1. The four parts of this problem are not related to one another.

   (a) Let $A = \begin{pmatrix} 8 & -3 \\ 1 & 2 \end{pmatrix}$. Compute A^{-1} (if it exists).

   (b) Let $B = \begin{pmatrix} 3 & -2 \\ 6 & x \end{pmatrix}$. For what value(s) of x (if any) is B not invertible?

   (c) Find a vector in $\mathbb{R}^6$ that has length 1 and is parallel to $\mathbf{a} = (3, -5, 4, 4, 5, -3)$.

   (d) Find a normal equation of the plane containing the three points $(5, -3, 0)$, $(2, 1, -3)$ and $(-3, 7, 1)$.

2. Throughout this problem, let $\mathbf{v} = (1, -2, 4)$, $\mathbf{w} = (2, -2, 1)$ and $\mathbf{x} = (3, 1, 1)$.

   (a) Compute $\mathbf{v} \cdot \mathbf{w}$.

   (b) Compute the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$.

   (c) Compute $\mathbf{v} \times \mathbf{w}$.

   (d) Compute the projection of $\mathbf{v}$ onto $\mathbf{x}$.

   (e) Compute the projection of $\mathbf{x}$ onto the plane spanned by $\mathbf{v}$ and $\mathbf{w}$.
   Hint: The number 105 should appear somewhere in your answer.

   (f) Let $W = Span(\mathbf{w})$. Find a basis of $W^\perp$.

3. In each part of this problem, you are given a function $T : V_1 \to V_2$, where V_1 and V_2 are vector spaces. Determine, with proof, whether or not T is a linear transformation.

   (a) $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (3x - y + 5z, z - x)$.

   (b) $T : M_3(\mathbb{R}) \to \mathbb{R}$ defined by $T(A) = tr(A)$.

4. The parts of this problem are not related to each other.

   (a) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation which first rotates the plane counterclockwise by $\frac{\pi}{3}$, then reflects points across the y-axis, then rotates the plane clockwise by $\frac{\pi}{3}$. Compute $T(4, 6)$.

   (b) Suppose $S : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation with $S(1, 0, 0) = (3, -4)$; $S(1, 2, 0) = (-1, -4)$; and $S(3, 0, 1) = (0, 7)$. Compute $S(-2, 3, -2)$.

5. In this problem, assume T_1, T_2 and T_3 are as follows (you may assume that each of these are linear transformations, without proof):

   (a) $T_1 : \mathcal{P}^2 \to \mathbb{R}^2$ defined by $T_1(f) = (f(0), f'(0))$;
   (*Note:* $\mathcal{P}^2$ is the space of polynomials of degree ≤ 2)

   (b) $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T_2(\mathbf{x}) = \pi_{(1,2)}(\mathbf{x})$;

   (c) $T_3 : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T_3(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{pmatrix} -2 & 3 \\ -5 & 7 \end{pmatrix}$.

   For each of these transformations,

   - find the dimension of $ker(T_j)$;
   - find the dimension of $im(T_j)$;
   - classify T_j as one of these four types:
     - injective but not surjective;
     - surjective but not injective;
     - bijective;
     - neither injective nor surjective;

- determine how many solutions there are to the equation $T_j(\mathbf{x}) = (1,1)$;
  (*Note:* for T_1, "$\mathbf{x}$" means "f", so the equation is really $T_1(f) = (1,1)$.)
- describe the solution set of $T_j(\mathbf{x}) = (1,1)$.
  (*Note:* for T_1, "$\mathbf{x}$" means "f", so the equation is really $T_1(f) = (1,1)$.)

6. (a) Suppose $T_1(\mathbf{x}) = A_1\mathbf{x}$, where A is a 7×3 matrix.
   i. What vector space is the domain of T_1?
   ii. What vector space is the codomain (i.e. space of outputs) of T_1?
   iii. Is it possible for T_1 to be injective?
   If the answer to this question is "No", ignore parts (iv) and (v).
   iv. If T_1 is injective, do you know how many linearly independent rows that A_1 has? If so, how many?
   v. If T_1 is injective, do you know what the dimension of $im(T_1)$ is? If so, what is that dimension?

   (b) Suppose $T_2(\mathbf{x}) = A_2\mathbf{x}$, where A_2 is a 3×7 matrix.
   i. Is it possible for T_2 to be injective?
   If the answer to this question is "No", ignore parts (ii) and (iii).
   ii. If T_2 is injective, do you know how many linearly independent rows that A_2 has? If so, how many?
   iii. If T_2 is injective, do you know what the dimension of $im(T_2)$ is? If so, what is that dimension?

   (c) Suppose $T_3(\mathbf{x}) = A_3\mathbf{x}$, where A_3 is a 7×3 matrix.
   i. Is it possible for T_3 to be surjective?
   If the answer to this question is "No", ignore parts (ii) and (iii).
   ii. If T_3 is surjective, do you know how many linearly independent rows that A_3 has? If so, how many?
   iii. If T_3 is surjective, do you know what the dimension of $ker(T_3)$ is? If so, what is that dimension?

   (d) Suppose $T_4(\mathbf{x}) = A_4\mathbf{x}$, where A_4 is a 3×7 matrix.
   i. Is it possible for T_4 to be surjective?
   If the answer to this question is "No", ignore parts (ii) and (iii).
   ii. If T_4 is surjective, do you know how many linearly independent rows that A_4 has? If so, how many?
   iii. If T_4 is surjective, do you know what the dimension of $ker(T_4)$ is? If so, what is that dimension?