A New Triangle Generation of Generalized Genocchi Numbers Using Rook Placements on Genocchi Boards

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Grand Valley State University

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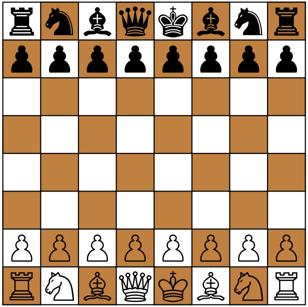
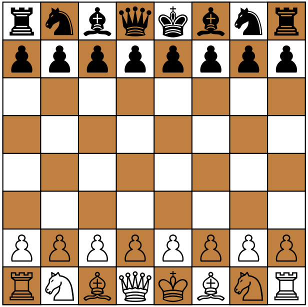
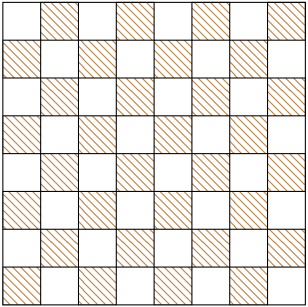
Background



Background



Boards



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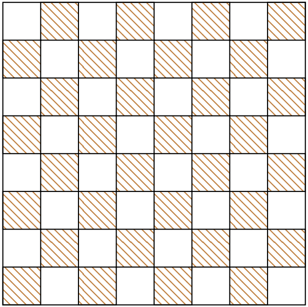
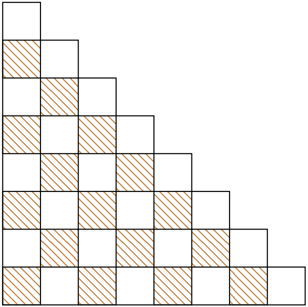
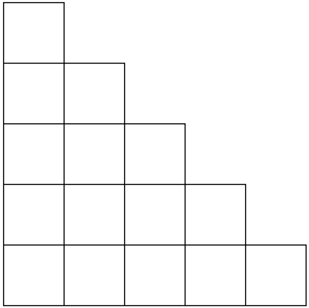




Triangle Board

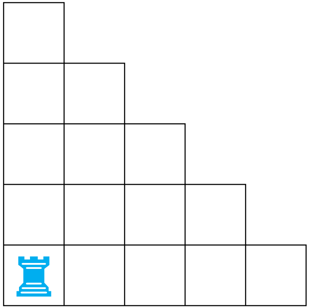


Size 5

Properties

- (i, j), where $j \leq i$ and $1 \leq i \leq n$

Triangle Board



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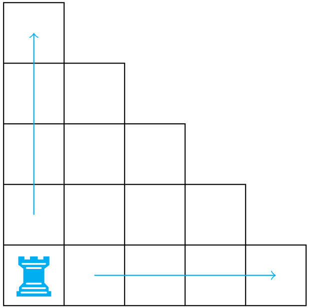
Triangle Board

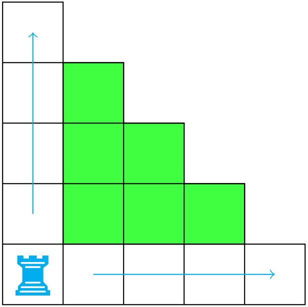


Size 5

Properties

- (i, j), where $j \leq i$ and $1 \leq i \leq n$
- Attack along rows and columns

Triangle Board



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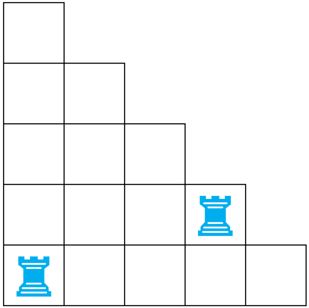

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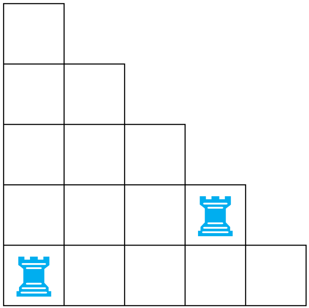

