Name:

Directions: This exam has five questions, spread across four pages (not counting this cover page). Answers must be justified appropriately on these pages; show all work and clearly mark your final answers. You may use a calculator, but you may not use a computer, notes or other study aids.

Grading:

Problem	Points Possible	Points Earned
1	55	
2	8	
3	13	
4	12	
5	12	
Total	100	

1. (11 pts each) Compute any five of the following six problems.

   Note: I will only grade your work on five of these six problems. If it is not clear from your work which five you want graded, draw an X through the one you don't want me to grade; otherwise I will grade the first five.

   (a) Find an antiderivative of the function $f(x) = 2x^2(x-2)$.

   (b) Compute $\int_1^2 (3 - 4x + 6x^2)\, dx$

   (c) Compute $\int \left(x^3 + 18x^{3/2} - \frac{3}{x^4} + 1\right)\, dx$

(d) Compute $\int 8x\sqrt{x^2+2}\,dx$

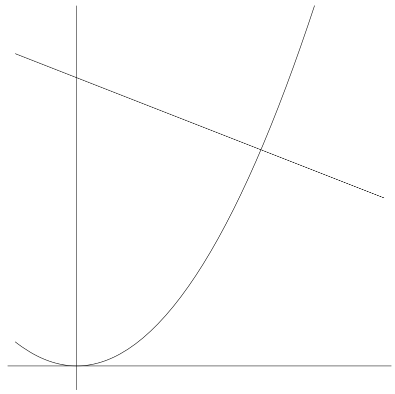
(e) Compute $\int_0^1 36(2x+1)^8\,dx$

(f) Compute $\int 10(8x^5-3)^{-6}x^4\,dx$

2. (8 pts) Suppose that the force applied to an object x units from its starting point is given by $f(x) = 4x + 3$ Newtons. Find the work done in moving the object 5 meters.

3. (13 pts) Suppose that an airplane's acceleration at time t is given by $a(t) = 12t + 48$ mi/hr^2. If the plane's velocity at time 0 is 300 mi/hr, find the distance the plane travels between time 0 and time 2.

4. (12 pts) Find the area of the region of points which lies above the graph of $y = x^2$, to the right of the y-axis and below the line $y = -x + 12$ (a picture of this region, **not to scale**, is shown below).



5. (12 pts) Find the volume of the solid obtained by revolving the region of points below the graph of $y = x^3$ from $x = 0$ to $x = 2$ around the y-axis.

1. Compute any five of the following six problems:
   (a) Find an antiderivative of the function $f(x) = 2x^2(x-2)$.
   (b) Compute $\int_1^2 (3 - 4x + 6x^2)\ dx$
   (c) Compute $\int \left(x^3 + 18x^{3/2} - \frac{3}{x^4} + 1\right)\ dx$
   (d) Compute $\int 8x\sqrt{x^2+2}\,dx$
   (e) Compute $\int_0^1 36(2x+1)^8\,dx$
   (f) Compute $\int 10(8x^5-3)^{-6}x^4\,dx$
2. Suppose that the force applied to an object x units from its starting point is given by $f(x) = 4x + 3$ Newtons. Find the work done in moving the object 5 meters.
3. Suppose that an airplane's acceleration at time t is given by $a(t) = 12t + 48$ mi/hr^2. If the plane's velocity at time 0 is 300 mi/hr, find the distance the plane travels between time 0 and time 2.
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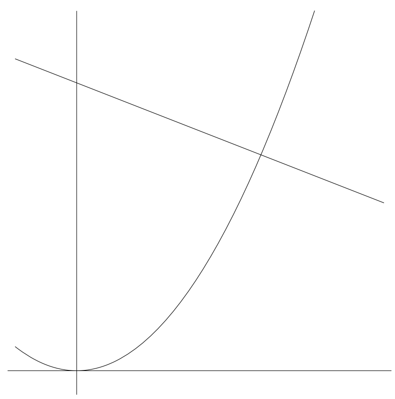

5. Find the volume of the solid obtained by revolving the region of points below the graph of $y = x^3$ from $x = 0$ to $x = 2$ around the y-axis.

1. (a) First, multiply out to get $f(x) = 2x^3 - 4x^2$. Then by usual antidifferentiation rules, $F(x) = \frac{2x^4}{4} - \frac{4x^3}{3} = \frac{1}{2}x^4 - \frac{4}{3}x^3$.

   (b) $\int_1^2 (3 - 4x + 6x^2)\, dx = \left[3x - 4 \cdot \frac{x^2}{2} + 2x^3\right]_1^2 = [6 - 8 + 16] - [3 - 2 + 2] = 14 - 3 = 11$.

