

Directions

In each of these problems you are given an equation with a single variable in it. Think of the equation as $f(x) = c$, where f is a function made up of a bunch of compositions of elementary functions. For each equation, you are to

- Draw an arrow diagram which decomposes f into its elementary components;
- Underneath your arrow diagram, draw arrows in the opposite direction labelled with the appropriate inverse or partial inverse of each component of f ;
- Solve the equation, simplifying your answer if appropriate.

Here are two worked out examples, so you can see what I mean:

Example 1: $5e^{3-x} = 12$

Arrow diagram:

$$\begin{array}{ccccccc} x & \xrightarrow{-x} & x+3 & \xrightarrow{e^x} & 5x & = & 12 \\ & \xleftarrow{-x} & & \xleftarrow{\ln x} & & & \\ & & x-3 & & \frac{x}{5} & & \end{array}$$

Solution of the equation: $x = -\left(\ln \frac{12}{5} - 3\right)$.

Example 2: $3(e^x - 8)^2 = 75$

Arrow diagram:

$$\begin{array}{ccccccc} x & \xrightarrow{e^x} & x-8 & \xrightarrow{x^2} & 3x & = & 75 \\ & \xleftarrow{\ln x} & & \xleftarrow{\pm\sqrt{x}} & & & \\ & & x+8 & & \frac{x}{3} & & \end{array}$$

Solution of the equation: $x = \ln\left(\pm\sqrt{\frac{75}{3}} + 8\right) = \ln(\pm 5 + 8) = \ln 3, \ln 13$.

Problems

- $\left(\frac{x^3}{4} - 3\right)^3 = 64$
- $5|x| - 3 = 8$
- $2|6 - 3x| + 4 = 12$
- $\log x^3 - 3 = 4$
- $\log(x^3 - 3) = 4$
- $\log^3(x - 3) = 4$
- $\log^3 x - 3 = 4$
- $3 = 5^{\ln(3x+2)}$
- $\sqrt{6 + 3\sqrt{x}} = 4$
- $(3x^4 - 5)^3 + 4 = 12$
- $16 = 2^{4 \arcsin x}$
- $\log_2(\log_3 x) = 4$
- $1 = \arctan e^x$
- $(3 - x)^{2/3} = 4$

Answers

1. $(\frac{x^3}{4} - 3)^3 = 64$

$$x \begin{array}{c} \xrightarrow{x^3} \\ \xleftarrow{\sqrt[3]{x}} \end{array} \quad \begin{array}{c} \xrightarrow{x/4} \\ \xleftarrow{4x} \end{array} \quad \begin{array}{c} \xrightarrow{x-3} \\ \xleftarrow{x+3} \end{array} \quad \begin{array}{c} \xrightarrow{x^3} \\ \xleftarrow{\sqrt[3]{x}} \end{array} 64$$

Solution: $x = \sqrt[3]{28}$

2. $5|x| - 3 = 8$

$$x \begin{array}{c} \xrightarrow{|x|} \\ \xleftarrow{\pm x} \end{array} \quad \begin{array}{c} \xrightarrow{5x} \\ \xleftarrow{x/5} \end{array} \quad \begin{array}{c} \xrightarrow{x-3} \\ \xleftarrow{x+3} \end{array} 8$$

Solutions: $x = -\frac{11}{5}, \frac{11}{5}$

3. $2|6 - 3x| + 4 = 12$

$$x \begin{array}{c} \xrightarrow{-3x} \\ \xleftarrow{\frac{x}{-3}} \end{array} \quad \begin{array}{c} \xrightarrow{x+6} \\ \xleftarrow{x-6} \end{array} \quad \begin{array}{c} \xrightarrow{|x|} \\ \xleftarrow{\pm x} \end{array} \quad \begin{array}{c} \xrightarrow{2x} \\ \xleftarrow{x/2} \end{array} \quad \begin{array}{c} \xrightarrow{x+4} \\ \xleftarrow{x-4} \end{array} 12$$

Solutions: $x = \frac{2}{3}, \frac{10}{3}$

4. $\log x^3 - 3 = 4$

$$x \begin{array}{c} \xrightarrow{x^3} \\ \xleftarrow{\sqrt[3]{x}} \end{array} \quad \begin{array}{c} \xrightarrow{\log x} \\ \xleftarrow{10^x} \end{array} \quad \begin{array}{c} \xrightarrow{x-3} \\ \xleftarrow{x+3} \end{array} 4$$

