# Everything but the kitchen sink

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# Explaining the title

## Things Matt Northrup suggested I talk about

- Advice for students in math-related fields
- Research opportunities for students
- Tips for internships or job searches
- How to approach professors and ask for help
- My college and grad school experience
- Research I did in grad school
- Math topics I find interesting
- Why I chose a career in math

# About me

- I'm from Babson Park, FL (population 1330)
- I went to high school in Frostproof, FL (population 2877; HS graduating class size 96)
- my parents went to college but grandparents did not;
- I have older siblings who had finished college when I was still young;
- my family members (and friends' family members) were almost always self-employed;
- the outside world was unknown (no social media or internet back then)

# My college experience

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I started out as a chemistry major. That lasted a year and a half (organic chemistry, yuck).

Then, I changed majors to mathematics.

# My college experience

**Degree requirements for my B.S. in Mathematics**

Intro CS
Calc 1,2,3
Diff. eqns
Linear algebra
Discrete / proofs
Real analysis
Complex analysis
Abstract algebra
Topology
Physics 1
Physics 2
Electives

# Detour: pure vs. applied math

## Pure mathematics

- Focuses on the development of abstract and theoretical concepts
  (in the research world, this means proving new theorems)
- Goal is to advance human understanding of mathematics, without regard for specific applications

## Applied mathematics

- Focuses on the use of math to solve applied problems in biology/physics/finance/industry/etc.
  (in the research world, this means identifying and developing *new* uses of mathematics)
- Goal is practical applications

# An argument for pure mathematics

Pure and applied mathematics are connected - without knowledge of one, you can't do the other very well.

Often, pure mathematics ends up having useful application that wasn't being thought of when the pure math was explored:

| Area of pure math study | | Application found later |
|---|---|---|
| group theory | $\rightarrow$ | X-ray crystallography |
| non-Euclidean geometry | $\rightarrow$ | relativity |
| spectral graph theory | $\rightarrow$ | data clustering |
| number theory | $\rightarrow$ | cryptography |
| dynamics of tilings | $\rightarrow$ | quasicrystals |

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I actually had to go to my professor's office hours (the horror!).

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1. The professor was shit. Nice guy, but a shit professor.
2. I lacked "mathematical maturity".

# Mathematical maturity

**Example from Real Analysis 2**

Compute

$$\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$$

where $S$ is the surface defined by $y = 10 - x^2 - z^2$ with $y \geq 1$, oriented with rightward-pointing normal (and $\mathbf{F}$ is some formula).

As a student, I'd approach a problem like this by:

1. finding a similar-looking problem in the book or my notes, and
2. trying to mimic the solution from the similar problem in this context.

# Mathematical maturity

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In other words, my instinct was to follow *recipes*.

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**Problem # 1:** Following recipes only worked some of the time.

Sometimes, the examples weren't close enough to the problem asked for me to adapt them to what I was assigned.

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**Problem # 2:** Even if I got the right answer, I often had no idea what the answer meant or why I was doing the problem.

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Put another way, *I was a "cook", not a "chef"*.

# Mathematical maturity

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Today, *I'm a chef:*

- I can recite formal definitions of all the symbols here;
- I can give a layman's description or a physical interpretation of what these symbols mean;
- I know ways to rewrite this integral, and why those ways make sense;
- etc.

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You will find that in most cases, solving problems becomes much easier, and you gain back all the time you spent learning course vocabulary by solving problems more quickly.

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# Learn from my mistakes!

**Definition**

I (you) have **command** of a mathematical concept when I (you) can explain that vocabulary to someone else, answering any (most) questions they have.

# How to learn mathematics

Let's say you are learning about some new mathematical thing for the first time.

Here is a big list of questions to ask about this new mathematical thing:

1. What is the exact definition of this word/phrase?
2. What is a layman's way of describing this word/phrase?
3. What part of speech is this word/phrase?
4. Why do I care about this word/phrase?
5. How does this word/phrase relate to words/phrases I learned earlier?
6. Is there an interpretation of this item outside mathematics (in physics or economics)?

