - 2x] = 32x^2 - 8x$.
   (c) $(f-g)(3) = f(3) - g(3) = [2(3^2) - 3] - \frac{3+3}{2} = 15 - 3 = 12$.
   (d) Let $y = g(x)$ so $y = \frac{x+3}{2}$. Solve for x to get $x = 2y - 3$, so $g^{-1}(x) = 2x - 3$.

5. (a) $f(-3) = 6$.
   (b) $f(2) = 8$.
   (c) $g^{-1}(-1) = 2$.
   (d) $g(x) = -3$ for no x, so x DNE.

(e) $f(x) = g(x)$ when $x \approx -6.5$.

(f) $\frac{h(4)-h(1)}{4-1} = \frac{2-(-2)}{3} = \frac{4}{3}$.

(g) The slope of the tangent line is about $\frac{-2}{3}$.

(h) $(f \circ g)(6) = f(g(6)) = f(-6) = 5$.

(i) $(fh)(3-6) = (fh)(-3) = f(-3)h(-3) = 6(5) = 30$.

(j) Find $(h^{-1} \circ g \circ h)(1) = h^{-1}(g(h(1)) = h^{-1}(g(-2)) = h^{-1}(1) = 2$.

6. (a) Let x be the width of the rectangle. Then, the dimensions of the rectangle are x by $x+4$ so the area of the rectangle is $x(x+4) = x^2 + 4x$. We cut out a piece of area 4, so the remaining shape has area $x^2 + 4x - 4$. Set this equal to 41 and solve for x:

$$x^2 + 4x - 4 = 41$$
$$x^2 + 4x - 45 = 0$$
$$(x+9)(x-5) = 0$$

Thus $x = -9$ or $x = 5$. Throw out the negative answer because it makes no sense in this story problem, leaving us with a width of 5 inches.

(b) The slope of the energy is $240 - 3005 - 0 = -12$ J/sec, and since the initial energy is 300, we obtain $J(t) = 300 - 12t$. Plugging in $t = 12$ (which is seven seconds after five seconds), we get $J(12) = 300 - 12(12) = 300 - 144 = 156$ J.

1. Sketch crude graphs of each of these functions:

   (a) $f(x) = |x|$
   (b) $f(x) = \ln x$
   (c) $f(x) = \cos x$
   (d) $f(x) = -2(x-4)^2 - 1$
   (e) $f(x) = \arctan x$
   (f) $f(x) = 4^x$
   (g) $f(x) = x^4$
   (h) $f(x) = \frac{1}{x}$
   (i) $f(x) = 2x^2 - x^3$
   (j) $f(x) = (x-5)(x-1)(x+4)$

2. Evaluate each of the following expressions:

   (a) $\log_9 3$
   (b) $\log 10000$
   (c) $\ln e^7$
   (d) $e^{3\ln 2}$
   (e) $\sin \frac{5\pi}{3}$
   (f) $\tan \frac{\pi}{2}$
   (g) $\tan \frac{5\pi}{4}$
   (h) $\cos \frac{-\pi}{6}$
   (i) $\arctan \sqrt{3}$
   (j) $\arcsin \frac{-1}{2}$

3. Find all horizontal and/or vertical asymptotes of the function

$$f(x) = \frac{2x^2 - x + 30}{x^2 + 2x - 15}.$$

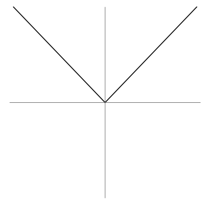
4. Find all solutions of each of the following equations:

   (a) $\sin x = \frac{\sqrt{3}}{2}$
   (b) $5 \tan x = 5$

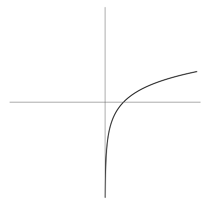
5. Classify each of the following statements as true or false:

   (a) $\log_4 \frac{A}{B} = \log_4 A - \log_4 B$.
   (b) The graph of a logarithmic function has a horizontal asymptote.
   (c) $\tan(\arctan x) = x$.
   (d) The function $f(x) = x^2 - 7x + 4$ is one-to-one.
   (e) $\sin(A + B) = \sin A + \sin B$.
   (f) If a polynomial has degree 8 and its leading coefficient is 3, then both of its tails point upward.
   (g) The function $f(x) = 2^x$ is an even function.
   (h) The statement $\log_y z = w$ is the same as the statement $z^w = y$.
   (i) $\ln 1 = 0$.
   (j) $\sin(-x) = -\sin x$.

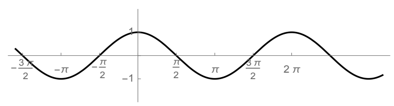
1. (a) $f(x) = |x|$



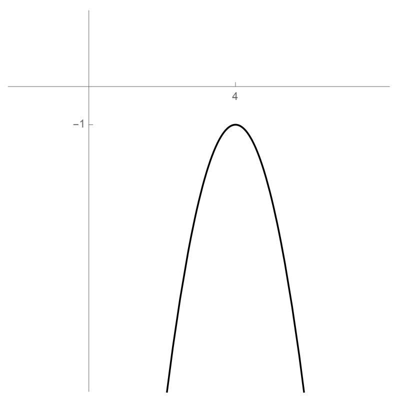
(b) $f(x) = \ln x$



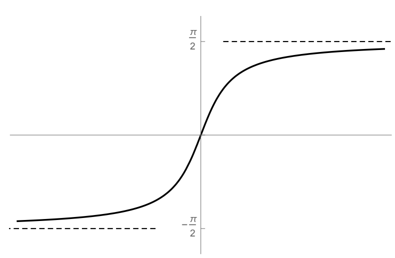
(c) $f(x) = \cos x$



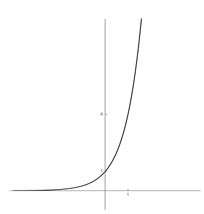
(d) $f(x) = -2(x-4)^2 - 1$ is a parabola opening downward with vertex $(4, -1)$:



