

TRANSFORMING TRADITION: FROM ONE INSTRUCTOR TO AN ENTIRE DEPARTMENT

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ABSTRACT. In this paper, we describe the evolution of calculus instruction at a mid-sized, career-oriented, comprehensive public university. Changes in practice, that began with a single faculty member implementing technology as a tool for student inquiry, have spread to other department faculty via support from an internal grant, workshops, and the use of shared “living” course materials rather than traditional texts. This evolution in ways of thinking led to formation of a working group charged with assessment of calculus instruction. By focusing its attention on an analysis of student outcomes, the working group has come up with further recommended reforms to calculus instruction, both in terms of pedagogical approaches and content coverage. These recommendations have the potential to support further cultural change in the department (including similar reform in other courses), ultimately leading to greater student success.

1. BACKGROUND

In this article we describe an ongoing process of change in two calculus courses in the mathematics department at Ferris State University, a mid-sized, career-oriented, comprehensive university in Michigan. The department consists of 14 tenure-line and 13 adjunct faculty, and enrolls about 60 majors in applied mathematics, actuarial science, computer science, and secondary education. Typically, no more than 1-2 graduates per year pursue graduate study; most enter careers immediately. Our calculus courses enroll some mathematics majors, but principally serve students in pre-professional STEM majors and engineering technology programs.

The department is currently reforming its approach to teaching its two-course single-variable calculus sequence (Calculus 1 (4 credits), Calculus 2 (4 credits)). Historically, faculty have used traditional texts, made minimal use of technology, and delivered instruction via lecture. With support from the university’s endowment foundation and teaching and learning center, the department has embarked on a reform agenda. This includes formation of an ad hoc committee of faculty called the Calculus Working Group (CWG) whose charge is to analyze and assess current departmental practice in teaching calculus with the goal of coming up with recommendations regarding content coverage, technology use, and pedagogy. In

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the next section, we describe the general departmental climate in the context of the four frames described in [17]. We then describe the specific evolutionary steps that gave rise to the CWG and discuss preliminary results of ongoing efforts to reform our single-variable calculus sequence.

2. THE FOUR FRAMES OF DEPARTMENTAL CULTURE

According to Reinholz and Apkarian [17], “culture is a historical and evolving set of *structures* and *symbols* and the resulting *power* relationships between *people*”. Our department strongly values faculty autonomy, evidenced in two ways: how courses are selected and how they are taught. Department faculty, using a rotation they develop and adjust, individually select their semester schedules. While courses have recommended texts, each faculty member, including adjunct faculty, determine both the materials and the instructional approach they use in meeting course learning outcomes.

As a result of numerous retirements, our department now includes several relatively new tenure-line faculty. With this transition in personnel, new ways of thinking, or *symbols*, as defined in [17], have arisen. This includes a change in pedagogy, from an emphasis on content coverage via lecture, toward the inclusion of small group collaboration and classroom discussion. Although most of the instruction in the department cannot be classified as purely inquiry, it strongly aligns with the “twin pillars” of inquiry-based mathematics education identified in [10]: active student engagement with rich mathematical ideas, and a learning environment where students collaborate and communicate mathematical ideas. For more on inquiry-based learning in mathematics, see [6] and [9].

Department faculty, to help ensure balanced “power relationships between people”, have generally respected differing viewpoints on teaching practice. As such, newer faculty have been empowered to teach from a more inquiry-oriented perspective. With this growth in inquiry-oriented instruction, faculty who previously felt reticent to use active approaches have since increased their level of student engagement. Conversations about pedagogy continue, as department faculty have started to support one another in ways that reflect what McDuffie and Graeber [14] describe.

Assessment practices have also evolved. Although originally driven by external forces (most notably accreditation), there have been changes in ways of thinking about assessment. Learning outcomes previously consisting of lists of topics are now stated in terms of what students should know and be able to do. In 2015, the faculty launched a year-long project to create a master course document (MCD) for each course. Each MCD collects relevant course information into a single document containing the course description and prerequisites, learning outcomes, suggested text materials, typical content, and guidelines for assessment (at general education, course and program levels). As a result of the MCD project, the department now has a systemic process for collecting data on program outcomes, and regularly meets to discuss that data.

It is in the context of the departmental culture outlined above that we tell the story of how our calculus sequence is being reformed, starting with the work of one faculty member and eventually leading to the CWG, a new structure that is now leading to new symbols impacting relationships among faculty.

3. INCORPORATION OF TECHNOLOGY INTO CALCULUS COURSES

In this section, we lay out the department's evolution in its approach to teaching calculus. This change started with one faculty member and gradually grew to include more members of the department. In Fall 2012, this faculty member was appointed to a tenure-track position; during his first semester, he taught Calculus 1 using a traditional lecture approach. While his approach worked well at his prior institutions, it seemed to him to be unsatisfactory at Ferris. Only 68% of his students finished the course with a 60% average or better, just 8% finished with a 90% average or better, and several students expressed displeasure with his approach on their course evaluations.