Size 5

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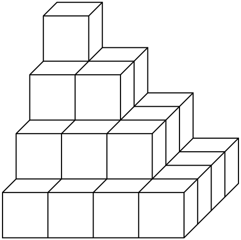
⋆ Significance ⋆

Number of ways to place k non-attacking rooks on a triangle board of size n corresponds to the Stirling numbers of the second kind

Triangle Boards in Three Dimensions

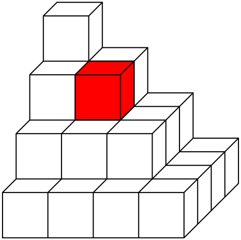
- (i, j, k) with $1 \leq i, j \leq k$ and $1 \leq k \leq m$.



Size 4

Triangle Boards in Three Dimensions

- (i, j, k) with $1 \leq i, j \leq k$ and $1 \leq k \leq m$.
- Attack along

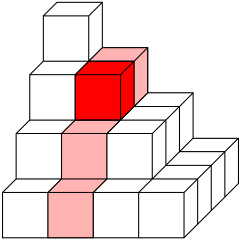


Size 4

1^{st} Generalization of Triangle Board

Triangle Boards in Three Dimensions

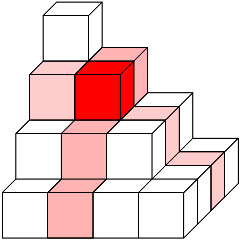
- (i, j, k) with $1 \leq i, j \leq k$ and $1 \leq k \leq m$.
- Attack along walls



Size 4

Triangle Boards in Three Dimensions

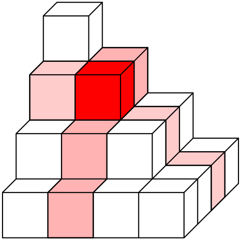
- (i, j, k) with $1 \leq i, j \leq k$ and $1 \leq k \leq m$.
- Attack along walls, slabs



Size 4

Triangle Boards in Three Dimensions

- (i, j, k) with $1 \leq i, j \leq k$ and $1 \leq k \leq m$.
- Attack along walls, slabs and layers



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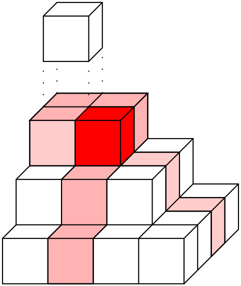
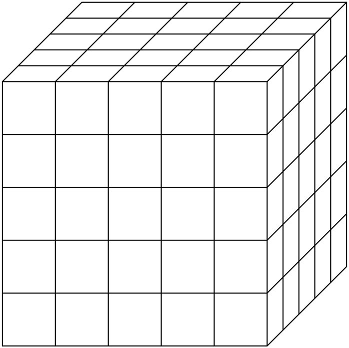


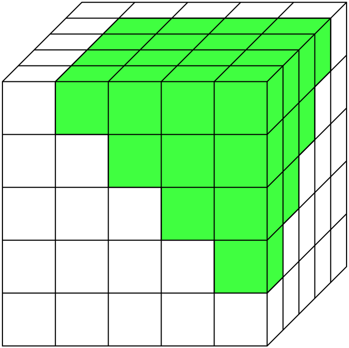
Size 4

Complement to 1^{st} Generalization of Triangle Board



Size 5

Complement to 1^{st} Generalization of Triangle Board



Size 5

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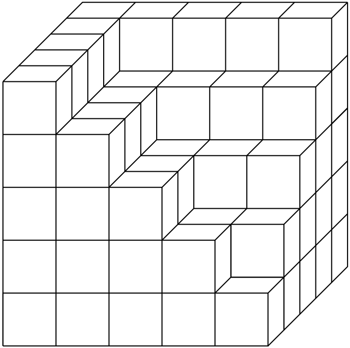
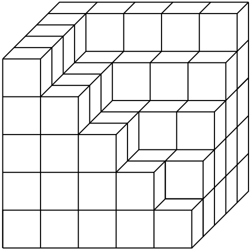


Size 5

Genocchi Board

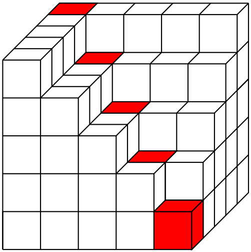
- (i, j, k) with $1 \leq k \leq \max\{i, j\}$ and $1 \leq i, j \leq m$.