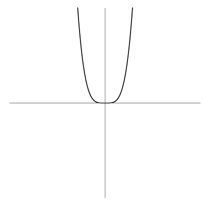
(e) $f(x) = \arctan x$



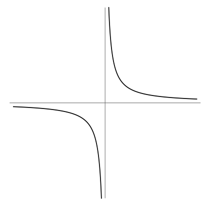
(f) $f(x) = 4^x$



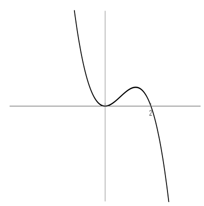
(g) $f(x) = x^4$



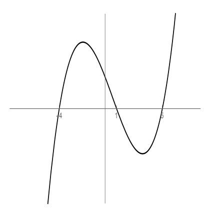
(h) $f(x) = \frac{1}{x}$



(i) $f(x) = 2x^2 - x^3 = x^2(2-x)$ has x-ints $(0,0)$ (which has multiplicity 2) and $(2,0)$ (which has multiplicity 1). Since the degree is 3 and the leading coefficient is -1, the right tail points down and the left tail points up. So the graph looks like



(j) $f(x) = (x-5)(x-1)(x+4)$ has x-ints $(5,0)$, $(1,0)$ and $(-4,0)$ (all multiplicity 1). Since the degree is 3 and the leading coefficient is 1, the right tail points up and the left tail points down. So the graph looks like



2. (a) $\log_9 3 = \frac{1}{2}$ (because $9^{1/2} = \sqrt{9} = 3$).
   (b) $\log 10000 = 4$ (because $10^4 = 10000$).
   (c) $\ln e^7 = 7$ (by a Cancellation Law).
   (d) $e^{3\ln 2} = 2^3 = 8$ (by a log/exp rule).
   (e) $\sin \frac{5\pi}{3} = \frac{\sqrt{3}}{2}$ (Quadrant II (+), ref. angle $\frac{\pi}{3} = 60°$)
   (f) $\tan \frac{\pi}{2}$ **DNE** (slope at $90°$ is undefined).
   (g) $\tan \frac{5\pi}{4} = 1$ (Quadrant III (−), ref. angle $\frac{\pi}{4} = 45°$ where the slope is 1)
   (h) $\cos \frac{-\pi}{6} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

(i) $\arctan\sqrt{3} = \frac{\pi}{3}$.

(j) $\arcsin\frac{-1}{2} = -\arcsin\frac{1}{2} = -\frac{\pi}{6}$.

3. For the vertical asymptotes, set the bottom equal to zero and solve for x:

$$x^2 + 2x - 15 = 0 \Rightarrow (x+5)(x-3) = 0 \Rightarrow x = -5, x = 3$$

Neither of these values of x make the numerator 0, so the VA are $x = -5$ and $x = 3$.

For the horizontal asymptotes, when x is large, $f(x) \approx \frac{2x^2}{x^2} = 2$ so the HA is $y = 2$.

4. (a) One solution is $x = \arctan\frac{\sqrt{3}}{2} = \frac{\pi}{3}$. A second solution is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Therefore all solutions are $\frac{\pi}{3} + 2\pi n$ and $\frac{2\pi}{3} + 2\pi n$ where n is any integer.

   (b) First, divide both sides by 5 to get $\tan x = 1$. Then one solution is $x = \arctan 1 = \frac{\pi}{4}$; all solutions are $\frac{\pi}{4} + \pi n$ where n is any integer.

5. (a) TRUE (log of a quotient is the difference of the logs)

   (b) FALSE (exponential functions have HA, logarithmic functions have VA)

   (c) TRUE (this is the "good" cancellation law for $\arctan$ and $\tan$)

   (d) FALSE (the graph is a parabola which won't pass the Horizontal Line Test)

   (e) FALSE (if $A = B = \frac{\pi}{2}$, the left-hand side is $\sin\pi = 0$ but the right-hand side is $1 + 1 = 2$)

   (f) TRUE (even degree, positive leading coefficient)

   (g) FALSE (the graph is not symmetric about the y-axis)

   (h) FALSE ($\log_y z = w$ is the same as $y^w = z$.

   (i) TRUE (since $e^0 = 1$).

   (j) TRUE ($\sin$ is an odd function).

1. Sketch the graph of each of the following functions:

   (a) $f(x) = \frac{1}{x+3} + 2$
   (b) $f(x) = \arctan(x+5)$
   (c) $f(x) = e^{-x}$
   (d) $f(x) = -\sin 2x$

2. Sketch the graph of each of the following functions:

   (a) $f(x) = -(x+2)^2 - 4$
   (b) $f(x) = -\log_3(x+6)$
   (c) $f(x) = \cos\frac{x}{3} + 2$
   (d) $f(x) = 3\sin\left(x + \frac{3\pi}{2}\right)$

3. Answer each question, writing your answers with correct notation. If any of these things fail to exist, say so.

   (a) What is/are the horizontal asymptote(s) of the function $f(x) = \ln x + 5$?
   (b) What is/are the vertical asymptote(s) of the function $f(x) = \ln(x+2) - 7$?
   (c) What is the period of the function $f(x) = 3\sin 4x - 1$?
   (d) What is the domain of the function $f(x) = \sqrt{x-4} + 2$?
   (e) Find the largest value of y obtained by this function:

   $$f(x) = 2\cos(x+\pi) - 5$$

   (f) What is/are the y-intercept(s) of the function $f(x) = e^{-x} - 3$?

4. Solve each equation:

   (a) $e^{2x-1} = 17$
   (b) $\frac{3}{x+2} + 1 = \frac{2}{x-1}$
   (c) $6\sin^2 x = 6$