Because of these results, this instructor sought to revamp his calculus courses, specifically, to engage students and to improve their performance. Drawing on his experience at his prior institutions, where he had taught a calculus course with an accompanying lab and performed research using *Mathematica*, he surmised that significantly incorporating technology into his classes could help accomplish his goals. Much of the literature on the efficacy of technology usage on student learning shows positive impacts (see [23] for a meta-analysis): studies by Cooley [3], Cunningham [4], and Palmiter [16] showed gains in conceptual understanding and computational skill when students regularly used a computer algebra system (CAS) such as *Mathematica* in their calculus course. On the other hand, a study by Melin-Conejeros [15] found that students who were required to use a CAS on calculus homework performed at a lower level on his final examination than a control group; he stated that when used only as a tool for doing homework (as opposed to having regular usage in instruction), a CAS could have harmful effects. No matter the result, research on CAS in calculus instruction emphasizes that one must carefully consider how the technology is used in the classroom to achieve positive effects (see [21, 19, 2]). Furthermore, a consistent theme is that technology works best when there is a culture of support [20]. That was certainly the case here, given the elements of our departmental culture described in Section 2. In particular, the department head and others immediately and enthusiastically supported this instructor's efforts at reform.

Toward that end, in Spring 2013 this faculty member applied for and received an internal university grant from the university's foundation to reform his calculus courses. The grant provided for purchase of a limited *Mathematica* site license, implementation of strategies to increase student engagement, and support for changes in practice among multiple members of the department. This included a transition in Calculus 1 and 2 from a 4-hour, lecture-only model to a "3+1" model, consisting of three hours of lecture and one hour of computer lab work per week. During the computer lab period, students work on inquiry-based, student-centered activities (a portion of one such activity is included in Appendix B). These activities are designed to reinforce and/or introduce calculus concepts, to foster independent use of technology, to increase meaningful work with applications, and to engage students with procedural elements of the courses, reducing time spent on these items during lecture. Put another way, the goal is to help students to develop mental schemes where technology is part of a broader solution strategy, similar to what is described in [7, 8, 1, 18, 24, 22, 5]. During the three weekly "lecture" hours, students sometimes listen to the instructor talk about course material, but the lectures make frequent use of *Mathematica* graphics and computations, and

students also spend “lecture” time working on group activities, practicing problems, etc. Lectures and labs are carefully constructed to complement one another, and there is a consistent tone between lecture days and lab days.

To increase innovative teaching practice, in Fall 2013 this faculty member conducted a series of workshops for department faculty on the use of *Mathematica* as a teaching tool. About a third of the tenure-line faculty and one adjunct participated. They learned how to use the software and studied best practices for classroom use. Faculty who participated in the workshops produced inquiry-based, *Mathematica*-focused activities for use in their courses. In return, they were awarded professional development incentive money from the University’s Center for Teaching and Learning. Workshop participants included an adjunct instructor, who built a file that uses animated graphics to teach systems of linear equations to lower-level students, and a tenured full professor, who developed a series of *Mathematica*-based materials for a calculus survey course taken by construction management students and elementary education majors. In total, courses for which activities were created have ranged from beginning algebra to differential equations. Moving forward, the department offers both models for teaching calculus. Faculty can elect a four-hour lecture model but will be encouraged to change to the 3+1 model and provided support to do so.

As additional faculty began to incorporate use of technology in their courses, particularly in calculus, the College of Arts and Sciences supported extension of the *Mathematica* license to a university-wide unlimited license. This inspired yet another faculty member to focus a sabbatical on the development of *Mathematica* labs for the department’s numerical analysis course. The department also expanded use of a lab approach to the third semester of calculus. As a result, the department now offers lab-enhanced sections for all three of its calculus courses, as well as a survey of calculus course, numerical analysis, mathematical modeling, differential equations, and beginning algebra.

The change to the 3+1 model led faculty to see the need for better coordination between the presentation of course topics on lecture days and computer activities on lab days. Concurrently, several newer faculty noticed that our students struggled with note-taking in ways that negatively impacted the meaning of what an instructor was trying to convey. For instance, students would transcribe what might be written on the board as

IMPORTANT: $(fg)'(x) \neq f'(x)g'(x)$

Ex: Let $f(x) = x^2$;
let $g(x) = x^3$.

Then $(fg)'(x) = (x^5)' = 5x^4$, but
 $f'(x)g'(x) = (2x)(3x^2) = 6x^3$.

to end up looking like the following:

Important: $fg'(x) \neq f'(x)g'(x)$. Ex: let $f(x) = x^2$ let $g(x) = x^3$
 $(fg)'(x) = (x^5)' = 5x^4$ $f'(x)g'(x) = (2x)3x^2 = 6x^3$

At a more selective institution, this might not be much of a problem, because students learn the material more readily on their own or refer to a textbook. But at Ferris State, students tend to rely on their transcriptions of lectures to the detriment of their success when their transcriptions are unclear or incorrect.