# How to learn mathematics

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7. What picture can I draw that represents this type of object or relates to this concept?
8. Does this word/phrase have synonyms? Does it have synonyms in specific contexts?
9. Does this word/phrase have antonyms? Antonyms in specific contexts?

Let's say you are learning about some new mathematical thing for the first time.

Here is a big list of questions to ask about this new mathematical thing:

10. If it is a noun, is it a type of another noun? Is another noun a specific type of this thing?
11. If it is a noun, what are some examples of things that are this item? What are some things that are not this item?
12. If it is a mathematical object, what type of object is it (number, vector, set, function, etc.)?
13. If it is a numerical quantity, what values can this quantity take (whole numbers? integers? real numbers? complex numbers?)

Let's say you are learning about some new mathematical thing for the first time.

Here is a big list of questions to ask about this new mathematical thing:

14. If it is an adjective, what noun(s) does it describe?
15. If it is an adjective, what are some examples of things described by this adjective? What are some basic examples of things NOT described by this adjective?

# How to learn mathematics

The two best ways to discover answers to these questions are:

1. read math (in 2023, this includes "watch math"), and
2. talk math.

# Reading math

Reading math isn't like reading a work of fiction.

You have to read math super-slowly and super-carefully. You need to break down complicated sentences into pieces that you can digest.

You should **always** reread definitions and theorems (not necessarily the proofs) presented in class, reading them carefully and slowly so that you understand each word in them.

Go to lunch and discuss things with your classmates.

Talk about your course material in math club or GIS.

Form a study group and ask each other the kinds of questions I listed earlier. Ask each other what theorems say or why they are important.

# Talking math

## When talking about math

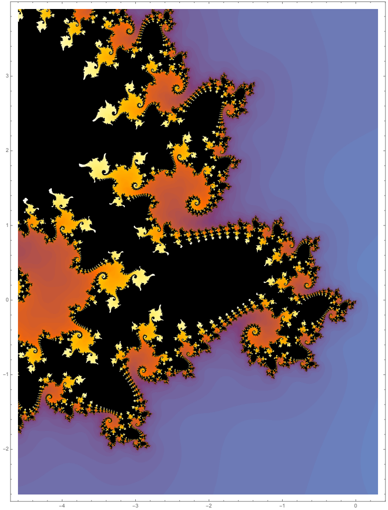
- be honest with yourself and others about what you do and don't know
- avoid the misconception that you know nothing
- avoid the misconception that you know everything
- be willing to put forth ideas that you aren't certain about
- be skeptical - ask others to explain their thoughts more thoroughly
- let the discussion be open-ended

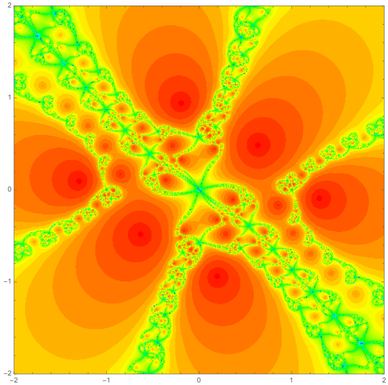
Whether reading math or talking math:

BE PATIENT!

If you come across questions like those I listed earlier that you can't answer, come to office hours or email your professor!

Another elective I took as an undergraduate was *Introduction to Dynamics*:



# My research

I really liked this class, and it inspired me to pursue research in a subfield of mathematics called *dynamical systems*.

I did an undergraduate research project in dynamics, focused on dynamics in grad school (after 2 years of generic graduate math courses) and have worked in the field ever since.

# My research

**Definition (for a layman)**

A **dynamical system** is a mathematical model for any quantity changes as time passes.

The quantity should evolve according to some function that tells you the value of the quantity one unit of time from now, in terms of the current value of the quantity.

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**Definition (more formal)**

A **dynamical system** is a set $X$ and a function $T : X \to X$.

**Connection:** $X$ is the set of possible values of the quantity being studied, and if $x \in X$ is the current value, $T(x)$ is the value one unit of time from now.