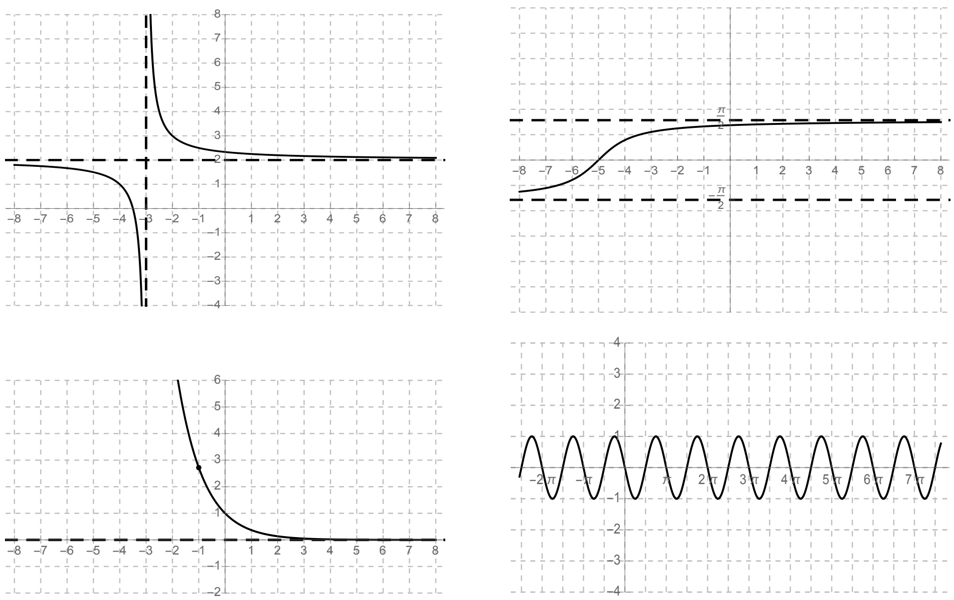
5. Solve each equation:

   (a) $\ln(e^{x/3} + 1) = 4$
   (b) $2\sqrt{2x^2 - 5} - 7 = -3$
   (c) $e^x + 30e^{-x} = 13$

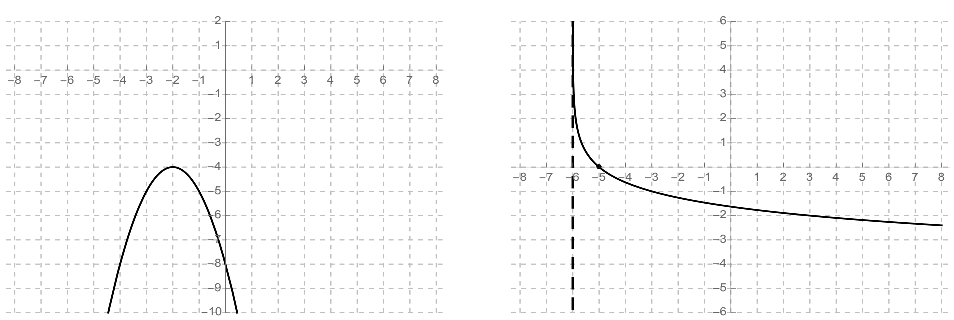
6. Choose any two of the following five equations and solve them.

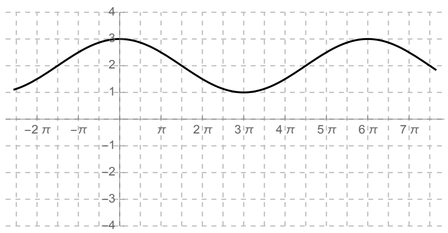
   (a) $\sin^2 x = 2\cos x + 1$
   (b) $5^{x-2} = 2^x$
   (c) $x^{5/2} - 7x^2 = 18x^{3/2}$
   (d) $\log_3(8x+3) + \log_3 x = 4$
   (e) $x + 4 = \sqrt{8-x}$

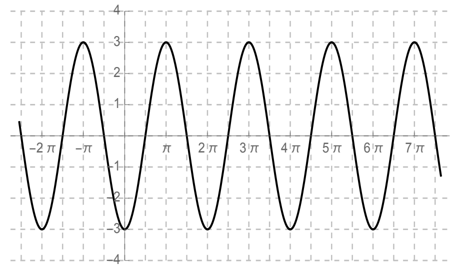
1. (a) Take the graph of $\frac{1}{x}$, shift it left 3 units and shift it up 2 units to get the top left graph below.
   (b) Take the graph of $\arctan x$, shift it left 5 units to get the top right graph below..
   (c) Take the graph of e^x and reflect it across the y-axis to get the bottom left below.
   (d) Take the graph of $\sin x$, smash it horizontally by a factor of 2, and then reflect across the x-axis to get the bottom right below.



2. (a) Take the graph of x^2, shift it left 2 units, reflect across the x-axis and then shift down 4 units to get the top left graph below.
   (b) Take the graph of $\log_3 x$, shift it left 6 units, then reflect across the x-axis to get the top right graph below.
   (c) Take the graph of $\cos x$, stretch it horizontally by a factor of 3, then shift it up 2 units to get the bottom left graph below.
   (d) Take the graph of $\sin x$, shift it left $\frac{3\pi}{2}$ units, then stretch by a factor of 3 to get the bottom right graph below.







3. (a) Logarithmic graphs do not have horizontal asymptotes, so there are none.
   (b) The VA of $\ln x$ is $x = 0$; since the graph is shifted two units left, the VA moves to $x = -2$.
   (c) The graph is smashed horizontally by a factor of 4, making the period $\frac{2\pi}{4} = \frac{\pi}{2}$.
   (d) Take the graph of $\sqrt{x}$, shift it right 4 units and up 2 units. The domain of f is the set of x-values covered by this graph, which is $[4, \infty)$.
   (e) The graph is stretched by a factor of 2 and shifted down 5 units, so its range is $[-2-5, 2-5] = [-7, -3]$, making the answer to the question -3.
   (f) The y-intercept of e^x is $(0, 1)$; reflecting through the x-axis does not change this, but shifting down by 3 units moves the y-intercept to $(0, -2)$.

4. (a) x only appears once, so we can use arrow diagrams:

$$\begin{array}{ccccccc} x & \xrightarrow{2x} & & \xrightarrow{x-1} & & \xrightarrow{e^x} & 17 \\ \frac{1}{2}(\ln 17 + 1) & \xleftarrow[\frac{1}{2}x]{} & \ln 17 + 1 & \xleftarrow[x+1]{} & \ln 17 & \xleftarrow[\ln x]{} & \end{array}$$

   Therefore $x = \frac{1}{2}(\ln 17 + 1)$.

