1. Let $\mathbf{v} = (2, -3, 5)$ and $\mathbf{w} = (1, 4, -2)$. Compute the following quantities:
   (a) $\mathbf{v} \cdot \mathbf{w}$
   (b) the norm of $\mathbf{v}$
   (c) $\mathbf{v} \times \mathbf{w}$
   (d) a vector which is orthogonal to both $\mathbf{v}$ and $\mathbf{w}$
   (e) the projection of $\mathbf{v}$ onto $\mathbf{w}$
   (f) the normal equation of the plane containing the points $\mathbf{v}$, $\mathbf{w}$ and $(5, 2, 0)$

2. In each part of this problem, you are given a function $T : V_1 \to V_2$, where V_1 and V_2 are vector spaces. Determine, with proof, whether or not T is a linear transformation.
   (a) $T : \mathbb{R}^2 \to \mathbb{R}^4$ defined by $T(x, y) = (x + y, x - y, x + y, y)$.
   (b) $T : \mathbb{R}^2 \to \mathbb{R}$ where $T(\mathbf{v}) = dist(\mathbf{v}, \mathbf{0})$.

3. Find the standard matrix of each of these linear transformations (you may assume that each transformation is linear, without proof):
   (a) $T : \mathbb{R}^4 \to \mathbb{R}^2$, where $T(1, 0, 0, 0) = (2, 1)$; $T(0, 1, 0, 0) = (-1, 3)$; $T(0, 0, 1, 0) = (4, 0)$ and $T(0, 0, 0, 1) = (-3, 7)$.
   (b) $S : \mathbb{R}^2 \to \mathbb{R}^3$, where $S(1, 5) = (3, -4, 0)$ and $S(-2, 1) = (0, 3, 1)$.
   (c) $S \circ T$, where S and T are as described in parts (a) and (b) of this problem.

4. In this problem, you are given these linear transformations (you do not need to prove that these transformations are linear):
   (a) $T_1 : \mathbb{R}^4 \to \mathbb{R}^2$ defined by $T_1(w, x, y, z) = (w + 2x, 2w + 4x)$
   (b) $T_2 : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T_1(x, y, z) = (y, z + 3y, z - y)$
   (c) $T_3 : \mathbb{R} \to \mathbb{R}^3$ defined by $T_1(x) = (4x, -x, 3x)$

   For each of these transformations,

   - if the kernel has a basis, give a basis of the kernel (otherwise, write "NO BASIS").
   - if the image has a basis, give a basis of the image (otherwise, write "NO BASIS").
   - Is the transformation injective?
   - Is the transformation surjective?
   - Is the transformation invertible?

5. In this problem, assume T_1, T_2 and T_3 are linear transformations whose standard matrices are A_1, A_2 and A_3 respectively. Assuming the information that is given in this chart, fill out the rest of the chart (no justification is required):

	T_1	T_2	T_3
domain of T_j	$\mathbb{R}^4$		$\mathbb{R}^3$
codomain of T_j (space of outputs)	$\mathbb{R}^5$		$\mathbb{R}^7$
size of A_j		7×7	
rank of T_j		7	
$\dim C(A_j)$			
$\dim R(A_j)$			
$\dim N(A_j)$	2		
$\dim N(A_j^T)$			
Does $T(\mathbf{x}) = \mathbf{b}$ have a solution?	No		
How many solutions does $T(\mathbf{x}) = \mathbf{b}$ have?	0		1

6. In each part of this problem, you are given a vector space V and a subspace W of V. Find a basis of the orthogonal complement $W^\perp$ (your answers should be appropriately justified).

(a) $V = \mathbb{R}^4$; $W = \{(w, x, y, z) \in \mathbb{R}^4 : x + 3y = 0$ and $w - 7z = 0\}$.

(b) $V = \mathbb{R}^3$; $W = Span((2, -3, 5))$.