Solution: $x = \sqrt[3]{10^7}$

5. $\log(x^3 - 3) = 4$

$$x \begin{array}{c} \xrightarrow{x^3} \\ \xleftarrow{\sqrt[3]{x}} \end{array} \quad \begin{array}{c} \xrightarrow{x-3} \\ \xleftarrow{x+3} \end{array} \quad \begin{array}{c} \xrightarrow{\log x} \\ \xleftarrow{10^x} \end{array} 4$$

Solution: $x = \sqrt[3]{10003}$

6. $\log^3(x - 3) = 4$

$$x \begin{array}{c} \xrightarrow{x-3} \\ \xleftarrow{x+3} \end{array} \quad \begin{array}{c} \xrightarrow{\log x} \\ \xleftarrow{10^x} \end{array} \quad \begin{array}{c} \xrightarrow{x^3} \\ \xleftarrow{\sqrt[3]{x}} \end{array} 4$$

Solution: $x = 10^{\sqrt[3]{4}} + 3$

7. $\log^3 x - 3 = 4$

$$x \begin{array}{c} \xrightarrow{\log x} \\ \xleftarrow{10^x} \end{array} \quad \begin{array}{c} \xrightarrow{x^3} \\ \xleftarrow{\sqrt[3]{x}} \end{array} \quad \begin{array}{c} \xrightarrow{x-3} \\ \xleftarrow{x+3} \end{array} 4$$

Solution: $x = 10^{\sqrt[3]{7}}$

8. $3 = 5^{\ln(3x+2)}$

$$\begin{array}{cccc} x \xrightarrow{3x} & \xrightarrow{x+2} & \xrightarrow{\ln x} & \xrightarrow{5^x} 3 \\ \xleftarrow{\frac{x}{3}} & \xleftarrow{x-2} & \xleftarrow{e^x} & \xleftarrow{\log_5 x} \end{array}$$

Solution: $x = \frac{1}{3} (e^{\log_5 x} - 2)$

9. $\sqrt{6 + 3\sqrt{x}} = 4$

$$\begin{array}{cccc} x \xrightarrow{\sqrt{x}} & \xrightarrow{3x} & \xrightarrow{x+6} & \xrightarrow{\sqrt{x}} 4 \\ \xleftarrow{x^2} & \xleftarrow{\frac{x}{3}} & \xleftarrow{x-6} & \xleftarrow{x^2} \end{array}$$

Solution: $x = \frac{100}{9}$

10. $(3x^4 - 5)^3 + 4 = 12$

$$\begin{array}{ccccc} x \xrightarrow{x^4} & \xrightarrow{3x} & \xrightarrow{x-5} & \xrightarrow{x^3} & \xrightarrow{x+4} 12 \\ \xleftarrow{\sqrt[4]{x}} & \xleftarrow{\frac{x}{3}} & \xleftarrow{x+5} & \xleftarrow{\sqrt[3]{x}} & \xleftarrow{x-4} \end{array}$$

Solution: $x = \sqrt[4]{\frac{7}{3}}$

11. $16 = 2^4 \arcsin x$

$$\begin{array}{ccc} x \xrightarrow{\arcsin x} & \xrightarrow{4x} & \xrightarrow{2^x} 12 \\ \xleftarrow{\sin x} & \xleftarrow{\frac{x}{4}} & \xleftarrow{\log_2 x} \end{array}$$

Solution: $x = \sin\left(\frac{\log_2 12}{4}\right)$

12. $\log_2(\log_3 x) = 4$

$$\begin{array}{cc} x \xrightarrow{\log_3 x} & \xrightarrow{\log_2 x} 4 \\ \xleftarrow{3^x} & \xleftarrow{2^x} \end{array}$$

Solution: $x = 3^{16}$

13. $1 = \arctan e^x$

$$\begin{array}{cc} x \xrightarrow{3^x} & \xrightarrow{\arctan x} 1 \\ \xleftarrow{\log_3 x} & \xleftarrow{\tan x} \end{array}$$

Solution: $x = \log_3(\tan 1)$

14. $(3 - x)^{2/3} = 4$

$$\begin{array}{cccc} x \xrightarrow{-x} & \xrightarrow{x+3} & \xrightarrow{x^2} & \xrightarrow{\sqrt[3]{x}} 4 \\ \xleftarrow{-x} & \xleftarrow{x-3} & \xleftarrow{\sqrt{x}} & \xleftarrow{x^3} \end{array}$$

Solution: $x = -5$