   (b) This is a rational equation with least common denominator $(x+2)(x-1)$. Multiply through by this common denominator to get

$$(x+2)(x-1)\frac{3}{x+2} + (x+2)(x-1)(1) = (x+2)(x-1)\frac{2}{x-1};$$

   cancel to get

$$\begin{aligned} 3(x-1) + (x+2)(x-1) &= 2(x+2) \\ 3x - 3 + x^2 + x - 2 &= 2x + 4 \\ x^2 + 4x - 5 &= 2x + 4 \\ x^2 + 2x - 9 &= 0 \end{aligned}$$

   Last, solve this with the quadratic formula to get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(-9)}}{2} = \frac{-2 \pm \sqrt{40}}{2}.$$

   (This answer simplifies to $x = -1 \pm \sqrt{10}$, but you need not simplify it.)

(c) x only occurs once, so we can use arrow diagrams:

$$\begin{array}{ccccccc} x & \xrightarrow{\sin x} & & \xrightarrow{x^2} & & \xrightarrow{6x} & 6 \\ \frac{\pi}{2}+2\pi n, \frac{-\pi}{2}+2\pi n & \xleftarrow[\substack{\arcsin x+2\pi n \\ \pi-\arcsin x+2\pi n}]{} & \pm 1 & \xleftarrow[\pm\sqrt{x}]{} & 1 & \xleftarrow[\frac{1}{6}x]{} & \end{array}$$

Therefore $x = \pm\frac{\pi}{2} + 2\pi n$.

5. (a) x only occurs once, so we can use arrow diagrams:

$$\begin{array}{ccccccccc} x & \xrightarrow{x/3} & & \xrightarrow{e^x} & & \xrightarrow{x+1} & & \xrightarrow{\ln x} & 4 \\ 3\ln(e^4-1) & \xleftarrow[3x]{} & \ln(e^4-1) & \xleftarrow[\ln x]{} & e^4-1 & \xleftarrow[x-1]{} & e^4 & \xleftarrow[e^x]{} & \end{array}$$

Therefore $x = 3\ln(e^4-1)$.

(b) x only occurs once, so we can use arrow diagrams:

$$\begin{array}{ccccccccccccc} x & \xrightarrow{x^2} & & \xrightarrow{2x} & & \xrightarrow{x-5} & & \xrightarrow{\sqrt{x}} & & \xrightarrow{2x} & & \xrightarrow{x-7} & -3 \\ \pm\sqrt{\frac{9}{2}} & \xleftarrow[\pm\sqrt{x}]{} & \frac{9}{2} & \xleftarrow[\frac{1}{2}x]{} & 9 & \xleftarrow[x+5]{} & 4 & \xleftarrow[x^2]{} & 2 & \xleftarrow[\frac{1}{2}x]{} & 4 & \xleftarrow[x+7]{} & \end{array}$$

Therefore $x = \pm\sqrt{\frac{9}{2}}$.

(c) Rewrite this as $e^x + \frac{30}{e^x} = 13$; since this equation contains "something" and "1 over something", multiply though by the "something" (in this case e^x) to get a quadratic-type equation:

$$\begin{aligned} e^x\left(e^x + \frac{30}{e^x}\right) &= e^x(13) \\ e^{2x} + 30 &= 13e^x \\ e^{2x} - 13e^x + 30 &= 0 \\ (e^x - 10)(e^x - 3) &= 0 \end{aligned}$$

Therefore $e^x - 10 = 0$ or $e^x - 3 = 0$, i.e. $x = \ln 10, \ln 3$.

6. (a) For the equation $\sin^2 x = 2\cos x + 1$, use the Pythagorean identity to rewrite $\sin^2 x$ as $1 - \cos^2 x$:

$$\begin{aligned} 1 - \cos^2 x &= 2\cos x + 1 \\ 0 &= \cos^2 x - 2\cos x \\ 0 &= \cos x(\cos x - 2) \end{aligned}$$

Therefore $\cos x = 0$ (i.e. $x = \pm\frac{\pi}{2} + 2\pi n$) or $\cos x = 2$ (no solutions). Altogether, we have $x = \pm\frac{\pi}{2} + 2\pi n$.

(b) For the equation $5^{x-2} = 2^x$, take the natural logarithm of both sides:

$$\begin{aligned} \ln 5^{x-2} &= \ln 2^x \\ (x-2)\ln 5 &= x \ln 2 \\ x\ln 5 - 2\ln 5 &= x\ln 2 \\ x\ln 5 - x\ln 2 &= 2\ln 5 \\ x(\ln 5 - \ln 2) &= 2\ln 5 \\ x &= \frac{2\ln 5}{\ln 5 - \ln 2}. \end{aligned}$$

(This could be rewritten as $x = \frac{\ln 25}{\ln \frac{5}{2}} = \log_{5/2} 25$.)

(c) For the equation $x^{5/2} - 7x^2 = 18x^{3/2}$, set one side equal to zero, and factor:

$$\begin{aligned} x^{5/2} - 7x^2 &= 18x^{3/2} \\ x^{5/2} - 7x^2 - 18x^{3/2} &= 0 \\ x^{3/2}(x - 7x^{1/2} - 18) &= 0 \\ x^{3/2}(x^{1/2} - 9)(x^{1/2} + 2) &= 0 \end{aligned}$$

From the first factor, we get $x^{3/2} = 0$, i.e. $x = 0$. From the second factor, we get $x^{1/2} = 9$, i.e. $\sqrt{x} = 9$, i.e. $x = 9^2 = 81$. From the last factor, we get $x^{1/2} = \sqrt{x} = -2$ which has no solution. Altogether, we have $x = 0, 81$.

(d) For the equation $\log_3(8x+3) + \log_3 x = 4$, combine using a log rule:

$$\begin{aligned} \log_3(8x+3) + \log_3 x &= 4 \\ \log_3[x(8x+3)] &= 4 \\ \log_3(8x^2 + 3x) &= 4 \end{aligned}$$

Thinking of the arrow diagram

$$\begin{array}{ccc} 8x^2 + 3x & \xrightarrow{\log_3 x} & 4 \\ 3^4 = 81 & \xleftarrow[3^x]{} & \end{array}$$

we get $8x^2 + 3x = 81$, i.e.

$$\begin{aligned} 8x^2 + 3x - 81 &= 0 \\ (x-3)(8x+27) &= 0 \end{aligned}$$

From the first factor, we get $x = 3$. From the second factor, we get $x = \frac{-27}{8}$. But the second solution doesn't work when plugged into the equation (you would have to take the log of a negative number), so we get only $x = 3$.