   (c) First, change the third term using an exponent rule:

$$\begin{aligned}\int \left(x^3 + 18x^{3/2} - \frac{3}{x^4} + 1\right) dx &= \int \left(x^3 + 18x^{3/2} - 3x^{-4} + 1\right) dx \\ &= \frac{x^4}{4} + 18\frac{x^{5/2}}{5/2} - 3\frac{x^{-3}}{-3} + x + C \\ &= \frac{x^4}{4} + 18 \cdot \frac{2}{5}x^{5/2} + x^{-3} + x + C \\ &= \frac{x^4}{4} + \frac{36}{5}x^{5/2} + x^{-3} + x + C.\end{aligned}$$

   (d) This integral requires a u-substitution: let $u = x^2 + 2$ so that $du = 2x\,dx$ and $\frac{du}{2x} = dx$. Now substituting into the integral, we get

$$\begin{aligned}\int 8x\sqrt{x^2+2}\, dx = \int 8x\sqrt{u}\frac{du}{2x} &= \int 4u^{1/2}\, du \\ &= 4\frac{u^{3/2}}{3/2} + C \\ &= 4 \cdot \frac{2}{3}u^{3/2} + C = \frac{8}{3}(x^2+2)^{3/2} + C.\end{aligned}$$

   (e) This integral requires a u-substitution: let $u = 2x + 1$ so that $du = 2\,dx$ and $\frac{du}{2} = dx$. Since this is a definite integral, we have to change the limits: when $x = 0$, $u = 2(0) + 1 = 1$ and when $x = 1$, $u = 2(1) + 1 = 3$.

   Putting all this into the integral, we get

$$\begin{aligned}\int_0^1 36(2x+1)^8\, dx = \int_1^3 36u^8\frac{du}{2} &= \int_1^3 18u^8\, du \\ &= \left[18\frac{u^9}{9}\right]_1^3 = [2u^9]_1^3 = 2(3)^9 - 2(1)^9 = 39364.\end{aligned}$$

   (f) This integral requires a u-substitution: let $u = 8x^5 - 3$ so that $du = 40x^4$ and $\frac{du}{40x^4} = dx$. Now substituting into the integral, we get

$$\begin{aligned}\int 10(8x^5-3)^{-6}x^4\, dx = \int 10u^{-6}x^4\frac{du}{40x^4} &= \int \frac{1}{4}u^{-6}\, du \\ &= \frac{1}{4} \cdot \frac{u^{-5}}{-5} + C = \frac{-1}{20}(8x^5-3)^{-5} + C.\end{aligned}$$

2. Work is the integral of force; since we move the object 5 meters the answer is

$$W = \int_0^5 f(x)\,dx = \int_0^5 (4x+3)\,dx = [2x^2+3x]_0^5 = (2\cdot 5^2 + 3\cdot 5) - 0 = 65\text{Nm}.$$

3. First, compute the velocity by integrating the acceleration: $v(t) = \int (12t+48)\,dt = 6t^2+48t+C$. Since the initial velocity is 300, we have $C = 300$ so $v(t) = 6t^2+48t+300$. Last, integrate again to get the distance travelled:

$$\begin{aligned} d = \int_0^2 v(t)\,dt &= \int_0^2 (6t^2+48t+300)\,dt \\ &= [2t^3+24t^2+300t]_0^2 \\ &= [2\cdot 2^3 + 24\cdot 2^2 + 300\cdot 2] - 0 \\ &= 16+96+600 = 712 \text{ miles.} \end{aligned}$$

4. First, find the intersection point in the first quadrant by setting the two equations equal to each other:

$$\begin{aligned} x^2 &= -x+12 \\ x^2+x-12 &= 0 \\ (x+4)(x-3) &= 0 \\ \Rightarrow x = -4, x &= 3 \end{aligned}$$

Now discard the negative answer (since it isn't to the right of the y-axis) to leave $x = 3$. Thus the region goes from $x = 0$ to $x = 3$, so its area is

$$\begin{aligned} A &= \int_0^3 (-x+12)\,dx - \int_0^3 x^2\,dx \\ &= \left[\frac{-x^2}{2} + 12x\right]_0^3 - \left[\frac{x^3}{3}\right]_0^3 \\ &= \frac{-3^2}{2} + 12(3) - \frac{3^3}{3} \\ &= -4.5 + 36 - 9 = 22.5. \end{aligned}$$

5. By the formula derived in class,

$$V = \int_a^b 2\pi x f(x)\,dx = \int_0^2 2\pi x(x^3)\,dx = \int_0^2 2\pi x^4\,dx = \left[\frac{2\pi x^5}{5}\right]_0^2 = \frac{64\pi}{5} \approx 40.212.$$