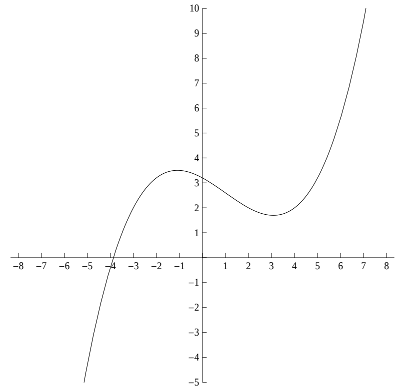
1. (a) Consider the equation $x^3 - 4x + 2 = 0$. Use Newton's method to approximate a solution of the equation by computing $x_2$ and using initial guess $x_0 = 0$.

   (b) Suppose you used Newton's method on the unknown function whose graph is given below with initial guess $x_0 = 1$. Use the graph to estimate the values of $x_1$ and $x_2$ you would obtain.




2. A hot air balloon is rising (directly upward) at a rate of 100 meters per minute. An observer is standing 300 meters from the point where the balloon lifted off. What is the rate of change of the distance from the observer to the balloon, at the instant where the balloon is 400 meters in the air?

3. If a truck travels at speed $x$ miles per hour, then its fuel costs (in dollars) are $C(x) = \frac{10000}{x} + 4x$. If the truck driver wants to minimize his fuel costs, at what speed should she drive?

4. The electric potential $V$ at a point $(x, y)$ is given by the equation $V = 6x^2 + 4y^2$. Find the point on the line $x + 4y = 50$ where the electric potential is minimized.

5. (a) Compute the linearization of the function $f(x) = \frac{1}{1+\sqrt{x}}$ at $a = 9$. Use the linearization to estimate $f(9.5)$.

   (b) (6 pts) Suppose $y = 3x^4 - 2x^2 + x$. Compute the differential $dy$.

   (c) (6 pts) Suppose $y = 12x^{1/3} - 3x^{3/5}$. Compute the differential $dy$ when $x = 1$ and $dx = \frac{1}{4}$.

1. (a) First, $f(x) = x^3 - 4x + 2$ so $f'(x) = 3x^2 - 4$. Then

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{0^3 - 4(0) + 2}{3(0)^2 - 4} = \frac{1}{2};$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{1}{2} - \frac{(1/2)^3 - 4(1/2) + 2}{3(1/2)^2 - 4} = \frac{7}{13} \approx .538462.$$

(b) Draw in the tangent line at 1; it appears to hit the $x$-axis at about 5.5 or so, so $x_1 \approx 5.5$. Now draw in the tangent line at 5.5; it appears to hit the $x$-axis at about 4 so $x_2 \approx 4$. (Answers may vary.)

2. Let $h$ be the height of the balloon and let $x$ be the distance from the balloon to the observer. By the Pythagorean Theorem, $h^2 + 300^2 = x^2$. Take $\frac{d}{dt}$ of both sides of this equation to get

$$2h\frac{dh}{dt} = 2x\frac{dx}{dt}.$$

Now we are given $h = 400$ and $\frac{dh}{dt} = 100$; we can solve for $x$ using the original equation:

$$400^2 + 300^2 = x^2 \Rightarrow x = 500$$

so by plugging all these in, we get

$$2(400)(100) = 2(500)\frac{dx}{dt}$$

i.e. $\frac{dx}{dt} = 80$ meters per minute.

3. The utility is $C(x) = \frac{10000}{x} + 4x$; take the derivative using the Quotient Rule on the first part to get $C'(x) = \frac{0x-1(10000)}{x^2} + 4 = \frac{-10000}{x^2} + 4$. Set the derivative equal to zero to get the equation $0 = \frac{-10000}{x^2} + 4$, and solve for $x$ to get $x = 50$ units. (This checks out as a minimum).

4. The utility is $V = 6x^2 + 4y^2$; this has two variables so we start with the constraint $x + 4y = 50$ and solve for $x$: $x = 50 - 4y$. Now plug into the utility to get $V(y) = 6(50-4y)^2 + 4y^2$; take the derivative (use the Chain Rule on the first part) to get $V'(y) = 6(2)(50-4y)(-4) + 8y = -2400 + 192y + 8y = -2400 + 200y$. Set this equal to zero to get $0 = -2400 + 200y$, i.e. $y = 12$. (This checks out as a minimum.) Then solve for $x$ to get $x = 50 - 4(12) = 2$ so the point with the minimum electric potential is $(x, y) = (2, 12)$.

5. (a) $f(x) = \frac{1}{1+\sqrt{x}} = (1+\sqrt{x})^{-1}$ is given; take the derivative using the Chain Rule to get $f'(x) = -(1+\sqrt{x})^{-2} \cdot \frac{1}{2\sqrt{x}}$. Now $f(a) = f(9) = \frac{1}{4}$ and $f'(a) = -4^{-2} \cdot \frac{1}{6} = \frac{-1}{96}$. So the linearization at $a = 9$ is

$$L = f(a) + f'(a)(x-a) = \frac{1}{4} + \frac{-1}{96}(x-9)$$

and therefore

$$f(9.5) \approx L(9.5) = \frac{1}{4} + \frac{-1}{96}(9.5-9) = .244792.$$

(b) $dy = f'(x)\,dx = (12x^3 - 4x + 1)\,dx$.

(c) $dy = f'(x)\,dx = (4x^{-2/3} - \frac{9}{5}x^{-2/5})\,dx$; now plug in $x = 1$ and $dx = \frac{1}{4}$ to get $dy = (4 - \frac{9}{5})(\frac{1}{4}) = \frac{11}{5} \cdot \frac{1}{4} = \frac{11}{20} = .55$.