(e) For the equation $x + 4 = \sqrt{8 - x}$, square both sides to get a quadratic:

$$\begin{aligned} x + 4 &= \sqrt{8 - x} \\ (x + 4)^2 &= 8 - x \\ x^2 + 8x + 16 &= 8 - x \\ x^2 + 9x + 8 &= 0 \\ (x + 8)(x + 1) &= 0 \end{aligned}$$

Therefore we get two solutions $x = -8, -1$. However, when plugging in $x = -8$ in the original equation, we get $-4 = \sqrt{16}$, which is false. This leaves only the valid solution $x = -1$.

1. Rewrite each expression in a form such that:
   - there are no radicals in your expression, and
   - there are no exponents in the denominator of any fraction in your expression.

   (a) $\frac{1}{x^3}$ (b) $2x\sqrt{x}$ (c) $\left(\sqrt[3]{x-2}\right)^2$ (d) $\sqrt{\frac{9}{x}}$

2. Rewrite each expression in terms of natural logarithms and/or natural exponentials:

   (a) $\log_5 73$ (b) 4^x

3. Evaluate each expression:

   (a) $\sin\frac{2\pi}{3}$
   (b) $\tan\pi$
   (c) $\cos\frac{-\pi}{4}$
   (d) $e^{\ln 4}$
   (e) $\log_5\frac{1}{5}$
   (f) $\log_6 9 + 2\log_6 2$
   (g) $\frac{13^6 13^2}{(13^2)^4}$
   (h) $4!$
   (i) $\arcsin\frac{1}{2}$
   (j) $\arctan -1$

4. Perform the indicated operation and simplify:

$$\frac{3}{x-2} + \frac{4}{x-1}$$

5. Classify the following statements as true or false:

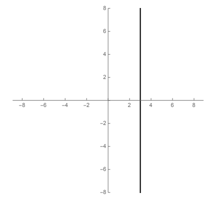
   (a) To compute $\sin^2 x$, first take the sine of x, then square the answer.
   (b) $\sin(x-y) = \sin x - \sin y$.
   (c) Two lines are parallel if their slopes are equal.
   (d) $(7^{19})^{25} = (7^{25})^{19}$.
   (e) $\sin(-x) = -\sin x$.
   (f) $\tan(\arctan x) = x$.
   (g) $\arctan(\tan x) = x$.
   (h) e^x has a graph which is continuous, but not smooth.
   (i) $\sqrt{x^2+1} = x+1$.
   (j) $\sqrt{x^2+1} = |x+1|$.

6. Suppose $f(x) = 3-2x$ and $g(x) = 1-x$. Find and simplify the rule for $(f \circ g)(x)$.

7. (a) Find the slope of the line passing through $(-4,3)$ and $(2,7)$.

(b) Write the equation of the line passing through $(-3, 5)$ with slope -2.

(c) Write the equation of the line whose graph is shown below:

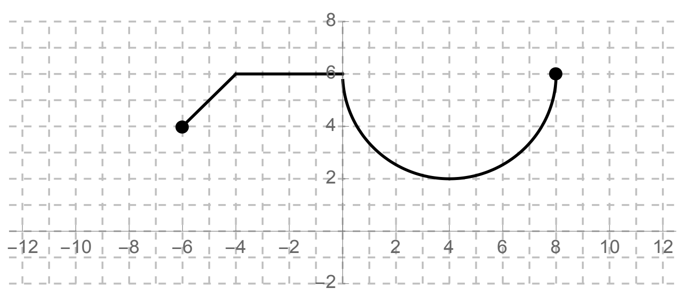


8. The function $\cos x$ is an example of an **even** function.

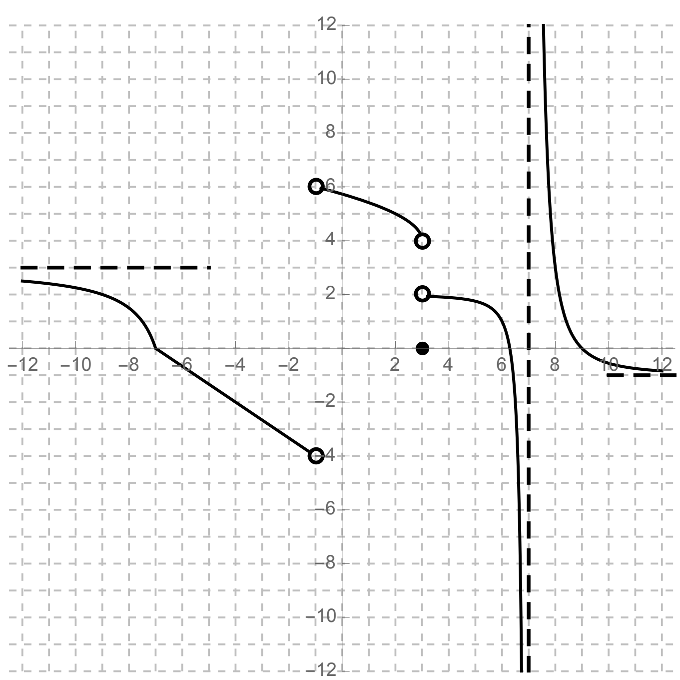
   (a) What property does the word "even" refer to about the graph of $\cos x$?

   (b) What algebraic fact / trig identity does the fact that $\cos x$ is even refer to?

9. Here is the graph of an unknown function f:



Here is the graph of an unknown function g:



Here is the rule for function h:

$$h(x) = \begin{cases} 2x+1 & x < 2 \\ 10 & x = 2 \\ 4-x & x > 2 \end{cases}$$

Here is a table of values for function j:

x	-4	-3	-2	-1	0	1	2	3	4
$j(x)$	7	2	-2	6	2	0	6	7	-1

Use that information to answer these questions:

(a) What is the domain of f?

(b) Find $g(3)$.

(c) Find $g(-1)$.

(d) Find all x such that $g(x) = -2$.

(e) How many x-intercepts does f have?

(f) Find the y-intercept of h.

(g) Find $j(4-2)$.

(h) Find $(h+j)(1)$.

