Name:

Circle your TA: Amanda Potts Paul Gibson

Directions: Write your name on every page. Show all work and clearly mark your final answers. All your work should be on these exam pages (do not use scrap paper). You may not use a calculator.

1. (9 pts) Calculate the indefinite integral

$$\int \left\langle 4e^t, \frac{3}{t^2+1}, \sin\left(\frac{t}{3}\right) \right\rangle dt.$$

2. (9 pts) Determine whether the two lines given below are parallel, intersecting, or skew:

$$\begin{cases} x = -2t - 2 \\ y = t - 3 \\ z = 4 \end{cases} \qquad \begin{cases} x = 2t - 1 \\ y = -t + 1 \\ z = 3t + 1 \end{cases}$$

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3. (12 pts) Write an equation of the plane containing the point $(3, -3, 1)$ and the line given by the symmetric equations

$$\frac{x-1}{3} = y + 1 = \frac{z}{2}.$$

4. (15 pts) Write down a definite integral (you do not need to evaluate the integral) which gives the circumference of the ellipse given by the equation

$$\frac{x^2}{5} + \frac{y^2}{3} = 1.$$

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5. (9 pts) Sketch a rough graph in 3−dimensional space of each of the following:

   (a) $x^2 = y^2 + z^2$

   (b) $y^2 = x^2 + z^2 + 1$

   (c) $y^2 + z^2 = 1$

6. (10 pts) Prove (using techniques from Math 230) that if the sides of a parallelogram all have the same length, then the diagonals of that parallelogram are perpendicular.

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7. (20 pts) Consider the vector-valued function $\overrightarrow{r}(t) =< \cos t, t^3, 4\sin t >$, where $-\infty < t < \infty$.

   (a) Sketch the graph of $\overrightarrow{r}(t)$ as seen from the positive $y-$axis.

   (b) Suppose $\overrightarrow{r}(t)$ gives the position vector for a particle moving in $3-$dimensional space. Find the particle's velocity, speed, and acceleration when $t = 0$.

   (c) Sketch the graph of $\overrightarrow{r}(t)$ as seen from the usual perspective. Indicate the velocity and acceleration vectors at $t = 0$ on your graph.

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8. (16 pts) To say that two planes in $\mathbb{R}^3$ are perpendicular means heuristically that the two planes meet at right angles. For example, the $xy-$plane is perpendicular to the $xz-$plane.

   (a) Here is an incorrect definition of what it means for two planes to be perpendicular:
   Wrong definition: Two planes $\mathcal{P}$ and $\mathcal{Q}$ are said to be *perpendicular* if every vector in $\mathcal{P}$ is perpendicular to every vector in $\mathcal{Q}$.
   Explain why this definition is wrong.

   (b) Give a precise and correct mathematical definition of what it means for two planes to be perpendicular (there are many possible answers). Your answer should be a complete sentence that starts "Two planes $\mathcal{P}$ and $\mathcal{Q}$ are said to be perpendicular if ...".

   (c) Verify using the definition you gave in part (b) above that the $xy-$ and $xz-$planes are perpendicular.