Teaching the n^{th} Derivative Test with inquiry-based *Mathematica* activities

David M. McClendon
mcclend2@ferris.edu

Ferris State University
Big Rapids, MI, USA

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The problem

Upon joining the faculty at Ferris State University, I initially taught calculus courses in a very traditional fashion (i.e. lectures). Although this method was successful where I taught previously, I quickly observed at Ferris State that:

- Student performance was much worse than I had encountered as a TA and postdoc.
- Classroom atmosphere was very dry.
- Students weren't happy.

My proposed solution

With the support of the FSU Math Dept., the FSU College of Arts and Sciences and the Ferris Foundation, I converted my Calculus I and II classes from purely traditional lecture courses to a “mixed” model where students spend 2 days a week in lecture and 1 day a week in a computer classroom.

On the day students meet in the computer classroom, they work on laboratory-style assignments which use the computer algebra system *Mathematica*.

Many of these assignments are inquiry-based in nature (although I had never heard of “inquiry-based learning” when I developed them).

Why *Mathematica* (more generally, why computers)?

1. It allows for student-centered activities: students *do* rather than *listen*;
2. *Mathematica* quickly generates pictures and data; commands can be tweaked and rerun easily
3. Students can see pictures, data, and computations all at the same time on their screen
4. Students do not get bogged down with algebra and trig issues (they can use the computer to solve equations and graph functions, etc.) and can focus on the calculus they are supposed to learn
5. It's easy for me to troubleshoot when students have problems;
6. *Mathematica* proficiency is useful in more advanced courses and in industry.

An example lab activity

I will now outline an example of a lab activity my Calculus I students complete:

Primary goal of the assignment

Students will discover the Second Derivative and n^{th} Derivative Tests, and apply these tests classify critical points as local maxima, local minima, or saddles.

Secondary learning outcomes

- Students gain practice making, testing and refining conjectures;
- they develop their ability to write with precise mathematical notation;
- they practice critical thinking skills; and
- they gain additional proficiency with the *Mathematica* software.

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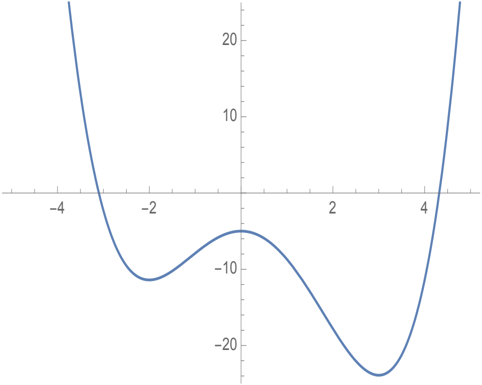
An initial motivating example

The assignment begins with an easy example:



An initial motivating example

Students are given the rule for this function and asked to:

1. plot the function;
2. estimate (from the graph) the locations of local maxima and/or local minima;
3. solve for the critical points (using *Mathematica* as an equation solver);
4. find $f''(x)$ for each critical point x; and
5. formulate a conjecture about how to use the second derivative to tell whether or not an x which is a solution of $f'(x) = 0$ is a local maximum or a local minimum of f.

An initial motivating example

I expect students to come up with the following conjecture:

Conjecture (Second Derivative Test)

For an x where $f'(x) = 0$:

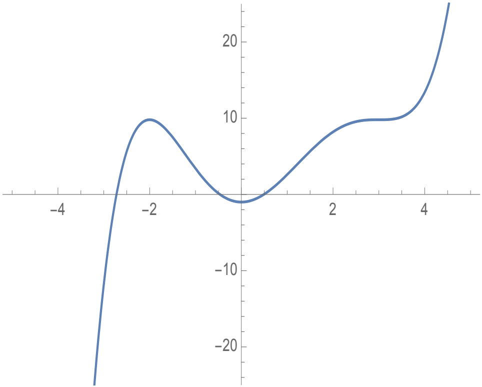
- if $f''(x) > 0$, then x is the location of a local min;
- if $f''(x) < 0$ then x is the location of a local max.

