1. (a) Find the determinant of the following matrix:

$$\begin{pmatrix} 2 & -3 & 1 \\ -4 & -1 & -2 \\ 5 & 1 & 2 \end{pmatrix}$$

   (b) Find the value of k so that the following matrix is not invertible:

$$\begin{pmatrix} 1 & 3 & -2 \\ 0 & 5 & -1 \\ 3 & 1 & k \end{pmatrix}$$

2. Suppose that a collection of data points is supposed to lie on a parabola of the form

$$y = a + bx + cx^2.$$

Suppose also that the data points collected are:

$$(-3, -13) \quad (-2, -8) \quad (-1, 0) \quad (0, 3) \quad (1, 4) \quad (3, 1) \quad (5, 0) \quad (6, -7)$$

   (a) Set up a matrix equation $A\mathbf{x} = \mathbf{b}$ which can be used to find the best fitting parabola. In particular, what are A, $\mathbf{x}$ and $\mathbf{b}$?
   (b) Compute (using least-squares) the model which best fits the data.
   (c) Use your model to predict the value of y when $x = 10$.

3. Find the inverse of the following matrix using the Gauss-Jordan method (please show all your steps).

$$\begin{pmatrix} 1 & 0 & -2 \\ -2 & 3 & 2 \\ -1 & 1 & 1 \end{pmatrix}$$

4. Solve the following systems of equations:

(a) $$\begin{cases} w & -4x & +2y & +z & = 1 \\ 2w & -3x & +y & +2z & = -4 \\ -5w & & +2y & -5z & = 15 \end{cases}$$

(b) $$\begin{cases} x & -4y & +2z & = 1 \\ 2x & -3y & +z & = -4 \\ -2x & +y & & = 3 \end{cases}$$

(c) $$\begin{cases} 3v & -w & -3x & +y & +2z & = 1 \\ 2v & -3w & +4x & -2y & +z & = -4 \\ 4v & -5w & -10x & +4y & +3z & = 6 \end{cases}$$

5. For each given $m \times n$ matrix A, fill out the rest of the chart, where the rows of the chart correspond to the following questions:
   (a) Give the dimension of the column space of A.
   (b) Give the dimension of the null space of A.
   (c) True or false: $A\mathbf{x} = \mathbf{b}$ has no solution for *some* $\mathbf{b} \in \mathbb{R}^m$.
   (d) True or false: $A\mathbf{x} = \mathbf{b}$ has no solution for *every* $\mathbf{b} \in \mathbb{R}^m$.
   (e) True or false: $A\mathbf{x} = \mathbf{b}$ has at least one solution for *some* $\mathbf{b} \in \mathbb{R}^m$.
   (f) True or false: $A\mathbf{x} = \mathbf{b}$ has at least one solution for *every* $\mathbf{b} \in \mathbb{R}^m$.
   (g) True or false: $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for *some* $\mathbf{b} \in \mathbb{R}^m$.
   (h) True or false: $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for *every* $\mathbf{b} \in \mathbb{R}^m$.

	A is an 18×14 matrix whose null space is six-dimensional	A is a 7×4 matrix whose columns are linearly independent	A is an 11×22 matrix that has 11 pivots	A is an invertible 5×5 matrix
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				
(h)				

6. Find the characteristic polynomial, the eigenvalues and corresponding eigenvectors of the following matrix:

$$\begin{pmatrix} 0 & -6 & 12 \\ -5 & 11 & -12 \\ -5 & 17 & -24 \end{pmatrix}$$

7. Suppose that the number $x(t)$ of sparrows at time t and the number $y(t)$ of cardinals at time t in a forest are modeled by this system of differential equations:

$$\begin{cases} x'(t) = \frac{4}{3}x(t) + \frac{2}{3}y(t) \\ y'(t) = \frac{1}{3}x(t) + \frac{5}{3}y(t) \end{cases}$$

If there are initially 30 sparrows and 120 cardinals in the forest, how many sparrows and how many cardinals will be in the forest at time $t = 20$? (I want an exact answer, not an approximation.)