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A **dynamical system** is a set $X$ and a function $T : X \to X$.

Furthermore, the value of the quantity two units of time from now is $T(T(x))$, the value three units of time from now is $T(T(T(x)))$, etc.

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**Definition (more formal)**

A **dynamical system** is a set $X$ and a function $T : X \to X$.

The value $n$ units of time from now would be $T(T(T(\cdots T(T(x)))))$, which in dynamics we denote as $T^n(x)$.

**Example (from MATH 450)**

Compound interest: $X = \mathbb{R}$; $T : \mathbb{R} \to \mathbb{R}$ defined by

$$T(x) = x(1 + i).$$

This gives the value of an account one year from now, if the account earns compound interest at annual rate $i$.

In this interest example, $T^n(x) = x(1 + i)^n$ (this is the so-called "future value formula" for compound interest).

# My research

## Big picture question # 1: prediction

Can you project the long-term value of the quantity? In other words, can you compute

$$\lim_{n \to \infty} T^n(x)?$$

If not, can you say something about probabilities associated to this quantity?

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**Big picture question # 1: prediction**

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If not, can you say something about probabilities associated to this quantity?

In the interest example, you can:

$$\lim_{n\to\infty} T^n(x) = \lim_{n\to\infty} x(1+i)^n = \infty.$$

But what if the interest rate, instead of being a constant $i$, is random (and sometimes negative if the market is bad)? Now, things are harder.

# My research

**Big picture question # 1: prediction**

Can you project the long-term value of the quantity? In other words, can you compute

$$\lim_{n\to\infty} T^n(x)?$$

If not, can you say something about probabilities associated to this quantity?

In fact, there are relatively simple models (motivated by biology but occurring in lots of fields) where even if all the parameters are constant, no predictions about the long-term future can be made.

# My research

## Big picture question # 2: classification

What are the essential mathematical properties of dynamical systems that tell us whether or not systems coming from different areas (say economics and biology) are “the same” or “different”, mathematically?

**Prototypes:**

- The essential properties of a circle are its center and radius.
- The essential properties of a line are its slope and y-intercept.
- One essential property of a vector space (or subspace) is its dimension.
- An essential property of a square matrix is its collection of eigenvalues and eigenvectors.

# My research

## Big picture question # 2: classification

What are the essential mathematical properties of dynamical systems that tell us whether or not systems coming from different areas (say economics and biology) are "the same" or "different", mathematically?

I work on problems in this area. In particular, I look for algebraic objects called *groups* (you first learn about these in MATH 420) that are associated to dynamical systems.

I work on proving theorems that show two dynamical systems are "the same" (or "similar") if something is true about the group(s) associated to them, and "different" otherwise.

# Undergraduate research

I got started with research by doing an undergraduate research project under the supervisor of one of my professors.

- I met with my advisor 1-2 times a week for a year;
- I sat in on a graduate class she taught;
- I did a bunch of reading;
- I worked on stuff she told me to work on; and
- eventually, I wrote a paper on what I did.

This project took a year (a typical project is 12-18 months).

# Why do research?

- It's a resume booster
  - You gain experience working on complex problems that don't have well-defined solutions
  - You gain practice with a long-term project
  - You get experience communicating technical ideas to a broad audience
- Gives you something unique to talk about with potential employers
- Improves your self-confidence
- Maybe you get a summer stipend and/or a free trip
- You learn more math
- You get experience useful in grad school or in an academic career

# Potential research topics

**Do you like games?** One project would be to develop strategies for European-style board games or certain pen-and-paper games (this could either involve a lot of coding, or involve no coding, depending on your interests)

**Are you prepping for a career in statistics? Are you a sports fan?** Statistical analysis of any big data set (I'm particularly interested in sports statistics)

**Are you interested in pure math?** Here, you could classify certain 2-D and 3-D configurations of 0s and 1s; you could study pictures like those I showed earlier; you can do a project in hard-core linear algebra; etc.

# The end