1. Let $\mathbf{v} = (1, 2, -1, 1, 1, -1)$ and $\mathbf{w} = (2, 3, -1, 0, 0, 2)$. Compute:

   (a) $\mathbf{v} \cdot \mathbf{w}$
   (b) a vector of length 4 which is parallel to $\mathbf{v}$
   (c) the projection of $\mathbf{w}$ onto $\mathbf{v}$
   (d) the distance from $\mathbf{v}$ to $\mathbf{w}$
   (e) the norm of $\mathbf{v} + \mathbf{w}$

2. Given each vector space V and each subspace W of V, find the dimension of $W^{\perp}$ and a basis of $W^{\perp}$.

   (a) $V = \mathbb{R}^3$; $W = Span(1, -5, 2)$
   (b) $V = \mathbb{R}^6$, $W = Span((1, 0, 0, 0, 0, 0), (2, 1, 0, 0, 0, 0), (-3, 4, 5, 0, 0, 0))$
   (c) $V = \mathbb{R}^4$; $W = Span((2, 1, -3, 0), (4, 2, -6, 0), (1, 1, 1, 1))$

3. Circle the letter of the correct answer to each question.

   (a) If A is a matrix with 9 columns and 5 rows, then $C(A)$ is...
      A. ... a subspace of $\mathbb{R}^9$
      B. ... a subspace of $\mathbb{R}^5$
      C. ... a subspace of $\mathbb{R}$
      D. ... not a subspace

   (b) If $T : \mathbb{R}^8 \to V_2$ is a surjective linear transformation, then $\dim(V_2)$...
      A. ... must be at least 8
      B. ... must be at most 8
      C. ... must be equal to 8
      D. ... could be anything

   (c) If $T : \mathbb{R}^4 \to V_2$ is an injective linear transformation, then $\dim(V_2)$...
      A. ... must be at least 4
      B. ... must be at most 4
      C. ... must be equal to 4
      D. ... could be anything

   (d) If A is a 11×14 matrix with 8 linearly independent columns, then $N(A)$ ...
      A. ... is a 3-dimensional subspace of $\mathbb{R}^{11}$
      B. ... is a 3-dimensional subspace of $\mathbb{R}^{14}$
      C. ... is a 6-dimensional subspace of $\mathbb{R}^{11}$
      D. ... is a 6-dimensional subspace of $\mathbb{R}^{14}$

(e) If the standard matrix of T is a 6×8 matrix with 6 linearly independent columns, then the equation $T(\mathbf{x}) = \mathbf{b}$ ...

A. ... either has no solution or infinitely many solutions, depending on what $\mathbf{b}$ is
B. ... always has one solution
C. ... always has no solution
D. ... always has infinitely many solutions

(f) If the standard matrix of T is a 7×7 matrix with 7 linearly independent rows, then the equation $T(\mathbf{x}) = \mathbf{b}$ ...

A. ... either has no solution or infinitely many solutions, depending on what $\mathbf{b}$ is
B. ... always has one solution
C. ... always has no solution
D. ... always has infinitely many solutions

4. (a) Let W be the subspace of $\mathbb{R}^4$ spanned by $(2,0,0,0)$ and $(2,-1,0,-2)$. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be defined by setting $T(\mathbf{v}) = \mathbf{v}^W$, the projection of $\mathbf{v}$ onto W. Find the standard matrix of T.

   (b) Use your answer to part (a) to compute the projection of $(2,-5,6,1)$ onto W.

5. Let $\mathbf{v}$ and $\mathbf{w}$ be nonparallel vectors in some real vector space V. Prove that if $\mathbf{v}$ and $\mathbf{w}$ have the same length, then the angle between $\mathbf{v}$ and $\mathbf{v} + \mathbf{w}$ is the same as the angle between $\mathbf{w}$ and $\mathbf{v} + \mathbf{w}$.

6. (a) Find the normal equation of the plane in $\mathbb{R}^3$ which contains the point $(2,-1,4)$ and the line whose parametric equations are

$$\begin{cases} x = 1 - 3t \\ y = 1 + 7t \\ z = -3 - t \end{cases}$$

   (b) Write the normal equation of any plane in $\mathbb{R}^3$ which is parallel to the plane you found in part (a). (Two planes in $\mathbb{R}^3$ are called *parallel* if they don't intersect.)

7. In each part of this question you are given a function $T : V_1 \to V_2$, where V_1 and V_2 are real vector spaces. Determine, with proof, whether or not T is a linear transformation:

   (a) $V_1 = \mathbb{R}^4$; $V_2 = \mathbb{R}^2$; $T(x_1, x_2, x_3, x_4) = (x_2 - x_1, x_1 + x_4)$.

   (b) $V_1 = M_{33}(\mathbb{R})$; $V_2 = M_{23}(\mathbb{R})$; $T(A) = BA$ where B is the matrix $B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \end{pmatrix}$.

8. Here are three linear transformations (you do not need to prove that these transformations are linear).

   (a) $R : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $R(x, y) = (x + y, x - 2y)$

   (b) $S : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $S(x, y, z) = (x - 2y + z, x + y + z, -3x + 6y - 3z, 0)$

   (c) $T : \mathbb{R}^5 \to \mathbb{R}$ defined by $T(\mathbf{x}) = \mathbf{v} \cdot \mathbf{x}$ where $\mathbf{v} = (1, 2, 3, 4, 5)$

   For each of these transformations,

   - if the kernel has a basis, write a basis of the kernel in the first row of the chart below (otherwise, write "NO BASIS").
   - if the image has a basis, write a basis of the image in the second row of the chart below (otherwise, write "NO BASIS").
   - Is the transformation injective? Answer "YES" or "NO" in the third row of the chart below.
   - Is the transformation surjective? Answer "YES" or "NO" in the fourth row of the chart below.
   - Is the transformation bijective? Answer "YES" or "NO" in the fifth row of the chart below.

	R	S	T
basis of kernel			
basis of image			
injective?			
surjective?			
bijective?			