1. Suppose that A is a 3×3 matrix whose determinant is 5.
   (a) Suppose that a matrix is made from A by switching the first two rows of A, then switching the first two columns. Find the determinant of this new matrix.
   (b) Suppose that a matrix is made from A by adding the first column to the second. Find the determinant of this new matrix.
   (c) Is A invertible? If so, give the determinant of A^{-1}. If not, explain why not.
   (d) Find the determinant of $3A$.
   (e) If A has eigenvalue 2 (repeated twice), what is the third eigenvalue of A?
   (f) Find the determinant of AB, where

$$B = \begin{pmatrix} 4 & -3 & -5 \\ 2 & -1 & -2 \\ 1 & 7 & -3 \end{pmatrix}.$$

2. Suppose that a collection of data points (x, y, z) is supposed to fit an equation of the form

$$axy + bxz + cyz = 1,$$

where a, b and c are constants. Suppose also that the data points collected are:

$$(1,2,5) \quad (1,7,10) \quad (2,3,8) \quad (3,1,2) \quad (3,5,14) \quad (4,2,9)$$

   (a) Set up a matrix equation $A\mathbf{x} = \mathbf{b}$ which can be used to find the equation. In particular, what are A, $\mathbf{x}$ and $\mathbf{b}$?
   (b) Compute (using least-squares) the model which best fits the data.

3. Solve the following system of equations. In this problem, you must use row reductions and show all your steps.

$$\begin{cases} -3x & +y & +z & = -2 \\ 5x & -2y & & = -4 \\ x & -4y & -3z & = -17 \end{cases}$$

4. Solve the following systems of equations. Write your answers in the appropriate form.

(a) $$\begin{cases} w & +x & -3y & +z & = -3 \\ 3w & -x & & -2z & = 7 \\ -2w & +2x & -3y & +3z & = -10 \end{cases}$$

(b) $$\begin{cases} 4x & -3y & +7z & = 6 \\ -6x & +5y & -4z & = -8 \\ -6x & +7y & +22z & = 5 \end{cases}$$

(c) $$\begin{cases} 2x & +5y & -z & = 8 \\ -x & +4y & -3z & = 4 \\ 5x & -2y & +7z & = 3 \end{cases}$$

5. Suppose the following matrices are row equivalent:

$$(A\,|\,\mathbf{b}) = \left(\begin{array}{ccccc|c} 2 & 1 & -2 & 1 & 4 & 2 \\ 5 & -1 & 5 & -4 & 7 & 0 \\ 0 & 7 & -22 & 13 & 6 & 10 \\ 1 & 4 & 3 & 0 & 1 & -3 \\ 1 & -17 & -6 & -4 & 3 & 12 \\ 8 & 4 & 7 & -3 & 11 & 9 \end{array}\right) \overset{\text{row ops}}{\longrightarrow} \left(\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{-17}{105} & 0 & \frac{268}{15} \\ 0 & 1 & 0 & \frac{41}{105} & 0 & \frac{-4}{15} \\ 0 & 0 & 1 & \frac{-7}{15} & 0 & \frac{-49}{15} \\ 0 & 0 & 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

(a) Find the rank of A.

(b) Find a basis of the column space of A.

(c) Find a basis of the row space of A.

(d) Find a basis of the null space of A.

(e) Find the dimension of the left nullspace of A.

(f) Classify the following statements as true or false:

i. $A\mathbf{x} = \mathbf{b}$ has at least one solution.

ii. $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

iii. The first, second and fourth columns of A are linearly independent.

iv. The first, second and fifth columns of A are linearly independent.

v. The first five rows of A are linearly independent.

6. Find the eigenvalues and eigenvectors of the following matrix (work must be shown). Be sure to indicate which eigenvector goes with which eigenvalue.

$$\begin{pmatrix} 6 & 6 & 6 \\ -3 & 3 & 9 \\ 5 & -1 & -7 \end{pmatrix}$$

7. Suppose that the sequence of numbers $\{x_n\}$ is defined recursively by setting $x_0 = 1$, $x_1 = 4$ and by defining, for $n \geq 2$, $x_n = 2x_{n-1} + 3x_{n-2}$. Find the exact value of x_{2016}, the 2016^{th} number in this sequence.