Here are the 19 theorems of Math 25:

Domination Law (aka Monotonicity Law) If f and g are integrable functions on $[a,b]$ such that $f(x) \leq g(x)$ for all $x \in [a,b]$, then $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$.

Positivity Law If f is a nonnegative, integrable function on $[a,b]$, then $\int_a^b f(x)\,dx \geq 0$. Also, if $f(x)$ is a nonnegative, continuous function on $[a,b]$ and $\int_a^b f(x)\,dx = 0$, then $f(x) = 0$ for all $x \in [a,b]$.

Max-Min Inequality If $f(x)$ is an integrable function on $[a,b]$, then

$$(\text{min value of } f(x) \text{ on } [a,b]) \cdot (b-a) \leq \int_a^b f(x)\,dx \leq (\text{max value of } f(x) \text{ on } [a,b]) \cdot (b-a).$$

Triangle Inequality (for Integrals) Let $f : [a,b] \to \mathbb{R}$ be integrable. Then

$$\left| \int_a^b f(x)\,dx \right| \leq \int_a^b |f(x)|\,dx.$$

Mean Value Theorem for Integrals Let $f : \mathbb{R} \to \mathbb{R}$ be continuous on $[a,b]$. Then there exists a number $c \in (a,b)$ such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x)\,dx.$$

Fundamental Theorem of Calculus (one part) Let $f : [a,b] \to \mathbb{R}$ be continuous and define $F : [a,b] \to \mathbb{R}$ by $F(x) = \int_a^x f(t)\,dt$. Then F is differentiable (hence continuous) on $[a,b]$ and $F'(x) = f(x)$.

Fundamental Theorem of Calculus (other part) Let $f : [a,b] \to \mathbb{R}$ be continuous and let F be any antiderivative of f on $[a,b]$. Then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Comparison Test (for Improper Integrals) Let f and g be two continuous functions on $[a,\infty)$. If $f(x) \leq g(x)$ for all $x \in [a,\infty)$, then:

1. If $\int_a^\infty f(x)\,dx$ diverges, so does $\int_a^\infty g(x)\,dx$.
2. If $\int_a^\infty g(x)\,dx$ converges, so does $\int_a^\infty f(x)\,dx$.

$n^{th}-$**Term Test** Let $\sum a_n$ be an infinite series. If $\lim_{n\to\infty} a_n \neq 0$ (or if this limit fails to exist), then the series diverges.

Integral Test Let $f : \mathbb{R} \to \mathbb{R}$ be a nonnegative, decreasing function such that $f(n) = a_n$ for all $n = 1, 2, 3, ...$ Then $\int_1^\infty f(x)\,dx$ and $\sum_{n=1}^\infty a_n$ either both converge or both diverge.

$p-$**series Test** Let p be a constant. Then the infinite series $\sum_{n=1}^\infty \frac{1}{n^p}$ converges if and only if $p \geq 1$.

Comparison Test (for Series) Suppose $0 \leq a_n \leq b_n$ for all n. Then:

1. If $\sum_{n=1}^\infty a_n$ diverges, so does $\sum_{n=1}^\infty b_n$.
2. If $\sum_{n=1}^\infty b_n$ converges, so does $\sum_{n=1}^\infty a_n$.

Alternating Series Test Let $\sum_{n=1}^\infty a_n$ be an alternating series such that $|a_n| \geq |a_{n+1}|$ for all n, and $\lim_{n\to\infty} a_n = 0$. Then $\sum_{n=1}^\infty a_n$ converges.

Triangle Inequality (for Series) If an infinite series $\sum a_n$ converges absolutely, then it also converges.

Rearrangement Theorem The terms of an absolutely convergent series can be regrouped or reordered without affecting the sum of the series.

Ratio Test Let $\sum_{n=1}^{\infty} a_n$ be an infinite series and define $\rho = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$. Then:

1. If $\rho > 1$, then $\sum a_n$ diverges.
2. If $\rho < 1$, then $\sum a_n$ converges absolutely.
3. If $\rho = 1$ or if the limit which defines ρ fails to exist, then the test is inconclusive.

Cauchy-Hadamard Theorem Let $\sum_{n=0}^{\infty} a_n(x-a)^n$ be a power series. Then there is a number $R \geq 0$ ($R = \infty$ is allowed) such that:

1. If $R = 0$, then the power series converges when $x = a$ and diverges otherwise.
2. If $R = \infty$, then the power series converges absolutely for all x.
3. If $R \in (0, \infty)$ then the series converges absolutely for $x \in (a-R, a+R)$; the series diverges for $x \in (-\infty, a-R) \cup (a+R, \infty)$; the behavior of the series when $x = a-R$ or $x = a+R$ is unknown.

Uniqueness of Power Series Suppose $f(x)$ is represented by two power series, each with positive radius of convergence, and each centered at the same point a, i.e.

$$f(x) = \sum_{n=0}^{\infty} a_n(x-a)^n = \sum_{n=0}^{\infty} b_n(x-a)^n.$$

Then all the coefficients of these power series must match, i.e. $a_n = b_n$ for all n, because $a_n = b_n = \frac{f^n(a)}{n!}$.

Taylor's Theorem Let $f(x)$ be an infinitely differentiable function; let $P_n(x)$ be its n^{th} Taylor polynomial centered at a and let $R_n(x) = f(x) - P_n(x)$ be the n^{th} remainder. Then for each x, there is some number z between a and x such that

$$R_n(x) = \frac{f^{n+1}(z)}{(n+1)!}(x-a)^{n+1}.$$

Here is a list of the terms which were defined in Math 25:

Terminology related to integrals Riemann sum, definite integral, average value, antiderivative, indefinite integral

Terminology related to infinite series partial sum, converges, diverges, converges absolutely, converges conditionally, geometric series, $p-$series

Terminology related to power series power series, radius of convergence, Taylor series, n^{th} Taylor polynomial, n^{th} remainder