Maximums and minimums of independent r.v.s: Suppose $X_1, ..., X_d$ are independent r.v.s with respective distribution functions $F_{X_1}(x), ..., F_{X_d}(x)$. Then:

$$F_{\min(X_1,...,X_d)}(x) = 1 - \prod_{j=1}^{d} \big(1 - F_{X_j}(x)\big)$$

i.e. *the survival function of the minimum is the product of the survival functions of the $X_j s$*; and

$$F_{\max(X_1,...,X_d)}(x) = \prod_{j=1}^{d} F_{X_j}(x)$$

i.e. *the distribution function of the maximum is the product of the distribution functions of the $X_j s$.*

1. A person trying to sell a piece of property accepts the highest of five sealed bids. If each bid is a random variable whose distribution function is $F(x) = 1 - 2/x$ for $x > 2$, and the bids are mutually independent, find the expected amount of the accepted bid.

2. The owners of ten homes file a claim after a storm; the size of each claim is a random variable with density function $f(x) = 5x^{-6}$ for $x > 1$. If the claim sizes are independent of one another, find the expected value of the smallest claim and the density function of the size of the largest claim.

3. Suppose $X_1, ..., X_d$ are independent exponential random variables with parameters $\lambda_1, ..., \lambda_d$. Let $U = \min(X_1, ..., X_d)$. Find the density function of U.

4. Suppose 8 random numbers are chosen uniformly and independently, each number being chosen from the interval $[0, 1]$. Find the probability that the maximum of the numbers chosen is greater than $\frac{1}{2}$ but less than $\frac{3}{4}$.