A universal model for Borel semiflows

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Prototype semiflow: Brownian motion

Let X be the space of continuous functions from $\mathbb{R}^+$ to $\mathbb{R}$ with the usual topology. Define for each $t \geq 0$ the *shift map* $\Sigma_t : X \to X$ by

$$\Sigma_t(f)(s) = f(t+s) - f(t).$$

(X, Σ_t) is a Borel semiflow which preserves Weiner measure (and other measures).

IDIs

Given a Borel semiflow (X, T_t), we say two distinct points x and x' are *instaneously and discontinuously identified (IDI)* by the semiflow if $T_t(x) = T_t(x') \, \forall t > 0$.

Define $IDI(T_t) = \{x : \exists\, y \neq x$ such that x and y are $IDI\}$

Define $IDI(x) = \{t \geq 0 : T_t(x) \in \text{IDI}(T_t)\}$.

Theorem (M) *For any x, $IDI(x)$ is countable.*

Brownian motion has no IDIs. To build a model which accounts for possible IDIs in a semiflow, we use the action of Brownian motion on a different space of functions.

A universal model

Theorem (M) *There exists a Polish space* Y *of left-continuous, increasing functions from* $\mathbb{R}^+$ *to* $\mathbb{R}^+$ *passing through the origin such that any Borel semiflow* $(X, \mathcal{F}, \mu, T_t)$ *is isomorphic to* $(Y, \mathcal{B}(Y), \nu, \Sigma_t)$ *for some* Σ_t- *invariant Borel probability measure* ν.