Check your instructor's name and section:

Spinolo	9:00		Kornfeld	12:00	
McClendon	9:00		Spinolo	12:00	
Nadler	11:00		Burslem	12:00	
Hooper	11:00		Burslem	2:00	

Question	Possible points	Score
1	20	
2	20	
3	10	
4	10	
5	10	
6	10	
7	10	
8	20	
9	10	
10	10	
11	20	
TOTAL	150	

Instructions:
Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. This exam has 15 pages, and 11 problems. Please make sure that all pages are included. No calculators, books, notes, etc. are allowed. Good luck!

Question 1. *(20 pts) The position of a dragonfly at time t is given by*

$$\mathbf{r}(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle$$

a) Compute the velocity $\mathbf{r}'(t)$ and acceleration $\mathbf{r}''(t)$ of the dragonfly at the time $t = 0$.

b) Compute the curvature of the dragonfly's path at the time $t = 0$.

Question ?? continued.

c) Compute the speed $\|\mathbf{r}'(t)\|$ *of the dragonfly as a function of time* t.

d) Compute the distance traveled by the dragonfly between times $t = 0$ *and* $t = 3$.

Question 2. *(20 pts) An astronomer would like to set up his instruments on the highest ground less than or equal to 2 miles from his home. If the astronomer lives at* $(0, 0)$ *and the height of the countryside is given by* $f(x, y) = x^2 + y^2 - 2x$*, where should the astronomer set up his instruments?*

Question 3. *(10 pts) Evaluate the following limits, or show that they do not exist. In either case, be sure to justify your answer.*

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y^2}{x^2+y^2}$

b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4y^2}{x^6+y^6}$

Question 4. *(10 pts) Consider the function*

$$f(x, y) = x^2y - \ln(x)$$

a) Compute the linear approximation to $f(x, y)$ at the point $(1, 2)$.

b) Use your answer from part a) to estimate the value of $f(1.1, 1.9)$.

Question 5. *(10 pts) An archer shoots an arrow with an initial speed of* s_0 *at an angle* $\pi/4$ *from the ground. If gravity acts on the arrow with an acceleration of* 10*, find the inital speed* s_0 *required for her to hit a target a distance* 100 *away.*

Question 6. *(10 pts)*

Match the given parametric equation to the description of its graph:

_____ 1. $\begin{cases} x = \cos u \cosh v \\ y = \sin u \cosh v \\ z = \sinh v \end{cases}$ *(A) sphere*

_____ 2. $\begin{cases} x = \cosh u \cosh v \\ y = \sinh u \cosh v \\ z = \sinh v \end{cases}$ *(B) hyperboloid of two sheets*

_____ 3. $\begin{cases} x = v \cos u \\ y = v \sin u \\ z = v \end{cases}$ *(C) hyperboloid of one sheet*

_____ 4. $\begin{cases} x = \cos u \cos v \\ y = \sin u \cos v \\ z = \sin v \end{cases}$ *(D) cylinder*

_____ 5. $\begin{cases} x = \cos u \\ y = \sin u \\ z = v \end{cases}$ *(E) cone*

Question 7. *(10 pts)*

a) Consider the functions

$$f(x,\, y) = \cos(xy^2) \qquad g(s,\, t) = f\Big(x(s,\, t),\, y(s,\, t)\Big).$$

Suppose that

$$x(0,0) = 1 \qquad y(0,0) = 0 \qquad x_s(0,0) = 2 \qquad y_s(0,0) = 1.$$

Find the partial derivative $g_s(0,0)$.

b) Consider the functions

$$f(x,\, y) = x - y^2 \qquad x(s,\, t) = e^t \sin s \qquad y(s,\, t) = st^2$$

$$g(s,\, t) = f\Big(x(s,\, t),\, y(s,\, t)\Big).$$

Find the partial derivative $g_t(s,\, t)$.

Question 8. *(20 pts) Consider the function*

$$f(x, y, z) = x^2 + y^2 - z^2 - 1.$$

a) Determine the unit vector in the direction of the maximum rate of increase of $f(x, y, z)$ *at the point* $(1, 1, 1)$.

b) Find the maximum rate of increase of $f(x, y, z)$ *at the point* $(1, 1, 1)$.

Question ?? continued. *Let $\mathcal{S}$ be the surface defined by*

$$f(x,\,y,\,z) = x^2 + y^2 - z^2 - 1 = 0.$$

c) Find a parametric equation for the normal line to $\mathcal{S}$ at the point $(1,\,1,\,1)$.

d) Find the equation of the tangent plane to $\mathcal{S}$ at the point $(1,\,1,\,1)$.

Question 9. *(10 pts) Consider the function*

$$f(x,\, y,\, z) = x^2 + 4y^2 + 9z^2 - 25.$$

a) Give the name of the surface $\mathcal{S}$ *defined by* $f(x,\, y,\, z) = 0$.

b) Consider the function $z(x,\, y)$ *which is implicitly defined by the equation*

$$f\Big(x,\, y,\, z(x,\, y)\Big) = 0$$

near the point $(0,\, 2,\, 1)$. *Find the partial derivative* $\dfrac{\partial z}{\partial x}(0, 2)$.

Question 10. *(10 pts) Let* $\mathbf{r}(t)$ *be a parametrized curve in space. Suppose that when* $t = 3$*, the magnitude of the acceleration vector* $\mathbf{r}''(3)$ *is equal to* 5 *and the angle between the velocity vector* $\mathbf{r}'(3)$ *and the acceleration vector* $\mathbf{r}''(3)$ *is equal to* $\pi/6$*. Find the tangential component of acceleration at the time* $t = 3$*.*

Question 11. *(20 pts) Consider the following four points in space:*

$$P(1,2,3), \quad Q(2,0,5), \quad R(3,1,-1), \quad S(0,4,2).$$

a) Find parametric equations for the line passing through P and Q and the line passing through R and S.

Question ?? continued.

b) Find an equation for the plane passing through the points P and Q and parallel to the line passing through the points R and S.