To increase coordination between lab and lecture, and to move students' focus in class away from transcribing and towards learning, department faculty, under the leadership of the aforementioned grant recipient, created downloadable lecture notes (see [12, 13]) in place of a textbook. When using these notes, students bring the appropriate section of notes to class. The notes incorporate in-class activities, highlight key facts, and include examples to be discussed during lecture. They also frequently use *Mathematica* computations to reinforce solutions obtained by hand. Some sections of the notes present material in a traditional style, others are set up for inquiry-based activities, and some are organized more flexibly to accommodate different approaches to presentation as well as time considerations. The \TeX files for these items are shared on the department's internal network drive. This enables users to revise sections as they see fit, tweaking them to fit their instructional approach by adding, subtracting, or editing modules as wanted. The process of using shared, custom materials has created a more collaborative approach in the teaching of our calculus courses. In fact, several faculty regularly use the materials in place of a standard text and have contributed to their ongoing revision, and more recently have created similar sets of notes now used in differential equations, abstract algebra, and other courses. These texts are living documents that change according to informal and formal assessment, as well as teaching approaches and new insights. Use of the texts and their availability as flexible documents enable faculty users to be more nimble in adapting to changing student needs and trends in mathematics instruction.

4. CALCULUS WORKING GROUP

The Foundation grant mentioned earlier was a success in terms of improving student performance (see Section 5) and generating an environment in the mathematics department where more progressive approaches to teaching (especially vis-à-vis the use of technology, where appropriate) were embraced. On the other hand, we offer roughly ten sections of Calculus 1 and 2 annually, and not all these are taught by the group of instructors working collaboratively as described in the previous section. This reflected a division in the approach to teaching calculus on the part of newer members of the department, who were using technology and implementing more engaged approaches to instruction, and the style of more senior faculty members, who relied more on a traditional approach. At the same time, some faculty members expressed concern when they observed promising advisees leaving the department's degree programs as a result of negative experience in their traditionally taught calculus courses. Furthermore, other faculty, while teaching upper-division courses, informally observed shortcomings in their students' knowledge of calculus.

With these issues in mind, five faculty formed an ad hoc committee called the Calculus Working Group (CWG) to assess the state of calculus practice and student performance in the context of answering five questions:

- (1) What are we teaching in our calculus classes? What are we emphasizing? What should we be emphasizing?

- (2) What pedagogies are we using? What are the strengths and weaknesses of the various pedagogical approaches?
- (3) How well prepared are our students in calculus? Is our placement mechanism for incoming students effective?
- (4) How much do our advanced students retain from our calculus courses? How well does our calculus sequence prepare students for future course-work in mathematics?
- (5) How well aligned is our calculus sequence with our program outcomes, the demographics of our calculus students, and our institutional mission?

Throughout the 2018-19 academic year, the CWG regularly met to discuss these and other issues. They surveyed instructors about their pedagogical methods and course content coverage. They audited exams, measuring the amount of trigonometry and sophisticated algebra required and cataloguing questions based on which course learning outcome(s) they assessed (essentially this classified questions as computational, conceptual/theoretical, or applied; for more on this, see Appendix C, which lists sample exam questions in each category). The group also analyzed the demographics (with respect to degree program and placement test score) of students taking calculus between 2013 and 2018, and studied student grades over the same time period, looking for trends in student performance related to degree program and/or placement test score. They also developed a survey administered to students who had already completed Calculus 1 and/or 2 to determine their basic grasp of calculus content. The survey asked some elementary, big-picture questions about calculus (one question was “Briefly describe the idea or concept of the Fundamental Theorem of Calculus”) and rote computations requiring very little work (example: “evaluate $\int e^{5x} dx$ ”).

The CWG quickly discovered a gulf in the type of assessments being used by calculus instructors in our department. For example, a student taking Calculus 1 from Professor A could earn 81% of the points on their exams by doing rote computations alone, but a student taught by Professor B might only be able to earn 53% of their points by similar computations. In any event, the assessments being used suggest that our instructors primarily assess students’ abilities to perform calculus computations rather than deep understanding of calculus concepts. Unsurprisingly, this approach has had negative long-term effects: only 12% of respondents to the CWG survey were able to say anything valid about the concept of the Fundamental Theorem of Calculus. Moreover, students have not been retaining computational ability: only 20% of those surveyed correctly integrated e^{5x} .