Size 5

Genocchi Board

- (i, j, k) with $1 \leq k \leq \max\{i, j\}$ and $1 \leq i, j \leq m$.
- Attack along walls, slabs and layers



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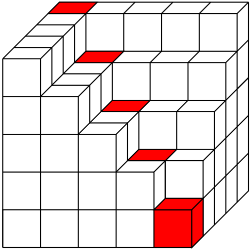
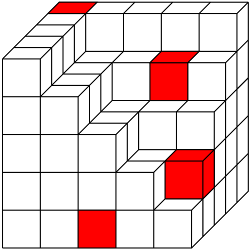
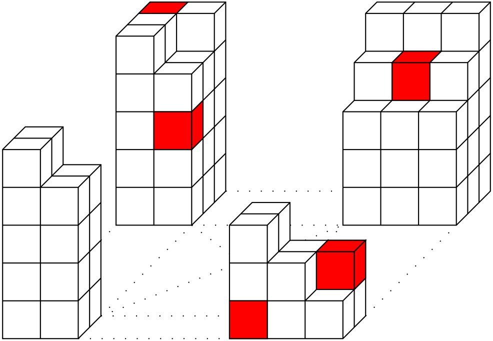




Size 5

2^{nd} Generalization of Triangle Board



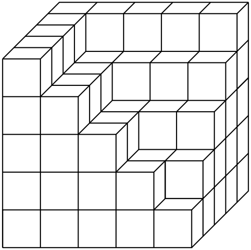
Size 5

Genocchi Board

Theorem (Alayont, Krzywonos)

Let $r_k(\Gamma_n)$ denote the k-rook number of the size n Genocchi board. Then $r_k(\Gamma_n)$ satisfies the recurrence relation

$$\begin{aligned} r_k(\Gamma_n) = & r_k(\Gamma_{n-1}) + \\ & r_{k-1}(\Gamma_{n-1})(2(n-k)+1)(n-k+1) + \\ & r_{k-2}(\Gamma_{n-1})(n-k+2)(n-k+1)^3 \end{aligned}$$



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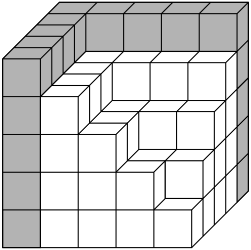

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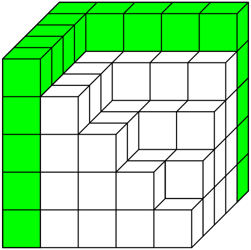

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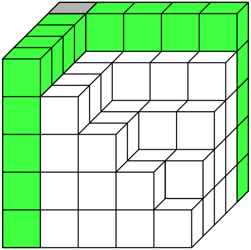
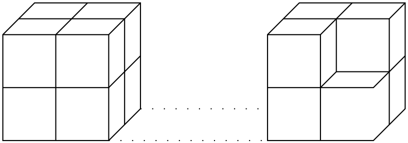


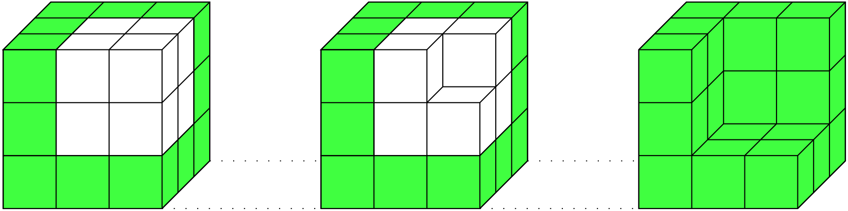
Size 5

4D *Genocchi* Board



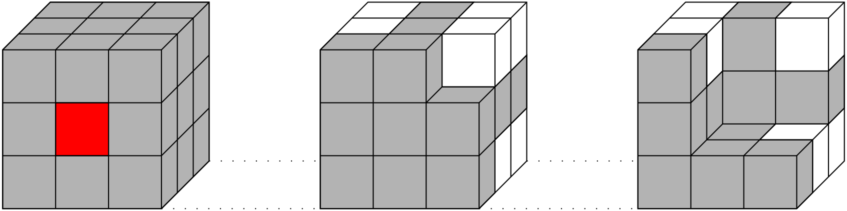
Size 2

4D *Genocchi* Board



Size 3

4D *Genocchi* Board



Attack along walls, slabs, layers and time (4D layers)

