1. Throughout this problem, let $A = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$ and let $\mathbf{x} = (4, -1)$. Compute each of the following expressions (unless the expression is nonsense, in which case you should indicate that it is nonsense):

   (a) the trace of A

   (b) $A \begin{pmatrix} 1 & -3 & 4 \\ 2 & 0 & -3 \end{pmatrix}$

   (c) $3A - A^T + 4I$

   (d) $3a_{12}\mathbf{x}$

   (e) $\mathbf{x}^T A^2 \mathbf{x}$

   (f) $\dim(Span(\mathbf{x}))$

2. Let $\mathbf{v} = (-2, -1)$. In each part of this problem, you are given a set S. If the set S makes sense, sketch a picture of S; however, if S is nonsense, indicate that it is nonsense.

   (a) $S = Span(\mathbf{v})$

   (b) $S = (-1, 2) + Span(\mathbf{v})$

   (c) $S = Span(\mathbf{v}, \mathbf{v} + (0, 1))$

   (d) $S = Span(\mathbf{v}, (0, 0, 1))$

3. In each part of this problem, you are given a vector space V and a set $\mathcal{S}$ of vectors in V. Determine whether $\mathcal{S}$ is linearly dependent or linearly independent. Your answers should be appropriately justified, either by appealing to some theory or by doing some algebra.

   (a) $V = \mathbb{R}^3$; $\mathcal{S} = \{(1, 2, -3), (5, -1, 4)\}$

   (b) $V = \mathbb{R}^5$; $\mathcal{S} = \{(3, 0, 1, 0, 0), (2, 4, -3, 0, 1), (-6, 0, -2, 0, 0), (4, 1, 3, -2, 1)\}$

   (c) $V = M_2(\mathbb{R})$; $\mathcal{S} = \{\begin{pmatrix} 1 & 2 \\ 5 & 7 \end{pmatrix}, \begin{pmatrix} 3 & 8 \\ 15 & 11 \end{pmatrix}, \begin{pmatrix} 2 & 9 \\ 6 & 19 \end{pmatrix}, \begin{pmatrix} 7 & 7 \\ 7 & 7 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 15 & 32 \end{pmatrix}\}$

   (d) $V = \mathbb{R}^3$; $\mathcal{S} = \{(a, b, c), (0, d, e), (0, 0, f)\}$
   Note: in this problem, letters a, b, c, d, e and f represent unknown nonzero constants.

4. (a) Suppose you know W_1 is a subspace of $\mathbb{R}^6$.

      i. What is the largest possible dimension of W_1?

      ii. What is the smallest possible dimension of W_1?

      iii. Is it possible that W_1 is spanned by four vectors?

      iv. Is it possible that W_1 contains a set of four linearly independent vectors?

   (b) Suppose you know W_2 is a subspace of $\mathbb{R}^6$ containing three linearly independent vectors.

      i. What is the largest possible dimension of W_2?

      ii. What is the smallest possible dimension of W_2?

      iii. Is it possible that W_2 is spanned by four vectors?

      iv. Is it possible that W_2 contains a set of four linearly independent vectors?

(c) Suppose you know W_3 is a subspace of $\mathbb{R}^6$ spanned by three vectors.

i. What is the largest possible dimension of W_3?

ii. What is the smallest possible dimension of W_3?

iii. Is it possible that W_3 is spanned by four vectors?

iv. Is it possible that W_3 contains a set of four linearly independent vectors?

(d) Suppose you know W_4 is a subspace of $\mathbb{R}^6$ spanned by three linearly independent vectors.

i. What is the largest possible dimension of W_4?

ii. What is the smallest possible dimension of W_4?

iii. Is it possible that W_4 is spanned by four vectors?

iv. Is it possible that W_4 contains a set of four linearly independent vectors?

5. Let $V = \mathbb{R}^3$ and suppose that $W \subseteq V$ is the plane containing the three points $(1, 1, -5)$, $(2, -1, 2)$ and $(-2, 1, -2)$.

(a) Write parametric equations for the plane W.

(b) Write parametric equations of the plane passing through the point $(-5, 3, -2)$ that is parallel to W.

(c) Is W a subspace of V? Prove or disprove your answer.

6. Let $V = \mathbb{R}^4$ and suppose that

$$W = \{(w, x, y, z) \in \mathbb{R}^4 : x = 2w, z = w + x + y\}.$$

Prove W is a subspace of V; find the dimension of W; and give a basis of W.

7. Let $B = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$ and suppose that

$$W = \{A \in M_2(\mathbb{R}) : AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\}.$$

Determine, with proof, whether or not W is a subspace of $M_2(\mathbb{R})$.

8. Let $V = C(\mathbb{R}, \mathbb{R})$ and suppose that W is the set of functions f in V that are equal to their second derivative.

(a) Find a nonzero element of W.

(b) Determine, with proof, whether or not W is a subspace of V.