The CWG also closely examined the content of Calculus 1 and 2 (similar to what was done in [11], but in greater detail), asking what content was most essential and what content could be de-emphasized (or outright deleted) while maintaining sufficient rigor consistent with program outcomes and with our university mission. Section by section, item by item, the group asked, “Given our calculus student demographics, why are we teaching this item?” As a result of its analysis, the CWG discovered a number of topics that could be excluded (related rates, the Limit Comparison Test, integration by inverse trigonometric substitution, sophisticated curve sketching, to name a few). Many of these topics had already been naturally abandoned by some instructors via the informal collaborative process of

editing lecture notes. However, they remained in others' Calculus 1 and Calculus 2 courses, perhaps because of inertia or perhaps because they appear in the calculus textbook adopted by the department years ago.

An additional rationale for this careful sifting of course content was to improve the culture of assessment. In 2015, the department revised its program and course-level learning outcomes. The purpose was twofold: first, to place greater focus on the development of conceptual understanding, and second, to create opportunities for useful assessment. For instance, in Calculus 1, this meant winnowing a list of 14 learning outcomes, each of which delineated a very narrow skill, to four outcomes that are more global in nature:

<p><u>Calculus 1 course learning outcomes</u></p> <p>LO1: Infer information about a function from a limit statement, derivative or integral.</p> <p>LO2: Estimate limits, derivatives, and integrals numerically and graphically (including situations where the limit, derivative or integral does not exist).</p> <p>LO3: Compute limits, derivatives, and integrals of algebraic, trigonometric and transcendental functions.</p> <p>LO4: Solve problems which apply limits, derivatives and integrals.</p>
<p><u>Calculus 2 course learning outcomes</u></p> <p>LO1: Compute definite, indefinite and improper integrals using different integration techniques.</p> <p>LO2: Solve problems which apply integrals.</p> <p>LO3: Determine whether an infinite series converges or diverges.</p> <p>LO4: Find the Taylor series of a function, and use that series to solve problems involving polynomial approximation.</p>

The advantage of these more global outcomes is that the actual process for collecting data for assessment purposes is made easier. However, this benefit comes at the cost of decreased clarity: in the third outcome of Calculus 1, exactly what limits, derivatives and integrals are important for students to compute? Similarly, what applied problems do we want students to solve? The CWG discovered the first outcome of Calculus 2 was being interpreted very differently from one instructor to the next, resulting in exams of greatly varying difficulty and ultimately leading to substantially different grade distributions depending on the instructor. Consequently, one recommendation of the CWG is to tighten this learning outcome, clarifying the types of integrals we expect our students to be able to compute.

All the work of the CWG was compiled into a preliminary report, which was shared with department faculty and client departments whose degree programs require calculus. After receiving feedback from these stakeholders, a final report was submitted to the department in October 2019. This report contains recommended changes to course content and spells out in detail what is meant by each of our course learning outcomes (in some cases, changes to the outcomes are suggested). It makes specific recommendations to instructors of calculus geared

toward increasing student understanding of the key ideas of calculus and de-emphasizing sophisticated symbolic manipulations, and lays out issues which require further assessment.

Since our university has a strong tradition of faculty autonomy, under which instructors have wide latitude to teach their courses as they see fit, imposing the use of *Mathematica* in all calculus courses or insisting on the use of an inquiry-based learning approach is not feasible. However, it is hoped that the analysis of the CWG will shrink differences in instructional practice and close the gap in student results and performance. There are three reasons for optimism: (1) the CWG has compiled substantial data to support its conclusions; (2) the CWG is driven by faculty; (3) the CWG report includes concrete ideas for teaching and specific ways to assess student understanding to aid instructors who are not used to a more progressive approach to teaching calculus.

5. RESULTS SO FAR

Upon transitioning his calculus courses to the 3+1 model and using the specially designed notes, the grant recipient noticed substantially increased performance from his Calculus 1 students. In the first three semesters he taught Calculus 1 with labs, 24% of his students (up from 8% previously) finished with a 90% average or better, and 93% (up from 68% previously) of his students finished with a 60% average or better. Despite no changes to exam content or format, students of all ability levels performed better on every exam in the course once labs were introduced. Student performance in the grant recipient's Calculus 2 was more mixed: while the percentage of students making 90% or better increased from 11% to 19% when labs were introduced, the percent making 60% or better was virtually unchanged, and while student performance increased greatly on assessments related to series, it decreased on assessments related to techniques of integration. (Figure 1 gives some more data from this instructor's Calculus 1 courses.)

Students were also happier with his calculus courses: adjusted overall ratings on course evaluations in the grant recipient's classes went from 3.5/5 (before labs, in Fall 2012) to 4.7/5 (with labs, in Spring 2014). This professor also surveyed his calculus students regarding their *Mathematica* usage; 77.4% of those surveyed either agreed or strongly agreed with the statement "I gained greater insight into course material through the lab assignments"; only 4.8% disagreed or strongly disagreed (see Figure 2). Three times as many students stated that they are "more interested" in taking math classes based on their experiences in his lab-focused calculus courses, as opposed to "less interested".

