1. Let A be the matrix
$$\begin{pmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \\ -4 & 1 & 0 \\ 0 & 2 & 0 \end{pmatrix}$$
and let $\mathbf{x}$ be the vector $(-2, 3, 2)$. Compute each of the following expressions (unless the expression is nonsense, in which case you should indicate that it is nonsense):
   (a) $A\mathbf{x}$
   (b) $a_{23}\mathbf{x} + a_{12}(1, -1, 3)$
   (c) $(\mathbf{x}^T\mathbf{x})A$
   (d) $\mathbf{x}^T(\mathbf{x}A)$
   (e) $\begin{pmatrix} 2 & 0 & -2 & 1 \\ 0 & 0 & 1 & 5 \end{pmatrix} A$

2. (a) Find the parametric equations of the line in $\mathbb{R}^4$ passing through the points $(1, 3, 5, 0)$ and $(5, -1, 4, 1)$.
   (b) Does the line described in part (a) of this problem pass through the point $(13, -9, 2, 2)$? Explain your answer.

3. Sketch a picture of the following subsets of $\mathbb{R}^2$:
   (a) $Span((1, -3)) + (5, -1)$
   (b) $Span((2, 3), (4, 3))$

4. In this question, you are given various subsets of $\mathbb{R}^3$. Determine whether each given subset is a point, line, plane, or all of $\mathbb{R}^3$:

(a)	POINT	LINE	PLANE	ALL OF $\mathbb{R}^3$	$Span((3, -2, 5))$
(b)	POINT	LINE	PLANE	ALL OF $\mathbb{R}^3$	$(2, -1, 5)$
(c)	POINT	LINE	PLANE	ALL OF $\mathbb{R}^3$	$Span((1, 0, 0), (0, 0, 1), (3, 0, -2))$
(d)	POINT	LINE	PLANE	ALL OF $\mathbb{R}^3$	$Span((2, 1, -3)) + (4, 0, 2)$
(e)	POINT	LINE	PLANE	ALL OF $\mathbb{R}^3$	$Span((4, 0, 0), (0, 2, 0), (0, 0, -3))$
(f)	POINT	LINE	PLANE	ALL OF $\mathbb{R}^3$	$\{(x, y, z) : x = 2t, y = 4t, z = 12t\}$
(g)	POINT	LINE	PLANE	ALL OF $\mathbb{R}^3$	$Span((2, 1, -1), (4, 2, -2), (-8, -4, 4))$
(h)	POINT	LINE	PLANE	ALL OF $\mathbb{R}^3$	$Span((1, 0, 0), (0, 1, 0)) + (0, 0, 1)$

5. Classify the following statements as true or false.

   (a) TRUE FALSE The set of points (x, y) lying on the line $y = mx + b$ is a subspace of $\mathbb{R}^2$.

   (b) TRUE FALSE The following set of vectors in $\mathbb{R}^4$ is linearly independent: $\{(1, 2, -1, 5), (2, -1, 6, -9), (0, 4, -1, 3), (-1, 3, 6, 8), (0, 2, -1, 6)\}$

   (c) TRUE FALSE The following set of vectors in $C(\mathbb{R}, \mathbb{R})$ is linearly independent: $\{\sin x, \cos x\}$

   (d) TRUE FALSE There is a basis of $\mathbb{R}^3$ consisting of four vectors.

   (e) TRUE FALSE The $xz-$plane (i.e. the set of points whose $y-$coordinate is zero) is a subspace of $\mathbb{R}^3$.

   (f) TRUE FALSE Any five linearly independent vectors in $\mathbb{R}^5$ must also span all of $\mathbb{R}^5$.

   (g) TRUE FALSE If W_1 and W_2 are subspaces of the same vector space V, then the union $W_1 \cup W_2$ must also be a subspace of V.

   (h) TRUE FALSE If W_1 and W_2 are subspaces of the same vector space V, then the intersection $W_1 \cap W_2$ must also be a subspace of V.

6. (a) Let $V = \mathbb{R}^3$ and let $W = \{(x, y, z) \in \mathbb{R}^3 : xyz = 0\}$. Determine, with proof, whether or not W is a subspace of V.

   (b) Let $V = \mathbb{R}^5$ and let $W = \{(x_1, x_2, x_3, x_4, x_5) : x_1 = x_3 \text{ and } x_4 = 2x_2 - x_5\}$. Determine, with proof, whether or not W is a subspace of V.

7. Let $V = M_3(\mathbb{R})$ and let $W = \{A \in M_3(\mathbb{R}) : A = A^T\}$. W is a subspace of V (you do not need to prove that W is a subspace).

   (a) Find a basis of W. (You do not need to justify that your answer is a basis.)

   (b) Give the dimension of W.

8. Let $V = \mathbb{R}^3$ and let $W = \{(x, y, z) : 3x + 5y - z = 0\}$. W is a subspace of V (you do not need to prove that W is a subspace).

   (a) Find a basis of W; completely justify that your answer is in fact a basis of W.

   (b) Give the dimension of W.

   (c) What kind of geometric object is W?