Northwestern University

Name: _____

Id #:

Math 230 Final Exam

Spring Quarter 2007

Monday, June 4, 2007

Check your instructor's name and section:

McClendon	9:00		Koshkin	12:00	
McClendon	12:00		Bode	1:00	

Question	Possible points	Score
1	12	
2	10	
3	12	
4	15	
5	12	
6	15	
7	20	
8	12	
9	18	
10	12	
11	10	
12	12	
13	20	
14	20	
TOTAL	200	

Instructions:
Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. This exam has 15 pages, and 14 problems. Please make sure that all pages are included. No calculators, books, notes, etc. are allowed. Good luck!

Question 1. *(12 points) Let* $A = (1, -1, 0)$*,* $B = (2, 1, -1)$*, and* $C = (-1, 1, 2)$*.*

a) Find an equation of the plane $\mathcal{P}$ *through* A*,* B *and* C*.*

b) Find the parametric equations of the line perpendicular to the plane $\mathcal{P}$ *in part a) through* A*.*

c) Find the area of the triangle ABC*.*

Question 2. *(10 points) Let* $A = (1, 2, 3)$*, and* $\mathcal{P}$ *be the plane:* $2x - y + z = 5$

a) Find the distance from A *to the plane* $\mathcal{P}$*.*

b) Find the equation of the plane $\mathcal{Q}$ *through* A*, parallel to the plane* $\mathcal{P}$*.*

Question 3. *(12 points) Find a) the line of intersection, and b) the angle between the two planes* $3x - 6y - 2z = 10$ *and* $2x + y - 2z = 5$.

Question 4. *(15 points) Describe and sketch the parametric surface given by:*

$$\begin{cases} x &= 2\cos u \cosh v \\ y &= 2\sin u \cosh v \\ z &= 2\sinh v \end{cases}$$

Hint: Find an equation relating $x, y,$ and z.

Question 5. *(12 points) Sketch the following surfaces given in spherical coordinates.*

a) $\phi = \frac{2\pi}{3}$

b) $\rho = 4$

c) $\theta = 0$

Question 6. *(15 points) A football is kicked at an angle of 45 degrees to the horizontal with an initial speed of 96 feet per second. Assuming the ball is launched from ground level, find the position function* $\vec{\mathbf{r}}(t)$ *of the ball, and compute the amount of time the ball is in the air (hang time). The magnitude of the acceleration due to gravity is given by* 32 *ft/sec*^{2}.

Question 7. *(20 points) The path of a glider is given by:*

$$\mathbf{r}(t) = \cos(4t)\mathbf{i} + \sin(4t)\mathbf{j} + (t+2)\mathbf{k}$$

a) Find the unit tangent vector of the curve $\mathbf{r}(t)$ *at the time* $t = 0$.

b) Find the curvature, and the tangential and normal components of acceleration of the glider at time $t = 0$.

c) Find how far the glider went from time $t = 0$ *to time* $t = 20$.

Question 8. *(12 points) Determine whether the following limits exist and compute the value of the limit if does exist. Be sure to justify your answer.*

a) $\lim_{(x,y)\to(0,0)} \dfrac{3x^2 - 2y^2}{x^2 + y^2}$

b) $\lim_{(x,y)\to(0,0)} \dfrac{3x^2 - 2y^2}{\sqrt{x^2 + y^2}}$

Question 9. *(18 points)*

a) Find $\dfrac{\partial z}{\partial x}$ *and* $\dfrac{\partial z}{\partial y}$ *at the point* $(1, 0, 1)$*, where* $3e^{xyz} - 4xz^2 + 2x\cos y = 1$.

b) Find an equation of the plane tangent to $3e^{xyz} - 4xz^2 + 2x\cos y = 1$ *at the point* $(1, 0, 1)$.

Question 10. *(12 points) Find the linear approximation* $L(x, y)$ *of* $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ *at the point* $(3, 2)$*, and use it to approximate the function value at the point* $(3.1, 1.9)$.

Question 11. *(10 points) Let* $f(x, y) = \ln(y - x^2)$.

a) Find and sketch the domain of f.

b) Sketch the level curve corresponding to $f(x, y) = 0$.

c) Sketch the gradient at the point $(1, 2)$. *Indicate correctly its position with respect to the level curve.*

Question 12. *(12 points) Suppose the elevation on a hill is given by* $f(x, y) = 200 - y^2 - 4x^2$.

a) If you put a ball on the hill at $(1, 2)$*, in which direction will the ball roll?*

b) If a level road is to be built at elevation 100, find the shape of the road. You may describe the shape in words, or sketch the level curve.

c) Find the rate of change of the elevation if a climber is walking from the point $(1, 2)$ *towards the point* $(4, 6)$.

Question 13. *(20 points) Find and classify all critical points of*

$$f(x,y) = \frac{1}{2}x^2 + 3y^3 + 9y^2 - 3xy + 9y - 9x$$

Question 14. *(20 points) Find the maximum and minimum values of the function* $f(x, y) = 2x + 4y$ *on the circle* $x^2 + y^2 = 4$.