Another instructor, who worked with *Mathematica* in 1993 when taking calculus as a high school senior, was excited about including labs in his calculus sections. In Fall 2014, he started teaching Calculus 1 with *Mathematica*. Since then, he has taught five more sections of Calculus 1 as well as two sections of Calculus 2, all using the 3+1 model. For this instructor, the labs have proven to be an effective and efficient way to explore computationally complicated, algebraically messy, and graphically cumbersome topics in a technologically savvy way. He has used labs to guide inquiry-based investigations of different types of discontinuities, computations using the definition of the derivative, graphical aspects of the derivative, the Chain Rule, Newton's method, optimization problems, and Riemann sums.

In Spring 2019, a third instructor taught two sections of Calculus 2 using the 3+1 model for the first time. Though he initially was concerned with losing one hour of lecture time each week, he found that the time spent in lab enhanced his lectures and enabled deeper learning of the content. As an example, instead of presenting in-class examples related to the Integral Test for series convergence, students used a *Mathematica* lab to compare infinite series and associated improper integrals. The lab also had students wrestle with series whose terms were not those coming from an integrable function, helping them discern the limitations of the Integral Test. In the end, use of labs provided students with a working knowledge of *Mathematica* as well as the opportunity to explore mathematical ideas in a collaborative setting without sacrificing content coverage.

All told, the 3+1 model has positively impacted student success: from Fall 2014 to Spring 2019, the overall DWF rate for Calculus 1 sections taught using the 3+1 model is 17%, much lower than the DWF rate of lecture-only Calculus 1 sections (36%). In Calculus 2, during the same time frame, the DWF rate in 3+1 sections is 17%, whereas the DWF rate in lecture-only sections is 23%.

The work of the CWG, even while still in progress, has also begun to have an impact. In Spring 2019, a CWG member taught two sections of Calculus 1. In previous iterations of the course, he devoted substantial class time to complicated algebraic techniques for evaluating limits, and presented a detailed theoretical discussion of how the Fundamental Theorem of Calculus is deduced from the Mean Value Theorem. Largely because of the discussions of course content within the CWG, this faculty member greatly scaled back the time spent on these topics, using the additional time on activities in which students estimate derivatives from tables, discuss Newton's method, and study integration from a more conceptual approach that blends lecture with group work. This professor felt that the revised course incorporated material better suited to the needs of his students, and he observed both informally and on assessments that students had greater ability to interpret the meaning of differentiation and integration in graphical and applied contexts.

6. REFLECTIONS AND CONCLUSION

In this paper we have described an ongoing process of change that started with one faculty member. Through an internal grant, which funded purchase of technology and workshops, what started as a singular vision grew to include roughly a third of department faculty, with about half of the calculus sections becoming lab-enhanced.

With support from the College of Arts and Sciences, the opportunity to revise instruction became even more accessible. The development of shared lecture notes, together with the work of the CWG, has further encouraged faculty collaboration and informed discussion. Using the department's electronic shared drive, faculty who develop labs or other materials can share their work with their colleagues. Our monthly departmental colloquium series has also seen an increased emphasis on teaching practice, providing faculty an opportunity to see what their colleagues are doing and to ask questions about how they might implement changes to their instructional practice.

Faculty conversations have started to change ways of thinking. Rather than holding strongly held positions and continuing to "do what has always been done",

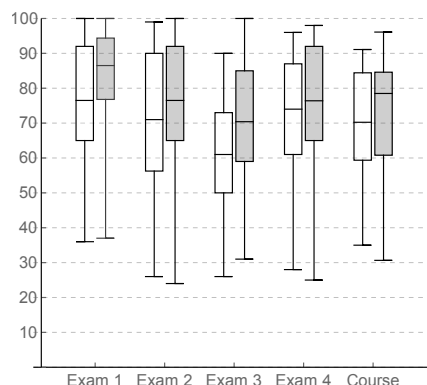


FIGURE 1. Box-and-whisker plots for student performance in one instructor's sections of Calculus 1. The white plots correspond to student performance in one section which was taught in a traditional style ($N = 28$); the gray plots measure student performance over three sections taught by the same instructor, with the same types of assessments, in a "3+1" format incorporating laboratory-style assignments ($N = 74$). Exam 1 refers to an exam on limits; Exam 2 covers derivatives; Exam 3 deals with applications of differentiation; Exam 4 covers integration. The last box plot of each color indicates student averages for the entire course.