Number of ways to place k rooks on size n 4D Genocchi Board

n \ k	0	1	2	3	4	5
1	1	**1**				
2	1	15	**7**			
3	1	72	505	**145**		
4	1	220	7525	33135	**6631**	
5	1	525	55445	1207260	3778201	**566641**

Recurrence Relation

$$r_k(\Gamma_n^{(4)}) = r_k(\Gamma_{n-1}^{(4)}) + r_{k-1}(\Gamma_{n-1}^{(4)})(n-k+1)(3(n-k+1)^2 - 3(n-k+1)+1) + r_{k-2}(\Gamma_{n-1}^{(4)})(n-k+2)3(n-k+1)^5 + r_{k-3}(\Gamma_{n-1}^{(4)})(n-k+3)(n-k+2)^4(n-k+1)^4$$

Recurrence Relation

$$r_k(\Gamma_n^{(4)}) = r_k(\Gamma_{n-1}^{(4)}) +$$
$$r_{k-1}(\Gamma_{n-1}^{(4)})(n-k+1)(3(n-k+1)^2 - 3(n-k+1)+1)+$$
$$r_{k-2}(\Gamma_{n-1}^{(4)})(n-k+2)3(n-k+1)^5 +$$
$$r_{k-3}(\Gamma_{n-1}^{(4)})(n-k+3)(n-k+2)^4(n-k+1)^4$$

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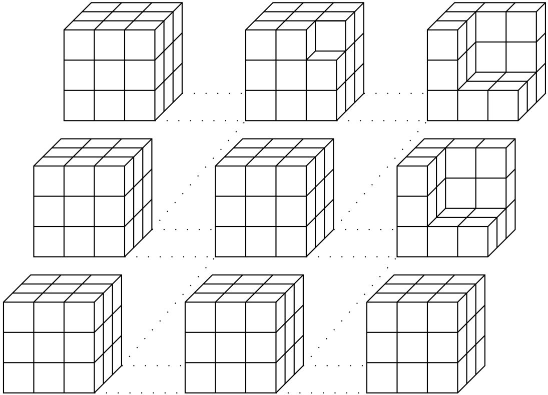
Attack along

- walls
- slabs
- layers
- time
- hyper-time



Size 3

Number of ways to place k rooks on size n 5D Genocchi Board

n \ k	0	1	2	3	4	5
1	1	**1**				
2	1	31	**15**			
3	1	226	3345	**1025**		
4	1	926	100875	954815	**209135**	
5	1	2771	1245715	87547640	598789745	**100482849**

Recurrence Relation for 5D Genocchi Board

Recurrence Relation

$$r_k(\Gamma_n^{(5)}) = r_k(\Gamma_{n-1}^{(5)}) +$$

$$r_{k-1}(\Gamma_{n-1}^{(5)})(n-k+1)\sum_{i=1}^{4}(-1)^{i+1}\binom{4}{i}(n-k+1)^{4-i} +$$

$$r_{k-2}(\Gamma_{n-1}^{(5)})(n-k+2)(n-k+1)^5(6(n-k+1)^2+1) +$$

$$r_{k-3}(\Gamma_{n-1}^{(5)})(n-k+3)(n-k+2)^5(n-k+1)^5(4(n-k+1)+2) +$$

$$r_{k-4}(\Gamma_{n-1}^{(5)})(n-k+4)\prod_{i=1}^{3}(n-k+i)^5$$

Future Work

- Generalize Recurrence Relation to n Dimensions
- Rook Placement on Different Types of Topological Boards

Thanks for your attention! Any questions?