(i) Write the equation of all vertical asymptotes of g.

(j) Find the net change in j from $x=0$ to $x=3$.

(k) Estimate the instantaneous rate of change in f when $x=-5$.

(l) Find $(g \circ h)(8)$.

10. Information about the functions f, g, h and j was provided in the previous problem. Use that information to answer these questions:

(a) Find $g(-9)+2$.

(b) Suppose $k(x) = 2j(2x)$. Find $k(-2)$.

(c) Of the four functions f, g, h and j, which has the greatest value when $x = 2$?

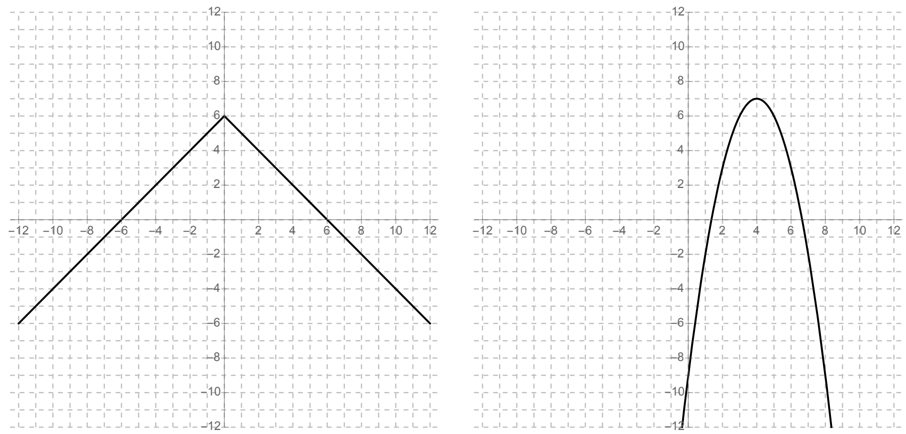
(d) Sketch the graph of the function $f(x-4)$.

(e) Sketch the graph of $f(2x)$.

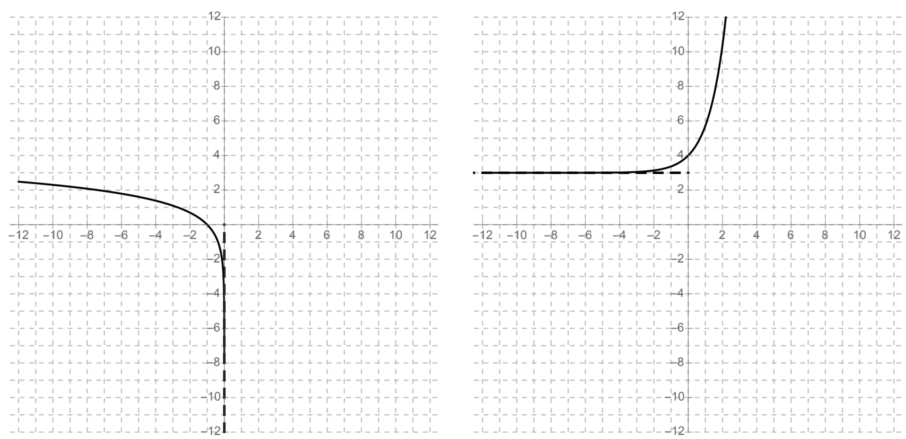
(f) Sketch the graph of $-f(x)$ on the provided axes.

11. Underneath each graph given below, write the rule for each function. Assume that any exponential and/or logarithmic functions are base e.

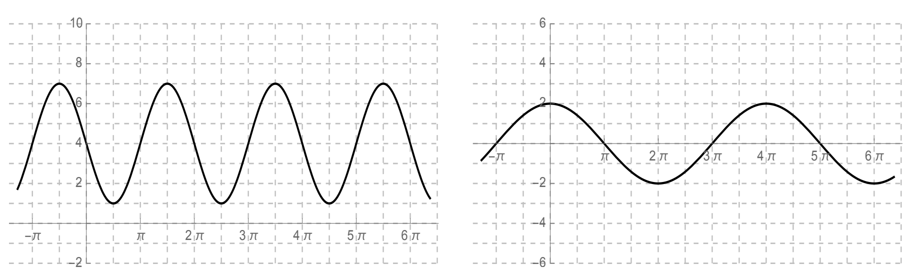
(a-b)



(c-d)



(e-f)



12. (a) One of these two functions is a polynomial. Circle the one that is, and give its degree and leading coefficient.

$$f(x) = 8x^2 - 5x^3 + 3x - 2 \qquad\qquad g(x) = \frac{2x^2+1}{x-2}$$

(b) One of these two functions is quadratic. Circle the one that is, and find its domain and range.

$$u(x) = (x-3)(x-9) \qquad\qquad v(x) = \sqrt{x^4+2}$$

(c) One of these two functions has a vertical asymptote. Circle the one that has a VA, and write the equation of its vertical asymptote.

$$F(x) = \ln(x-2) \qquad\qquad G(x) = e^{x-2}$$

13. (a) One of these two functions has a horizontal asymptote. Circle the one that has a HA, and write the equation of its horizontal asymptote.

$$h(x) = \frac{x}{x^2+2} \qquad\qquad k(x) = \frac{x^3-3x+1}{x-5}$$

(b) One of these two functions is one-to-one. Circle the one that is, and compute the rule for its inverse.

$$f(x) = \ln(x+5) - 2 \qquad\qquad g(x) = |x+3| - 2$$

(c) One of these two equations represents a circle. Circle the one that represents a circle, and give its radius.