	SA	A	N	D	SD	Average
I gained greater insight into course material by doing lab assignments.	28.6%	48.8%	17.9%	3.6%	1.2%	4.000
I would prefer a lab-based calculus course to a traditional calculus course.	27.4%	38.1%	19.0%	10.7%	4.8%	3.393

FIGURE 2. Responses to two questions, measured on a Likert scale, on a survey administered to 84 students who took a lab-enhanced calculus course during Spring and Fall 2013. The last column averages the responses on a scale of 1 = strongly disagree, to 5 = strongly agree.

we have seen a growing interest amongst our faculty in learning what others do in the classroom and incorporating new ideas in their own teaching practice. The most ardent advocates of a lecture-oriented approach are less skeptical about student collaboration, and those who emphasize the development of computational fluency are beginning to see the value of technology. By placing the focus of its analysis on the results of student learning, rather than on promoting particular ideological positions, the CWG has ushered in a new operational framework: faculty collaboration rooted in a process of continual improvement based on student learning, where goals for student learning are defined in accordance with student demographics and institutional values.

As one component of this framework, we hope and expect that in our department, other working groups will be formed to assess the quality of other courses. (One CWG member has already expressed intent to establish a Pre-calculus Working Group next year.) For readers of this article who may be interested in starting their own working groups (or in crystallizing the charge of already existing course committees), we offer the following reflections on our experience:

- Five members was an ideal size for our working group—this allowed for sufficient diversity in opinion, while not being so large that members felt unneeded. Group members do not actually have to have taught the course in question recently to have valuable input.
- One way to get started is with three tasks our group undertook: examine student demographics with regard to degree program, audit exams from an array of instructors, and assess the value of each item of course content. Beyond this, the group's focus should evolve naturally, depending on what is found in an initial analysis.
- When auditing exams, it is easy to make value judgments such as “this question is good” or “this question is too hard.” Avoid these types of judgments. Rather, categorize assessments in the context of what kind of prerequisite material they require, whether or not they require understanding versus mechanical work, etc.
- We encountered very little resistance when asking instructors to share their exams with us. This is in part because our calculus instructors are almost exclusively tenured faculty, and in part because we made it clear that the purpose was to gather data and objectively classify questions, as opposed to judging the quality of the exams.
- Decisions made regarding course content, etc. were relative to the particular needs of our students. For instance, we have very few majors pursuing graduate study in mathematics, and those with interest in graduate school tend to have a close relationship with a faculty member (through advising or an undergraduate research project) who helps them prepare for the GRE. As such, ensuring that our calculus courses contain all the content needed for the GRE is a relatively low priority for us. At a different institution, this might be a more important consideration.
- Be willing to question everything! In our working group, one member suggested deleting integration by parts entirely. While the subsequent discussion did not lead to the removal of integration by parts from our curriculum, it highlighted that the primary subset of our calculus students needing integration by parts are actuarial science majors. This led us to see a need for incorporating some continuous probability into Calculus 2, and to suggest to instructors that they connect integration by parts with expected value computations.
- Keep in mind the needs of client departments. After parsing the content of our calculus courses, the CWG wrote a preliminary report with an initial list of topics slated for de-emphasis or removal. This work was shared with every department across campus housing a degree program requiring calculus, and these departments responded with valuable feedback. In particular, we found that students in several of our engineering technology programs often need to compute centers of mass, but are not asked to compute

work or fluid pressure. Additionally, to our surprise, we also found that manufacturing engineering courses used continuous probability models in their advanced coursework. These findings informed our final report, in which we concluded that moments, centers of mass, and probabilistic applications needed to be prioritized in our Calculus 2 courses.

Last, we discovered that the process of reform is extremely slow, and that careful assessment leads to as many new questions as it does answers. For example, the CWG found that Calculus 1 students taking the course in any fall semester performed significantly worse than students taking the same course in the spring, even after allowing for variables such as placement test score or mode of instruction. At present, we do not know what causes this phenomenon. Is students' weak performance correlated with them being new to Ferris? Is this discrepancy reflecting the performance of students who took calculus in high school or community college, versus those who didn't? While we have established a process for collecting program assessment data, we lack sufficient course-level assessment data regarding our learning outcomes, and have not collected the specific information needed to address these questions.

In response to these challenges, the CWG plans to collect more robust data on students taking calculus (particularly with regard to whether they are new or returning students, whether or not they have previously taken calculus, and what prerequisite coursework they have completed at our institution). The CWG's report begs instructors to provide course assessment data related to learning outcomes. Additionally, the CWG has directed the department, through its course committees, to examine if the same fall/spring grade disparity exists in other courses.

The efforts of the CWG have been successful because of the group's respect for the cultural structure of faculty autonomy, their focus on student outcomes rather than on individual faculty practices, and their effort to maintain confidentiality in working with data. Their work has garnered the full support of the department head, who encourages experimentation and, through his own evolution, has begun to emphasize faculty finding their authentic voices as instructors rather than pushing for adoption of certain pedagogical approaches. These changes in symbols, or ways of thinking, have helped reduce typical concerns over promotion and tenure issues while, at the same time, minimizing ideological arguments. There is still much to be done, but these changes, supported by the work of the CWG, are helping to support development of a departmental culture rooted in continuous improvement, informed by careful assessment.