Students then asked to invent their own fourth- or fifth-degree polynomial and verify that their conjecture holds for that polynomial.

Then students are asked to explain why (on a theoretical level) they think this conjecture is true. (The expectation is that students make the connection between the sign of f'' and the concavity of f, concavity having been discussed earlier in the course)

A more complicated example

Then students consider an example of a function with a saddle:



A more complicated example

Repeating the basic procedure I described with the earlier function, I expect students to refine their earlier conjecture as follows:

Conjecture (incorrect)

For an x where $f'(x) = 0$:

- if $f''(x) > 0$, then x is the location of a local min;
- if $f''(x) < 0$ then x is the location of a local max;
- if $f''(x) = 0$ then x is neither a local min nor a local max.

Students then study an example of a function which shows the last statement of this conjecture to be incorrect.

The general situation

Last, students study several functions of the form

$$f(x) = \pm(x-2)^p \sin^q(\pi x)$$

where $p, q \in \mathbb{N}$. For each of these functions, students are asked to

1. plot the function;
2. identify (from the graph) whether $x = 2$ is a local max, local min, or saddle;
3. find the smallest n such that $f^{(n)}(2) \neq 0$;
4. find the value of $f^{(n)}(2)$ where n is as in the previous line; and
5. arrange all this information into a chart.

They then formulate a conjecture about how to use higher-order derivatives to tell whether or not an x which is a solution of $f'(x) = 0$ is a local maximum, a local minimum, or a saddle.

The general situation

A large majority of students are able to make the following claim, by looking at patterns in the data:

Conjecture (n^{th} Derivative Test)

For any x such that $f'(x) = 0$, let n be the smallest number where $f^{(n)}(x) \neq 0$.

- if n is even and $f^{(n)}(x) > 0$, then x is the location of a local min;
- if n is odd and $f^{(n)}(x) < 0$ then x is the location of a local max;
- if n is odd, then x is neither a local min nor a local max.

The general situation

Next, there are some problems which ask students to apply the n^{th} Derivative Test to classify various critical points as local maxima, local minima, or saddles.

Last, I students to explain why they think this conjecture (the n^{th} Derivative Test) is true (on a theoretical level).

I expect (and usually get) nonsense here, but occasionally a student says something that has the flavor of Taylor's Theorem in the background.

Impacts

Student performance is improved

Some data from my Calculus I courses at Ferris State:

	traditional approach	lab-based approach
Median course average	70.8%	78.5%
Percentage of students finishing course with 90% average or better:	8%	24.3%
Percentage of students finishing course with 60% average or better:	68%	90%

Impacts

Student performance is improved

- In both Calculus I and II, "strong", "medium" and "weak" students have all seen gains in performance
- Performance gains are consistent across each module of the courses
  (with the notable exception of integration techniques in Calculus II, where student performance has been flat)

Impacts

Students are happier

Average student course ratings (5-point scale):

traditional approach:	3.60
lab-based approach:	4.28

During the first year I used the lab-based approach, I surveyed students as to their thoughts on the labs:

- 50% of students surveyed said they would prefer a lab-based course to a traditional calculus course
- 25% preferred a traditional course
- 25% were neutral

77% were able to identify a specific course topic in which they gained greater insight via lab activities.

Impacts

Classroom atmosphere is improved

I spend most lab days walking around interacting with students, and most students work on the labs in groups.

This has helped to create a more collaborative atmosphere in the course which has overflowed into lecture days - students engage more when I ask them questions in lecture, and are asking more questions in lecture.

Impacts

More material can be covered

I have found that I cover more material in a Calculus I or II semester than I could before I started using lab assignments. This is a combination of

- the additional material arising naturally in a lab assignment (i.e. n^{th} Derivative Test), or
- having extra time because the labs help me cover material more quickly.