$$(x-2)^2 + (y+3)^2 = 4 \qquad\qquad (x-1)^2 + 3y^2 = 9$$

14. Solve each of the following equations:

(a) $e^{3-x} = 6$

(b) $|x+3| - 2 = 0$

(c) $\sqrt{x^2+4} = 3$

(d) $2\sin 4x = 1$

15. Solve each of the following equations:

(a) $e^{2x} - e^x - 6 = 0$

(b) $x^5 + 4x^4 = 32x^3$

(c) $\sin x - \cos x = 0$

1. (a) $\frac{1}{x^3} = x^{-3}$.
   (b) $2x\sqrt{x} = 2xx^{1/2} = 2x^{1+1/2} = 2x^{3/2}$.
   (c) $\left(\sqrt[3]{x-2}\right)^2 = (x-2)^{2/3}$.
   (d) $\sqrt{\frac{9}{x}} = \frac{\sqrt{9}}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$.
2. (a) $\log_5 73 = \frac{\ln 73}{\ln 5}$.
   (b) $4^x = e^{x\ln 4}$.
3. (a) $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ (Quadrant II; ref. angle $\frac{\pi}{3} = 60°$)
   (b) $\tan\pi = 0$ (slope at $180°$ is zero)
   (c) $\cos\frac{-\pi}{4} = \frac{\sqrt{2}}{2}$ (Quadrant IV; ref. angle $\frac{\pi}{4} = 45°$)
   (d) $e^{\ln 4} = 4$
   (e) $\log_5\frac{1}{5} = -1$ (since $5^{-1} = \frac{1}{5}$)
   (f) $\log_6 9 + 2\log_6 2 = \log_6 9 + \log_6 2^2 = \log_6(9\cdot 2^2) = \log_6 36 = 2$.
   (g) $\frac{13^6 13^2}{(13^2)^4} = \frac{13^{6+2}}{13^{2\cdot 4}} = \frac{13^8}{13^8} = 1$.
   (h) $4! = 4\cdot 3\cdot 2\cdot 1 = 24$.
   (i) $\arcsin\frac{1}{2} = \frac{\pi}{6}$.
   (j) $\arctan -1 = -\arctan 1 = -\frac{\pi}{4}$.
4. Find a common denominator and add:

$$\begin{aligned}\frac{3}{x-2} + \frac{4}{x-1} &= \frac{3(x-1)}{(x-2)(x-1)} + \frac{4(x-2)}{(x-1)(x-2)}\\ &= \frac{3x-3}{(x-2)(x-1)} + \frac{4x-8}{(x-2)(x-1)}\\ &= \frac{3x-3+4x-8}{(x-2)(x-1)}.\end{aligned}$$

5. (a) TRUE (this is the definition of $\sin^2 x$)
   (b) FALSE (try $x = \pi, y = \frac{\pi}{2}$; then $\sin(x-y) = \sin\frac{\pi}{2} = 1$ but $\sin x - \sin y = 0 - 1 = -1$)
   (c) TRUE (definition of parallel lines)
   (d) TRUE (both are $7^{19\cdot 25}$)
   (e) TRUE ($\sin$ is an odd function)
   (f) TRUE ($\tan$ inverts $\arctan$)
   (g) FALSE (only true if $\frac{-\pi}{2} < x < \frac{\pi}{2}$)
   (h) FALSE (since this graph has no sharp corners, it is smooth as well as continuous)

(i) FALSE (there is no nice way to simplify $\sqrt{x^2+1}$ in general)

(j) FALSE (there is no nice way to simplify $\sqrt{x^2+1}$ in general)

6. $(f \circ g)(x) = f(g(x)) = f(1-x) = 3-2(1-x) = 3-2+2x = 1+2x$.

7. (a) $m = \frac{y_2-y_1}{x_2-x_1} = \frac{7-3}{2-(-4)} = \frac{4}{6} = \frac{2}{3}$.

   (b) From the point-slope formula, this is $y = -2(x+3)+5$.

   (c) This vertical line has equation $x = 3$.

8. (a) "Even" means the graph is symmetric about the y-axis.

   (b) $\cos(-x) = \cos x$.

9. (a) $[-6, 8]$ (the set of x-values covered by the graph).

   (b) $g(3) = 0$ (the height of the dot at $x = 3$).

   (c) $g(-1)$ DNE (no dot above or below $x = -1$).

   (d) $x = -4, x \approx 6.5$.

   (e) None (the graph does not cross the x-axis).

   (f) $h(0) = 2(0) + 1$ so the y-intercept is $(0, 1)$.

   (g) $j(4-2) = j(2) = 6$.

   (h) $(h+j)(1) = h(1) + j(1) = [2(1)+1] + 0 = 3$.

   (i) $x = 7$ is the only VA of g.

   (j) This is $j(3) - j(0) = 7 - 2 = 5$ units.

   (k) This is the slope of f near $x = -5$, which is 1.

   (l) $(g \circ h)(8) = g(h(8)) = g(4-8) = g(-4) = -2$.

10. (a) $g(-9) + 2 = 2 + 2 = 4$.

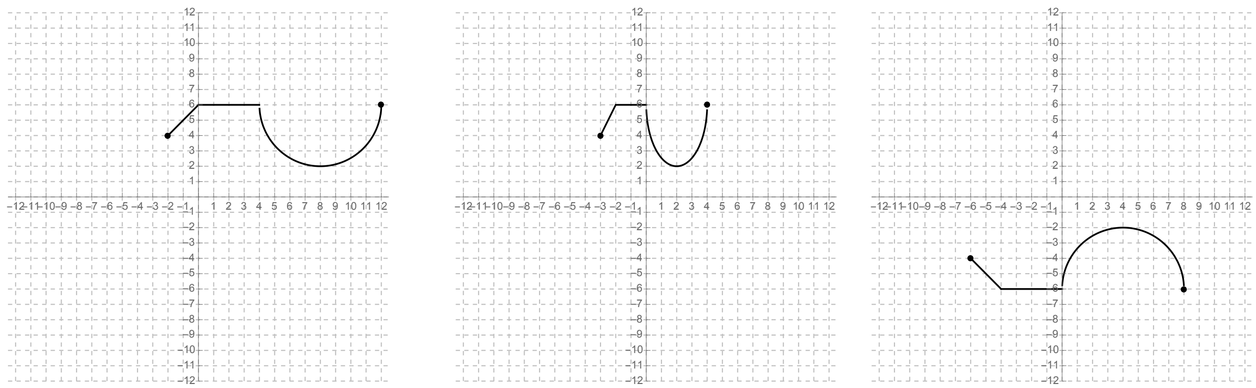
    (b) $k(-2) = 2j(2(-2)) = 2j(-4) = 2(7) = 14$.

    (c) $f(2) \approx 2.5$, $g(2) = 5$, $h(2) = 10$ and $j(2) = 6$, so h has the biggest value.

