

One of the things David Hilbert was famous for was giving a modern axiomatization of (Euclidean) (3-dimensional) geometry. Here are his axioms (it is interesting to compare these with Euclid's axioms):

Undefined primitives: point, line, plane

Primitive relations: betweenness, containment, congruence (of segments and angles)

I. Incidence Axioms

1. Two distinct points determine a line.
2. Any two distinct points on a line determine *that* line.
3. Three non-collinear points determine a plane.
4. Any three non-collinear points on a plane determine *that* plane.
5. If two points lie in a plane, then the line they determine lies in that plane.
6. If two planes have a point in common, then they have a second point in common.
7. Every straight line contains at least two points, every plane contains at least three non-collinear points, and space contains at least four non-coplanar points.

II. Order Axioms

1. If point B is between points A and C , then B is between C and A and B lies on the line determined by A and C .
2. If A and C are on line l , then there is a point B on l between A and C and a point D on l so that C is between A and D .
3. Given three distinct points on a line, exactly one point lies between the other two.
4. (Pasch's Axiom) Given a triangle and a line intersecting one edge of the triangle, either the line intersects a vertex of the triangle or the line intersects a second edge of the triangle.

III. Parallels

1. Given a plane, a line l in that plane, and a point in that plane not on the given line, there is *one and only one* line through the point not intersecting l .

IV. Congruence

1. Given a line segment AB and a point X on line l , then upon a given side of X on line l there is one and only one point Y on l so that the segment XY is congruent to AB .

2. Congruence of line segments is reflexive, symmetric and transitive.
3. Suppose B is between A and C on line l , and suppose Y is between X and Z on line l . Then if segments AB and XY are congruent, and if segments BC and YZ are congruent, then the segments AC and XZ are congruent.
4. Given angle α , a line l in a plane \mathcal{P} and a point X on line l , upon a given side of l in \mathcal{P} there is one and only one ray starting at X such that the angle formed by the ray and the line l is congruent to α and that all the interior points of the angle lie on the given side of l .
5. Angle congruence is reflexive, symmetric, and transitive.
6. SAS postulate for congruent triangles.

Continuity Axioms

1. (Archimidean Axiom) Given a segment AB on line l and a segment XY , if one lays off segments AA_1, A_1A_2, \dots with A_i between A and A_{i+1} , and each A_iA_{i+1} congruent to XY , eventually B is between A and some A_i .
2. (Line completeness) An extension of a set of points on a line with its order and congruence relations that would preserve the relations existing among the original elements as well as the fundamental properties of line order and congruence (Axioms I-III and V-1) is impossible.