REFERENCES

- [1] M. Artigue. Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning* 7 (2002), 245-274.
- [2] M. Berger. Using CAS to solve a mathematics task: A deconstruction. *Computers & Education* 55 (2010), 320-322.
- [3] L. Cooley. Evaluating student understanding in a calculus course enhanced by a computer algebra system. *PRIMUS* 7 (1997), 308-316.
- [4] R. Cunningham. The effects of achievement on using computer software to reduce hand-generated symbolic manipulation in freshman calculus (unpublished doctoral dissertation, Temple University) (1991).

- [5] P. Drijvers and L. Trouche. From artifacts to instruments: A theoretical framework behind the orchestra metaphor. In G.W. Blume and M.K. Heid (Eds) *Research on technology in the teaching and learning of mathematics: Cases and perspectives 2*, Information Age Publishing, Charlotte, NC (2008), 363-391.
- [6] D. Ernst, A. Hodge and S. Yoshinobu. What is inquiry-based learning? *Notices Amer. Math. Soc.* **64** (2017), 570-574.
- [7] F. Hitt and C. Kieran. Constructing knowledge via a peer interaction in a CAS environment with tasks designed from a task-technique, theory perspective. *International Journal of Computers for Mathematical Learning* **14** (2009), 121-152.
- [8] C. Kieran, P. Drijvers, A. Boileau, F. Hitt, D. Tanguay, L. Saldanha, and J. Guzmán. The co-emergence of machine techniques, paper-and-pencil techniques, and theoretical reflection: A study of CAS use in secondary school algebra. *International Journal of Computers for Mathematical Learning* **11** (2006), 205-263.
- [9] S.L. Laursen and C. Rasmussen. I on the prize: inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education* **5** (2019), 129-146.
- [10] S. Laursen, M.-L. Hassi, M. Kogan, A.B. Hunter and T. Weston. *Evaluation of the IBL Mathematics Project: Student and instructor outcomes of inquiry-based learning in college mathematics* University of Colorado (2011), available at <https://www.fitchburgstate.edu/uploads/files/AcademicAffairs/IBLmathReportALL.pdf>
- [11] A. D. Lauten, K. Graham and J. Ferrini-Mundy. Increasing the dialogue about calculus with a questionnaire. In B. Gold, S.Z. Keith and W.A. Marion (Eds) *Assessment Practices In Undergraduate Mathematics*, The Mathematical Association of America (1999), 237-240.
- [12] D. McClendon. Calculus I Lecture Notes (Spring 2019 edition). Available at <http://mcclendonmath.com/m220/lecturenotes220.pdf>
- [13] D. McClendon. Calculus II Lecture Notes (2016 edition). Available at <http://mcclendonmath.com/m230/lecturenotes230.pdf>
- [14] A.R. McDufie and A.O. Graeber. Institutional norms and policies that influence college mathematics professors in the process of changing to reform-based practices. *School Science and Mathematics* **103** (2003), 331-344.
- [15] J. Melin-Conejeros. The effect of using a computer algebra system in a mathematics laboratory on the achievement and attitude of calculus students (unpublished doctoral dissertation, University of Iowa) (1992).
- [16] J.R. Palmiter. Effects of computer algebra systems on concept and skill acquisitions in calculus. *Journal for Research in Mathematics Education* **22** (1991), 151-186.
- [17] D.L. Reinholz and N. Apkarian. Four frames for systemic change in STEM departments. *International Journal of STEM Education* **5** (2018), 1-10.
- [18] K. Ruthven. Instrumenting mathematical activity: Reflections on key studies of the educational use of computer algebra systems. *International Journal of Computers for Mathematical Learning* **7** (2002), 275-291.
- [19] T.S.A. Salleh and E. Zakaria. Integrating computer algebra systems (CAS) into integral calculus teaching and learning at the university. *International Journal of Academic Research* **3** (2011), 397-401.
- [20] N. Selinski and H. Milbourne. The institutional context. In D. Bressoud, V. Mesa, and C. Rasmussen (Eds) *Insights and Recommendations from the MAA National Study of College Calculus*, The Mathematics Association of America, Washington, DC (2015) 31-44.
- [21] D. Tall, D. Smith and C. Paez. Technology and calculus. in M.K. Heid and GW. Blume (Eds) *Research on technology and the teaching and learning of mathematics: Research syntheses 1* Information Age Publishing, Charlotte, NC (2008), 207-258.
- [22] T.K. Tiwari. Computer graphics as an instructional aid in an introductory differential calculus course. *International Electronic Journal of Mathematics Education* **2** (2007), 32-48.
- [23] C.L. Tokpah. The effects of computer algebra systems on students' achievement in mathematics (unpublished doctoral dissertation, Kent State University) (2008).
- [24] L. Trouche. Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning* **9** (2004), 281-307.