    (d) Shift f right by 4 units to get the graph below at left:

    (e) Smash f horizontally by a factor of 2 to get the graph below in the middle:

    (f) Reflect f across the x-axis to get the graph below at right:



11. Reading the three rows of graphs as (a) and (b), then (c) and (d), and last (e) and (f):

    (a) This is $|x|$, reflected across the x-axis and shifted up 6 units, i.e. $f(x) = -|x| + 6$.

    (b) This is x^2, reflected across the x-axis, shifted right 4 units and up 7 units, i.e. $f(x) = -(x-4)^2 + 7$.

    (c) This is $\ln x$, reflected across the y-axis, i.e. $f(x) = \ln(-x)$.

    (d) This is e^x, shifted up 3 units, i.e. $f(x) = e^x + 3$.

    (e) This is $\sin x$, reflected across the x-axis, shifted up 4 units and stretched vertically by a factor of 3, i.e. $f(x) = -3\sin x + 4$.

    (f) This is $\cos x$, stretched vertically by a factor of 2 and stretched horizontally by a factor of 2, i.e. $f(x) = 2\cos\frac{x}{2}$.

12. (a) f is a polynomial (since it is non-negative powers of x, multiplied by constants and added together) of degree 3 (the highest power) and leading coefficient -5 (the coefficient on the highest-power term).

    (b) By multiplying out u, we see that u is quadratic: $u(x) = u^2 - 12x + 27$. The domain of any quadratic is all real numbers. To find the range, we need to find the vertex: $h = \frac{-b}{2a} = \frac{-(-12)}{2(1)} = 6$ and $k = u(6) = 6^2 - 12(6) + 27 = -9$. Since the graph of u opens upward, its range is $[-9, \infty)$.

    (c) $F(x) = \ln(x-2)$ has VA $x = 2$ (its graph is the graph of $\ln x$, shifted right by 2 units).

13. (a) $h(x)$ has HA $y = 0$ (since the degree of the numerator is less than the degree of the denominator).

    (b) f is $1-1$ (the graph of g fails the Horizontal Line Test because it looks like a V). To get the inverse of f, set $y = f(x)$ and solve for x:

$$\begin{array}{ccccccc} x & \xrightarrow{x+5} & & \xrightarrow{\ln x} & & \xrightarrow{x-2} & y \\ e^{y+2}-5 & \xleftarrow[x-5]{} & e^{y+2} & \xleftarrow[e^x]{} & y+2 & \xleftarrow[x+2]{} & \end{array}$$

Therefore $f^{-1}(x) = e^{x+2} - 5$.

(c) $(x-2)^2+(y+3)^2 = 4$ is a circle (because it has form $(x-x_0)^2+(y-y_0)^2 = r^2$) with center $(2,-3)$ and radius $\sqrt{4} = 2$.

14. In each of these equations, x only appears once, so we can solve each equations with arrows:

(a) $e^{3-x} = 6$:

$$\begin{array}{ccccccc} x & \xrightarrow{-x} & & \xrightarrow{x+3} & & \xrightarrow{e^x} & 6 \\ -\ln 6 + 3 & \xleftarrow[-x]{} & \ln 6 - 3 & \xleftarrow[x-3]{} & \ln 6 & \xleftarrow[\ln x]{} & \end{array}$$

Therefore $x = -\ln 6 + 3$.

(b) $|x+3| - 2 = 0$:

$$\begin{array}{ccccccc} x & \xrightarrow{x+3} & & \xrightarrow{|x|} & & \xrightarrow{x-2} & 0 \\ -1,-5 & \xleftarrow[x-3]{} & \pm 2 & \xleftarrow[\pm x]{} & 2 & \xleftarrow[x+2]{} & \end{array}$$

Therefore $x = -1, -5$.

(c) $\sqrt{x^2+4} = 3$:

$$\begin{array}{ccccccc} x & \xrightarrow{x^2} & & \xrightarrow{x+4} & & \xrightarrow{\sqrt{x}} & 3 \\ \pm\sqrt{5} & \xleftarrow[\pm\sqrt{x}]{} & 5 & \xleftarrow[x-4]{} & 9 & \xleftarrow[x^2]{} & \end{array}$$

Therefore $x = \pm\sqrt{5}$.

(d) $2\sin 4x = 1$:

$$\begin{array}{ccccccc} x & \xrightarrow{4x} & & \xrightarrow{\sin x} & & \xrightarrow{2x} & 1 \\ \frac{\pi}{24}+\frac{\pi}{2n}, \frac{5\pi}{24}+\frac{\pi}{2n} & \xleftarrow[\frac{1}{4}x]{} & \frac{\pi}{6}+2\pi n, \frac{5\pi}{6}+2\pi n & \xleftarrow[\substack{\arcsin x + 2\pi n \\ \pi - \arcsin x + 2\pi n}]{} & \frac{1}{2} & \xleftarrow[\frac{1}{2}x]{} & \end{array}$$

Therefore $x = \frac{\pi}{24}+\frac{\pi}{2n}, \frac{5\pi}{24}+\frac{\pi}{2n}$.

15. (a) This equation is quadratic-type, so factor it and set each factor equal to zero:

$$e^{2x} - e^x - 6 = 0$$
$$(e^x - 3)(e^x + 2) = 0$$

From the first term, $e^x = 3$ so $x = \ln 3$; from the second term we get $e^x = -2$ which has no solution. Thus the only solution is $x = \ln 3$.

(b) $x^5 + 4x^4 = 32x^3$ Move all terms to the left-hand side, then factor and set each factor equal to zero:

$$x^5 + 4x^4 = 32x^3$$
$$x^5 + 4x^4 - 32x^3 = 0$$
$$x^3(x^2 + 4x - 32) = 0$$
$$x^3(x+8)(x-4) = 0$$

Therefore $x = 0, x = -8, x = 4$ are the three solutions.

(c) Add $\cos x$ to both sides, divide through by $\cos x$:

$$\begin{aligned} \sin x - \cos x &= 0 \\ \sin x &= \cos x \\ \frac{\sin x}{\cos x} &= 1 \\ \tan x &= 1 \end{aligned}$$

Therefore $x = \frac{\pi}{4} + \pi n$.