1. Suppose A is a 4×4 matrix whose trace is 2 and whose determinant is 4. Suppose further that A has two different real eigenvalues, both of which are of multiplicity 2.

   (a) Suppose that matrix B is formed from A by multiplying the second row by 3, then swapping the first and third columns. What is the determinant of B?

   (b) What is $\det(-3A)$?

   (c) How many solutions does the equation $A\mathbf{x} = (2, 3, -5, 1)$ have?

   (d) What are the eigenvalues of A?

2. Parts (a) and (b) of this question are related, but (c) has nothing to do with (a) or (b).

   (a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 & 2 \\ 6 & -3 \end{pmatrix}$. Be sure to tell me which eigenvector goes with which eigenvalue. (To get credit, you must show your work.)

   (b) Compute and simplify A^{2019}, where A is the matrix given in part (a) of this problem. (To get credit, you must show enough work so that I can tell how your answer comes from your work in part (a).)

   (c) Find all functions $y(t)$ which satisfy this differential equation:

$$y''(t) + 6y'(t) - 16y(t) = 72e^{-2t}$$

3. Compute the determinant of each matrix. (You may (and should) use technology to check your answers, but to get credit, I need to see appropriate work.)

   (a) $\begin{pmatrix} 2 & -1 & -2 \\ -3 & 7 & 4 \\ 1 & 5 & 3 \end{pmatrix}$

   (b) $\begin{pmatrix} 3 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 5 & 0 \\ 0 & 0 & 4 & 0 & -1 \\ 0 & 0 & -2 & 0 & -3 \\ 0 & 8 & 0 & 0 & 0 \end{pmatrix}$

4. Consider the following system of equations:

$$A\mathbf{x} = \mathbf{b} \quad \Leftrightarrow \quad \left\{ \begin{array}{rrrrrl} -2v & +w & -2x & +4y & +5z & = 2 \\ v & +2w & & -y & +3z & = 4 \\ 7v & +4w & 4x & -11y & -z & = 8 \end{array} \right.$$

   (a) Solve the system using row reductions. You must show all your steps.

   (b) Find a basis of the null space of A.

   (c) Find a basis of the column space of A.

   (d) Find a basis of the row space of A.

5. For each given system of equations:
   - If the system has a solution, find its solution set.
   - If the system has no solution, say so, and find the least-squares solution of the system.

(a) $\begin{cases} 2x - y + 3z = 4 \\ -5x + 2z = -1 \\ x - 3y + 11z = 2 \\ 4x + y = 5 \end{cases}$

(b) $\begin{cases} 3x - 7y + 2z = -26 \\ 2x - y + 4z = -1 \\ x - 3z = -9 \end{cases}$

(c) $\begin{cases} x - 3y = -8 \\ -4x + 2y + z = 7 \\ -3x + 2z = 7 \\ 2x + 7y - 4z = 3 \end{cases}$

6. Each column of this chart corresponds to a matrix A whose entries are real numbers. Use the given information to fill in all the blank spaces:

	(a)	(b)	(c)
Size of A		8×6	5×7
Number of equations in system $A\mathbf{x} = \mathbf{b}$	3		
Number of components in each solution of the system $A\mathbf{x} = \mathbf{b}$			
Domain of T, where $T(\mathbf{x}) = A\mathbf{x}$			
$\dim C(A)$			
$\dim N(A)$			4
Number of pivots in an echelon form of A		6	
Number of solutions of the system $A\mathbf{x} = \mathbf{0}$			
Possible number of solutions of the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b}$ is arbitrary	always 1 solution		
Is A invertible?			