APPENDICES

APPENDIX A: SAMPLE MATHEMATICA ACTIVITY

Here, we give an excerpt from a *Mathematica*-based lab assignment given to Calculus 1 students at Ferris State University. In this assignment students explore how to use the sign of the second derivative to classify critical points as local maxima, local minima or saddles:

- (1) Throughout this problem, let $f(x) = 3x^4 - 4x^3 - 36x^2 - 50$.
 - (a) Have *Mathematica* sketch a graph of f , where x ranges from -5 to 5 . From looking at the graph of f , list the values of x where you think $f'(x) = 0$ (these x -values should be integers). Why did you choose these values of x ?
 - (b) Use *Mathematica* to solve the equation $f'(x) = 0$ (remember that to solve an equation using *Mathematica*, you have to type the equation with two equals signs). Write down the solutions you get. Do these agree with the estimates you made in part (a)?
 - (c) Which of the x -values you found in parts (a) and/or (b) are locations of local maxima of f ?
 - (d) Which of the x -values you found in parts (a) and/or (b) are locations of local minima of f ?
 - (e) Compute $f''(x)$ at each value of x you found in parts (a) and/or (b).
 - (f) Based on the values you obtained in part (e), make a general conjecture ("conjecture" means "educated guess") about how to use the second derivative to tell whether or not an x which is a solution of $f'(x) = 0$ is a local maximum or local minimum of f .
 - (g) Use the conjecture you wrote in part (f) to study the function $Q(x) = 126x^4 - 1642x^3 - \frac{165}{2}x^2 + 27225x - 1542$. More specifically, find all values of x where $Q'(x) = 0$, and classify them as local maxima or local minima using the conjecture you wrote down in part (f).

This assignment continues by inviting students to consider functions with points c such that $f'(c) = f''(c) = 0$, making and testing conjectures until they ultimately discover the n^{th} Derivative Test.

APPENDIX B: SAMPLE EXAM QUESTIONS

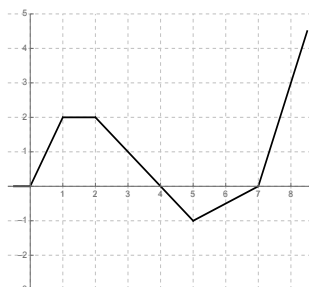
As described earlier, the CWG audited instructors' exams, classifying questions according to which learning outcome(s) the question fell under. Some of these exam questions, listed by outcome, are as follows:

LO1 (infer information from a calculus statement)

- (1) Sketch the graph of a function f which has all of the following properties: $\lim_{x \rightarrow 2^+} f(x) = 3$; $\lim_{x \rightarrow 2^-} f(x) = -2$; $f(2) = 3$; $\lim_{x \rightarrow \infty} f(x) = -1$; $\lim_{x \rightarrow 7^-} f(x)$ does not exist.
- (2) Suppose f is an unknown function and c is a number such that $f'(c) = f''(c) = 0$ but $f'''(c) = -2$. Is c the location of a local maximum of f , a local minimum of f , or neither?

LO2 (numerical/graphical estimation of calculus quantities)

- (1) The graph of some unknown piecewise linear function f is given below:



- Evaluate $\int_0^8 f(x) dx$; find $f'(3)$; and find a value of x where $f'(x) = \frac{1}{2}$.
- (2) An investor keeps track of the value of her investments regularly, recording her data in the following table:

t (years after initial investment)	0	1	3	5	8
$v(t)$ (value of investment, in dollars)	800	824	875	929	1017

- (a) Given this data, estimate $v'(4)$. Show the computations leading to your answer, and write your answer with correct units.
- (b) What does your answer to part (a) mean, in the context of this problem?

LO3 (compute limits, derivatives and integrals)

- (1) Find $\lim_{x \rightarrow -1} \frac{x^2 - 1}{3x + 3}$.
- (2) Find the second derivative y'' if $y = 2x + \sqrt{x}$.
- (3) Find $\frac{dy}{dx}$ if $y + 3xy - 4 = 0$.
- (4) Evaluate the integral $\int_1^4 x\sqrt{x} dx$.

LO4 (applications)

- (1) The efficiency of a factory that employs x workers is given by $E(x) = \frac{1}{5}x(30 - x)$. Find the number of workers that maximizes the efficiency, assuming that the factory needs at least 10 workers to operate and that there are only 40 workers available in the town.
- (2) Suppose that an object is moving back and forth along a line so that its velocity at time t is given by the function $v(t) = t(2t - 1)$. Suppose also that at time 0, the object is at position -3 .
- (a) What is the acceleration of the object at time 3?
- (b) What is the position of the object at time 4?
- (c) At time $t = 0$, is the object speeding up or slowing down? Explain your answer.