1. Let A, B and $\mathbf{x}$ be:

$$A = \begin{pmatrix} 1 & 4 & 2 & -5 \\ 0 & -3 & -2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & -1 \\ 2 & -3 \end{pmatrix} \qquad \mathbf{x} = (-1, -3, 7, 0).$$

Compute each of the following expressions (unless the expression is nonsense, in which case you should indicate that it is nonsense):

(a) the size of A
(b) BA
(c) $B^T BA$
(d) $B^T BAB$
(e) $3A\mathbf{x}$
(f) $a_{12}\mathbf{x} + \begin{pmatrix} 0 \\ 2 \\ -5 \\ 1 \end{pmatrix}$

2. Sketch a picture of the following subsets of $\mathbb{R}^3$:

(a) $Span((0, -1, 0))$
(b) $Span((0, -1, 0)) + (0, 0, 2)$
(c) $Span((1, 0, 0), (0, 0, 2))$
(d) $Span((1, 0, 0), (0, 0, 2)) + (0, 0, 3)$

3. (a) Find the parametric equations of the line in $\mathbb{R}^3$ passing through the points $(2, -3, 5)$ and $(1, 0, 4)$.
(b) Find the parametric equations of the plane in $\mathbb{R}^3$ passing through the points $(1, 4, -3)$, $(2, -3, 0)$ and $(-1, 5, -1)$.
(c) Does the line described in part (a) of this problem intersect the plane described in part (b), or is the line parallel to the plane? Explain.

4. Fill in each blank with the word ALWAYS, SOMETIMES or NEVER, so that the sentence is as accurate as possible.

(a) A set of six linearly independent vectors in $\mathbb{R}^6$ ________________ spans $\mathbb{R}^6$.
(b) A set of three linearly independent vectors in $\mathbb{R}^6$ ________________ spans $\mathbb{R}^6$.
(c) If W is a subspace of V, then the dimension of W is ________________ greater than the dimension of V.
(d) A square matrix ________________ has the same size as its transpose.
(e) A plane containing the origin is ________________ a subspace of $\mathbb{R}^3$.
(f) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors, then $\{\mathbf{v}_1, \mathbf{v}_2\}$ is ________________ a linearly independent set.
(g) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is ________________ a linearly independent set.
(h) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors and $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a linearly independent set of vectors, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is ________________ a linearly independent set.
(i) If $\mathcal{B}$ and $\mathcal{B}'$ are two different bases of the same finite-dimensional vector space V, then the union $\mathcal{B} \cup \mathcal{B}'$ is ________________ a basis of V.
(j) If $\mathcal{B}$ and $\mathcal{B}'$ are two different bases of the same finite-dimensional vector space V, then the intersection $\mathcal{B} \cap \mathcal{B}'$ is ________________ a basis of V.

5. In each part of this problem, you are given a vector space V and a subset W of V. For each problem:
   - Determine, with proof, whether or not W is a subspace of V.
   - If W is a subspace of V, find a basis of W and compute $\dim W$.

   (a) $V = \mathbb{R}^4$; $W = \{(w, x, y, z) \in \mathbb{R}^4 : w - 2x + 5y - z = 0\}$.

   (b) $V = C(\mathbb{R}, \mathbb{R})$; $W = \{f \in C(\mathbb{R}, \mathbb{R}) : \int_0^1 f(x)\, dx = 2\}$.

   (c) $V = M_2(\mathbb{R})$; $W = \{A \in M_2(\mathbb{R}) : tr(A) = 0\}$.

6. In each part of this problem, you are given a vector space V and a set $\mathcal{S}$ of vectors in V. Determine whether $\mathcal{S}$ is linearly dependent or linearly independent. Your answers should be appropriately justified, either by appealing to some theory or by doing some algebra.

   (a) $V = \mathbb{R}^3$; $\mathcal{S} = \{(1, 2, 3), (4, -2, 7), (8, -1, 4), (5, 0, 0), (5, 2, -6)\}$

   (b) $V = \mathbb{R}^6$; $\mathcal{S} = \{(1, 0, 2, -3, 1, 0), (2, 1, 5, -4, 0, 0)\}$

   (c) $V = M_{34}(\mathbb{R})$; $\mathcal{S} = \{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \}$

   (d) $V = \mathbb{R}^3$; $\mathcal{S} = \{(1, 2, 3), (1, -1, 1), (8, 7, 18)\}$